New Developments of Frequency Domain Acoustic Methods in LS-DYNA®

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Abstract

This paper presents the new developments of finite element methods and boundary element methods for solving vibro-acoustic problems in LS-DYNA. The formulation for a frequency domain finite element method based on Helmholtz equation is described and the solution for an example of a simplified compartment model is presented. For boundary element method, the theory basis is reviewed. A benchmark example of a plate is solved by boundary element method, Kirchhoff method and Rayleigh method and the results are compared. A dual boundary element method based on Burton-Miller formulation is developed for solving exterior acoustic problems which were bothered by the irregular frequency difficulty. Application of the boundary element method for performing panel contribution analysis is discussed. These acoustic finite element and boundary element methods have important application in automotive, naval and civil industries, and many other industries where noise control is a concern.

Introduction

This paper presents the recent developments of the finite element method (FEM) and boundary element method (BEM) in LS-DYNA for solving vibro-acoustic problems in frequency domain.

Vibro-acoustics has many important practical applications. A set of FEM and BEM have been implemented in LS-DYNA for solving vibro-acoustic problems in frequency domain [1]. For vibro-acoustic problems involving light acoustic fluid materials, such as air, a weak acoustic-structure interaction can be assumed. This means that the structure is not affected by the propagating acoustic wave. As the first step of the simulation, the vibration response on the surface of the acoustic volume is computed by the following two methods. In the first method, the transient structural response can be computed by the explicit time domain finite element methods. By using the FFT technique, the velocity or acceleration boundary condition is transformed to frequency domain. In the second method, one can use the recently implemented steady state dynamics (SSD) feature in LS-DYNA to compute the frequency response directly (see *FREQUENCY_DOMAIN_SSD [2]). The frequency domain velocity or acceleration response obtained by either way is taken as the boundary condition for the FEM or BEM acoustic solver. As the second step, the FEM or the BEM acoustic solver is used to compute the radiated acoustic pressure (Pa) at any point in the acoustic volume. The acoustic pressure (Pa) can be transformed to sound pressure level (dB) if a reference pressure is provided.

Frequency domain FEM for acoustics

1. Theory basis

The governing equation for the acoustic problem is the Helmholtz equation [3],

$$\nabla^2 p + k^2 p = 0 \tag{1}$$

where p is the acoustic pressure; $k = \omega / c$ is called the wave number; $\omega = 2\pi f$ is the circular frequency of the acoustic wave; and c is the wave speed.

For vibro-acoustic problems, the boundary condition is given as

$$\partial p / \partial n = -i\rho\omega v_n \tag{2}$$

where n is the normal vector pointing outside from the acoustic volume; $i = \sqrt{-1}$ is the imaginary unit; ρ is the acoustic fluid density and v_n is the normal velocity.

Using the weighted residue technique and taking the shape function N_i as the weighting function, the governing equation can be written as

$$\int_V \nabla^2 p N_i dV + \int_V k^2 p N_i dV = 0 \tag{3}$$

Using the Green's theorem, equation (3) can be written as

$$-\int_V \nabla p \nabla N_i dV + k^2 \int_V p N_i dV = -\int_\Gamma \frac{\partial p}{\partial n} N_i d\Gamma \tag{4}$$

With the substitution of the boundary condition (2) into equation (4), and taking the nodal pressure as the unknown variables, a linear equation system can be established and solved in frequency domain. Since there is only one variable on each node, this approach is very fast.

This frequency domain acoustic FEM can be used to solve interior acoustic problems. It can be activated by the keyword *FREQUENCY_DOMAIN_ACOUSTIC_FEM [2].

2. Example

A simplified compartment shown in Figure 1 is considered.

