

# An Improved 3D Adaptive EFG Method for Forging and Extrusion Analysis with Thermal Coupling in LS-DYNA<sup>®</sup>

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## Abstract

*The 3D adaptive EFG method using conventional moving least-square approximation or fast transformation method [1] has been successfully applied to metal forging and extrusion analysis thanks to its high accuracy in dealing with large material deformation [2] in LS-DYNA. Recently, a meshfree convex approximation [3-5] was developed to be an alternative in the large deformation analysis. However, its application to the adaptive method has not been investigated.*

*In this paper, an improved version of 3D adaptive EFG method with emphasizing on the modified maximum entropy approximation, whose approximation is non-negative and owns Korncker-Delta propriety at the boundary, is presented. The thermal effect in forging and extrusion problem is considered, and a scheme to interpolate the thermal state variables during the adaptive procedure is proposed.*

## Introduction

Adaptivity is a key ingredient in many non-linear solid and structural simulations involving large deformation and moving boundaries analysis. The object of introducing adaptivity is to control the global accuracy due to large topology change as well as to enhance the local accuracy due to high gradient or strong discontinuity which is difficult to achieve by the conventional method with fixed number of degrees of freedom.

Meshfree methods are the topic of recent research in many areas of computational science and engineering. One of the early incentives to develop meshfree method was its ability to handle large deformation problems. Well-known meshfree interpolation methods include the moving least-squares (MLS) approximation [6] and the reproducing kernel (RK) approximation [7]. In general, meshfree approximations are non-convex functions and do not satisfy the Kronecker-delta property at the boundary. Another well-known meshfree interpolation method is the radial basis functions (RBF), which are invariant under all Euclidean transformations. However, they are limited to small-scale problems because of their nonlocality. Recently, the maximum entropy (ME) approximation was developed based on the Shannon's entropy concept and ME principle, which is a convex approximation and satisfies the Kronecker-delta property at the boundary.

More recently, this method was further extended to a generalized approximation and called it a “generalized meshfree approximation” (GMF) [3-5]. The method can be used to construct various convex and non-convex meshfree approximations with the ease of essential boundary condition treatments. Under GMF framework, the ME approximation can be considered as a subset of GMF approximation while still maintains the weak Kronecker-delta property at the boundary.

Adaptive meshfree method [9-10] has shown its advantages in non-linear solid and structural simulations thanks to its high accuracy and natural conforming property in the adaptive computation. In this study, we apply this ME approximation constructed from GMF framework in the large deformation analysis using meshfree adaptivity.

### Construction of Maximum Entropy (ME) Approximation Using GMF Framework

This section describes the fundamental derivation and the basic equations in the construction of Maximum Entropy approximation in the GMF framework. First of all, a partition function is defined. The partition function consists of two parts. The first part is the exponential basis function which is adopted to enforce the convexity of the approximation. The second part is a kernel function that is used to introduce the locality as well as the smoothness of the approximation.  $\lambda$  is a scalar in one-dimension and a vector in multi-dimensional which needs to be solved implicitly. They are formulated as follows:

$$\text{Define the partition function } Z: Z(\mathbf{x}, \lambda) = \sum_{i=1}^N \phi_i(\mathbf{x}) e^{\lambda(\mathbf{x}-\mathbf{x}_i)/r_i}$$

where  $\phi_i(\mathbf{x})$  is the kernel function at node  $i$   
 $r_i$  is the supportsize of kernel at node  $i$

The unique solution of MAXENT is proven to be

$$p_i(\mathbf{x}, \lambda) = \frac{\phi_i(\mathbf{x}) e^{f_i(\mathbf{x}, \lambda)}}{Z(\mathbf{x}, \lambda)} \quad \forall p_i \geq 0, i = 1, \dots, N$$

$$\text{satisfying } \sum_{i=1}^N p_i = 1$$

$$\sum_{i=1}^N p_i(\mathbf{x}_i - \mathbf{x}) = 0$$

$$\text{where } f_i(\mathbf{x}, \lambda) = \lambda \cdot |(\mathbf{x} - \mathbf{x}_i)/r_i|$$

The constraint equation  $\sum_{i=1}^N p_i = 1$  is the well-known partition of unity. Another constraint

equation  $\sum_{i=1}^N p_i(\mathbf{x}_i - \mathbf{x}) = 0$  is the first-order completeness condition which is required in the non-

linear solid and structure analysis.  $p_i(\mathbf{x}, \lambda)$  is probability in the information theory and can be considered as the shape function in the interpolation theory as shown in Figure 1. As shown in

Figure 1, the constructed Maximum Entropy shape function is convex and possesses weak Kronecker-delta property at the boundary.

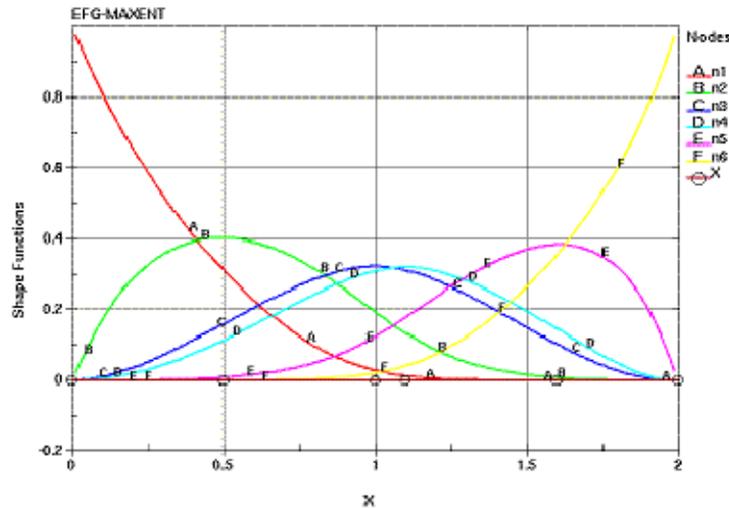


Figure 1: One-dimensional ME Shape functions

### Data Mapping in Adaptivity

Two sets of data mapping are considered in the meshfree adaptivity, i.e. node to node and stress point to stress point as shown in Figure 2. Each set of particles carries different support sizes and physical variables including the thermal effects. The location of stress point is calculated and updated from the position of its neighbor nodes. Therefore stress point moves as the material is deformed and is allowed to move outside the integration cell. The support size of stress point is defined internally in the adaptive procedure. A special boundary detection algorithm is developed to assure that the stress point near the boundary is covered by enough neighbor points as required in the mapping scheme. The accuracy of the data mapping scheme is controlled by the order of basis function in meshfree approximation defined by the user. Since the data mapping in explicit time integration can not be supplemented by the equilibrium condition as in the implicit analysis, the quality of data mapping will determine the quality of adaptive procedure.

The neighbor searching scheme in data mapping determines the efficiency of meshfree adaptive procedure especially in the context of