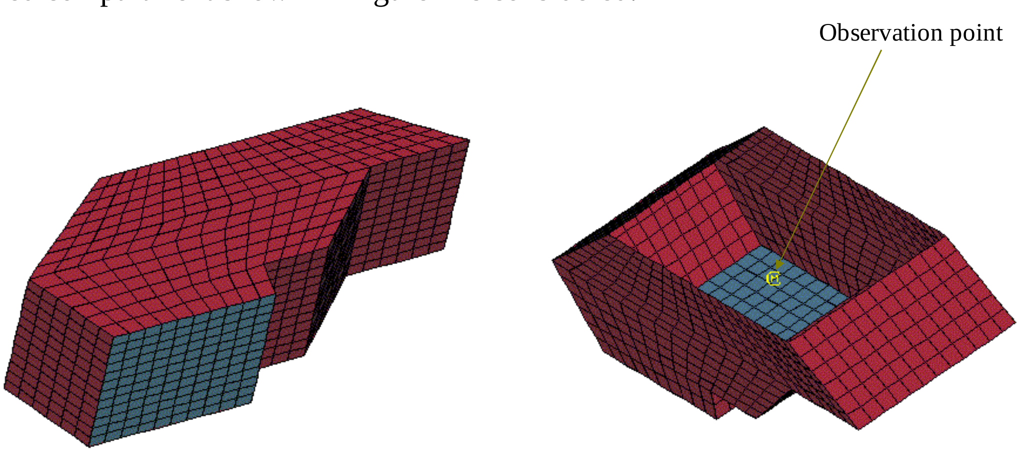

Figure 1 – A simplified compartment model of an automobile

The size of the compartment is $1.4 \times 0.5 \times 0.6$ m^3 and it is filled with air (ρ=1.23 kg/m^3, c=340 m/s). The surface of the compartment is assumed to be rigid and the bottom plate is excited vertically by a uniform velocity 7 mm/s in the frequency range of 10-500 Hz. Nastran, BEM of LS-DYNA and FEM of LS-DYNA are used to compute the Sound Pressure Level at the observation point in the compartment. The results are shown in Figure 2.

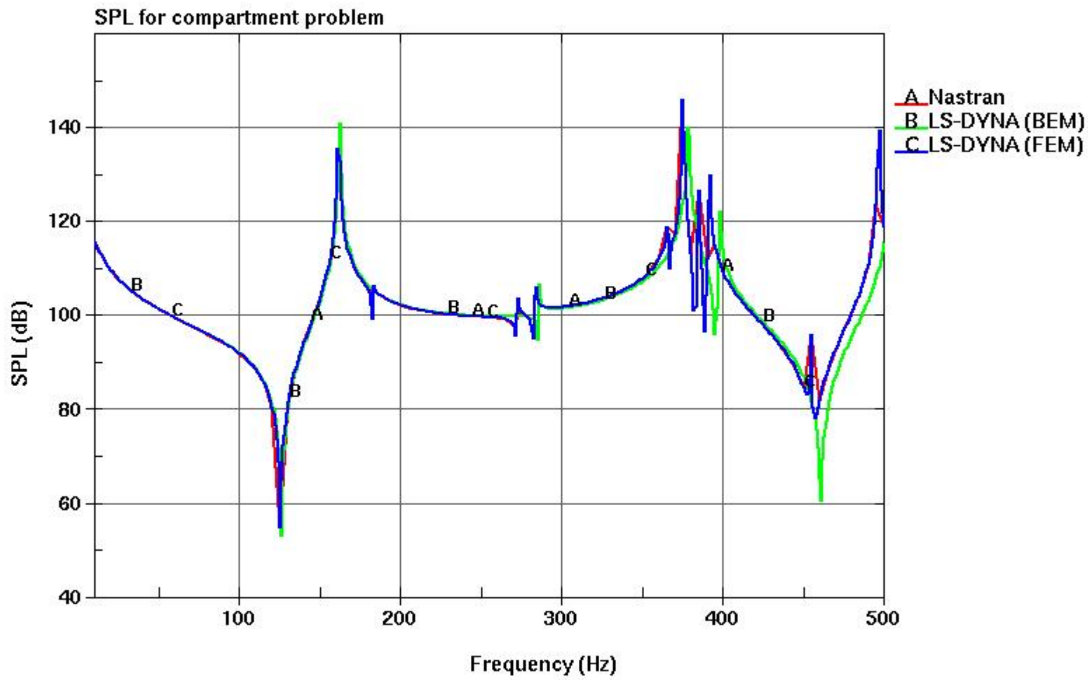

Figure 2 – Sound Pressure Level at the field point by 3 methods

The numerical results given by the three methods match reasonably well and each of them can predict the peak response frequencies well.

Frequency domain BEM for acoustics

1. Theory basis

Boundary element method and approximate methods are available in LS-DYNA to solve vibro-acoustic problems and they have been introduced in [1]. Comparing to FEM, the chief advantage of BEM is that only the surface of acoustic domain needs to be meshed. Thus the dimension of the problem is reduced by one. In addition, the radiation in an infinite medium given by Sommerfeld condition is automatically satisfied. Thus the external domain doesn't need to be bounded.

The governing equation for BEM is obtained by transforming equation (1) to an integral equation by using the Green's theorem [3],

$$p(P) = \int_{\Gamma} \left(G \frac{\partial p}{\partial n} - p \frac{\partial G}{\partial n} \right) d\Gamma \tag{5}$$

where

$$G = \frac{e^{-ikr}}{4\pi r} \tag{6}$$

is the singular fundamental solution, and r is the distance between the field point P and surface integration point.

The variational BEM and collocation BEM solve the Helmholtz equation as a linear system. A fast procedure based on domain decomposition and low rank approximation of the influence coefficient matrices has been implemented to both methods to accelerate the solution. MPP version of the methods is also available for solving large scale problems. Besides the BEM, two approximate methods, Rayleigh and Kirchhoff methods are also available in LS-DYNA [1]. The approximate methods do not require a system of equations to be assembled and solved. Consequently, they are faster than BEM. Rayleigh method assumes that the radiating structure is a plane surface clamped into an infinite rigid plane. In Kirchhoff method, the BEM is coupled with the transient FEM used for acoustics in LS-DYNA. The radiating boundary condition at infinity is satisfied by prescribing a non-reflecting boundary condition. In this case, at least one fluid layer needs to be merged to the vibrating structure. Comparing with the frequency domain acoustic FEM, the BEM is more versatile and can solve both the interior and exterior problems. Rayleigh and Kirchhoff methods can only solve exterior acoustic problems. However Kirchhoff method can consider strong interaction between acoustic fluid and structures. This is because in Kirchhoff method, during the transient FEM structural analysis phase, one or more finite element fluid layers (*MAT_ACOUSTIC) are attached with structures. In terms of precision, the solution by BEM can reach a high accuracy since it solves the singular integral equation and get the primary unknown variables on each node without any assumption. Rayleigh and Kirchhoff methods are each based on some assumptions thus they are less accurate. But Rayleigh and Kirchhoff methods may be employed as the first attack when solving some large scale problems because they are faster than BEM. One can see for the following example of a rectangular plate, the Rayleigh and Kirchhoff methods can still provide satisfactory results. This is because the geometry of the problem is simple and satisfies the assumption of the two approximate methods.

The BEM and the approximate Rayleigh and Kirchhoff methods can be activated by the keyword *FREQUENCY_DOMAIN_ACOUSTIC_BEM [2].

2. A benchmark example of a plate

This example considers a rectangular elastic plate subjected to an impulsive 1 Newton nodal force excitation (Figure 3). The material properties of the plate are given as follows. The density $\rho = 7800\ \text{kg/m}^3$, Young's modulus $E = 210\,\text{GPa}$, Poisson's ratio $\gamma = 0.3$. The sound pressure level (dB) at the observation point is computed using three methods: BEM, Kirchhoff method and Rayleigh method. The results can be found in Figure 4. This example can be used as cross validation of the three methods.

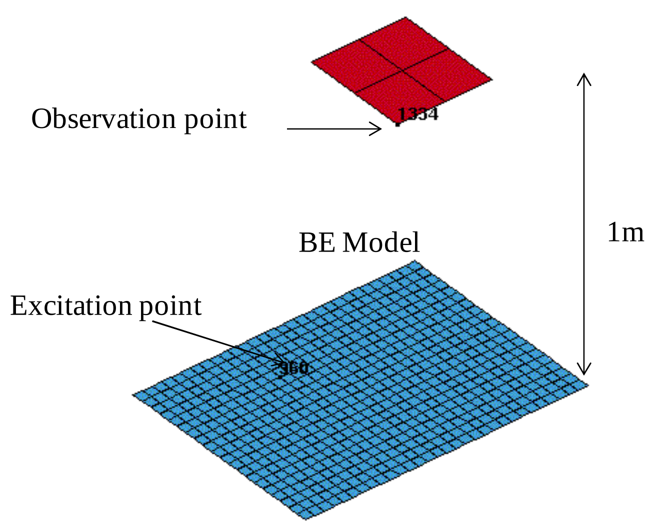

Figure 3 – A rectangular plate subjected to nodal force excitation

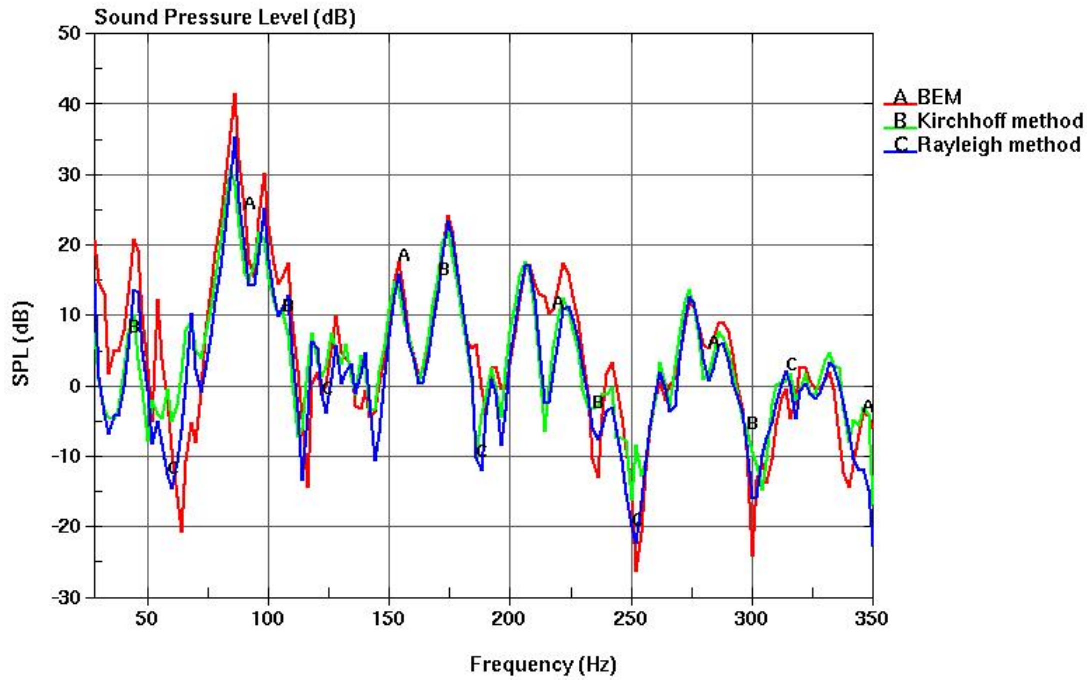

Figure 4 – SPL at observation point for the plate problem

3. A dual BEM for irregular frequency problems

Conventional BEM fails to yield unique solution for exterior acoustic problems at the eigen-frequencies. A dual BEM based on Burton-Miller formulation has been implemented to solve the irregular frequency problem for exterior acoustic problems [4]. A benchmark example of a pulsating sphere of a unit radius (r = 1m) surrounded by air and excited by unit velocity at the frequency range 1-300 Hz is considered. Analytical solution for this problem is available [3]. The acoustic pressure at radius 1.4m is shown in Figures 5. One can notice the non-unique results by conventional BEM at around frequency 175 Hz. With the dual BEM based on the Burton-Miller formulation, this non-uniqueness problem is solved.

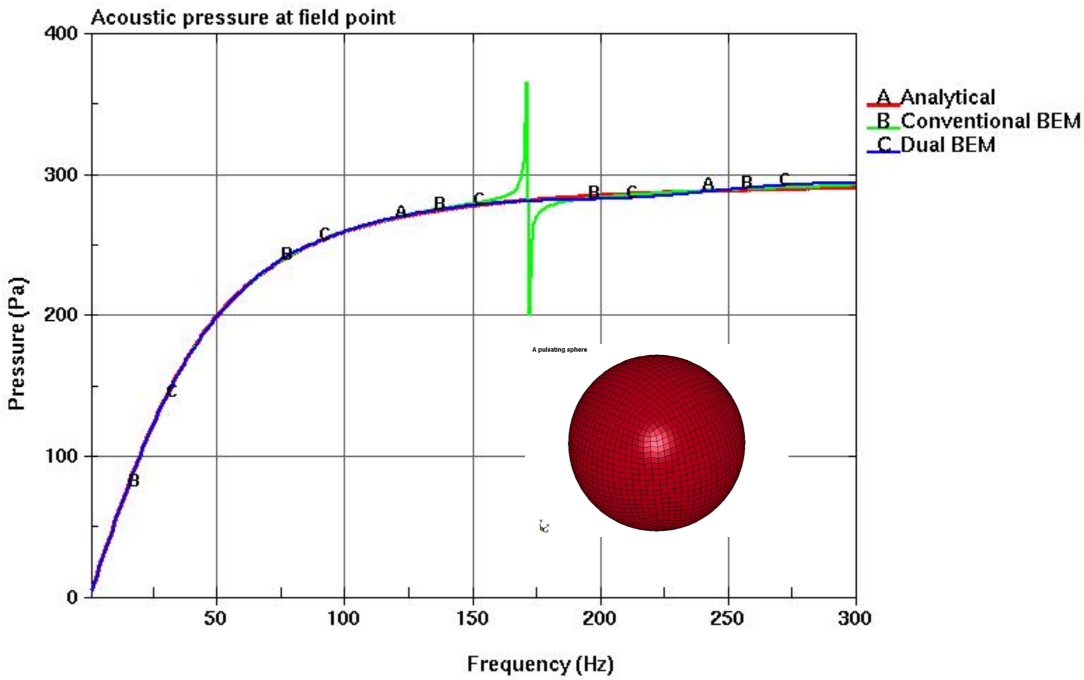

Figure 5 – Acoustic pressure Level at the field point by 3 methods

4. Panel contribution analysis

Panel contribution analysis is conducted to identify the panels which have a high contribution on the acoustic response. Countermeasures can then be applied to suppress the vibration of those panels, and then reduce the noise level. Panel contribution analysis can be performed with the BEM acoustic solver in LS-DYNA and it gives the contribution percentage of panels (given as part, set of parts or set of segments) on the acoustic results at observation points.

Suppose that the whole surface of the acoustic volume is composed with N panels. The integral equation (5) can be rewritten as

$$p(P) = \sum_{j=1}^{N} \int_{\Gamma_j} \left(G \frac{\partial p}{\partial n} - p \frac{\partial G}{\partial n} \right) d\Gamma_j = \sum_{j=1}^{N} p_j(P) \qquad (7)$$

Where, Γ_j represents the area of the j-th panel. The panel contribution percentage c_j for the j-th panel is the ratio of the pressure vector contributed by the j-th panel on the total pressure vector p, and it is expressed as

$$c_j = 100 \times \frac{p_j \cdot p}{p \cdot p} \qquad (8)$$

Where,"." represents the inner production of two vectors. So c_j is the ratio of the length of the projected contribution pressure vector (in the direction of the total pressure vector) to the length of the total pressure vector.

The panel contribution analysis can be activated by the keyword *FREQUENCY_DOMAIN_ ACOUSTIC_BEM_PANEL_CONTRIBUTION [3].

A simplified tunnel model is employed to illustrate the panel contribution analysis with LS-DYNA. The model is shown in Figure 6. The size of the tunnel is $1.5 \times 1.4 \times 3.0$ m^3. The tunnel is composed with 4 panels which are assumed to be rigid. A uniform normal velocity 10 mm/s in the frequency range of 150-300 Hz is applied to excite the whole model. The observation point is selected to be located at the center of the tunnel. The sound pressure level (dB) at the observation point is computed (see Figure 7). Figure 8 shows the panel contribution percentage of the 4 panels respectively. One can notice that the top panel (panel 3) makes the largest contribution for the noise for most frequencies. The contribution from panel 1 and panel 2 is almost identical, due to the symmetry of the structure.

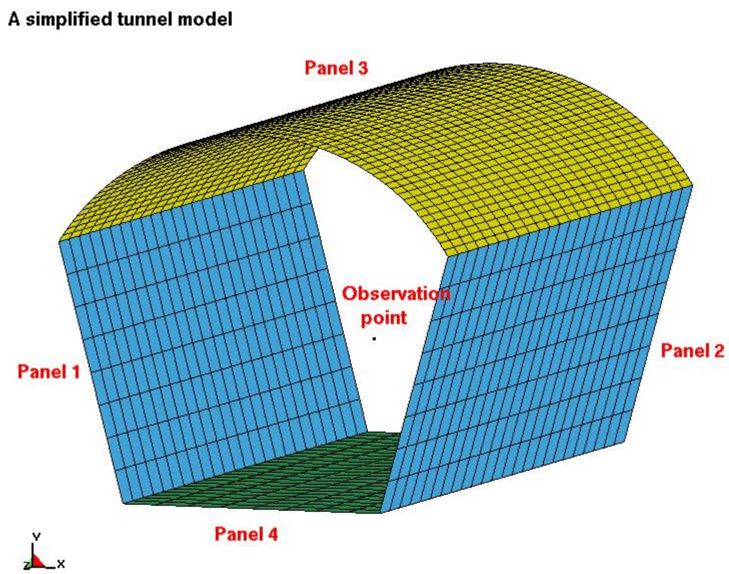

Figure 6 – A tunnel model composed with 4 panels

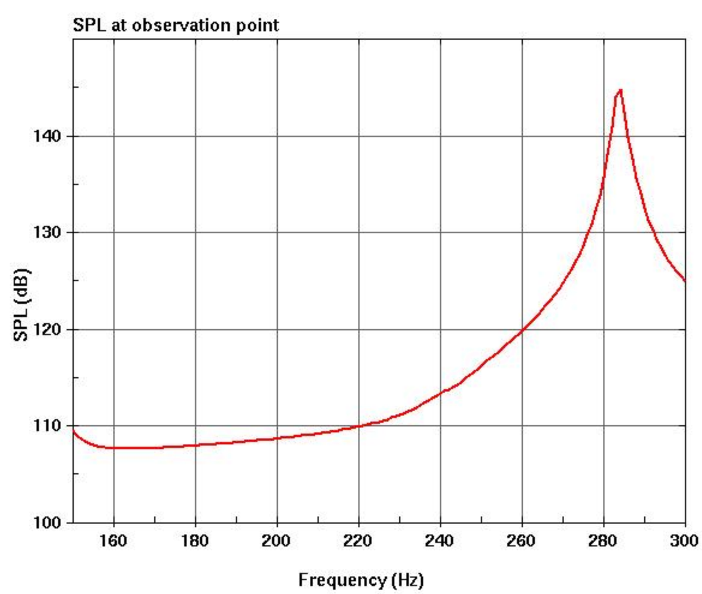

Figure 7 – Sound Pressure Level at observation point

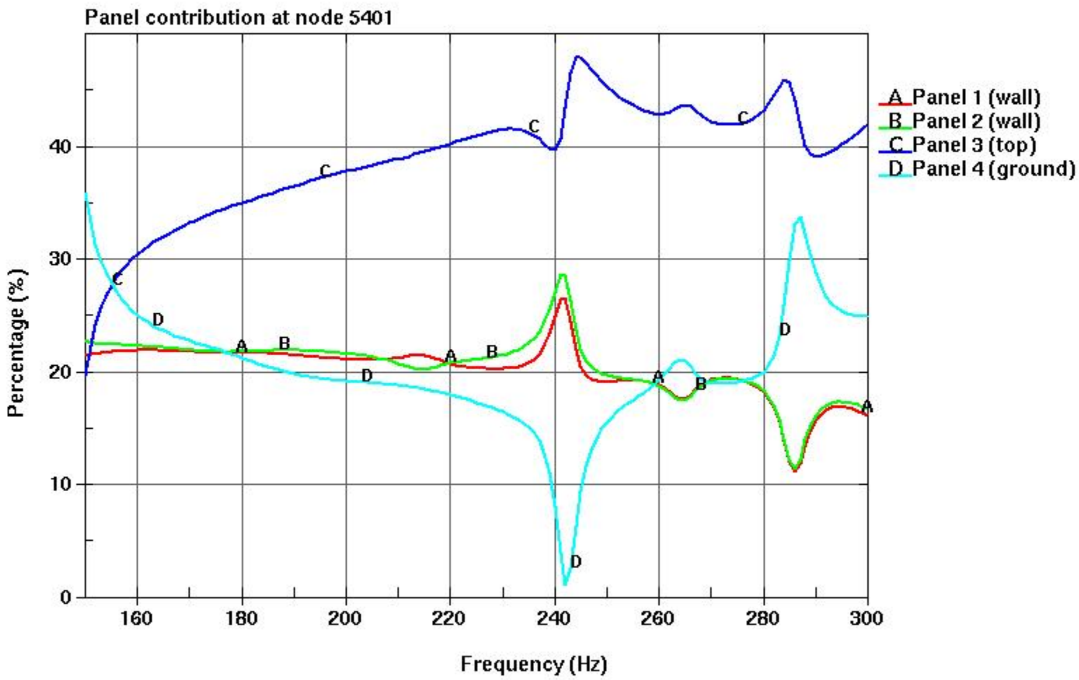

Figure 8 – Panel contribution percentage

Conclusions

A set of FEM and BEM have been implemented in LS-DYNA to solve vibro-acoustic problems in frequency domain. The general rules for using these methods are discussed. Theory bases are reviewed. Several examples are given to demonstrate the effectiveness and accuracy of the methods.

These frequency domain acoustic methods may find application in many industries where noise control is a concern, such as automobile, naval and civil industries.

References

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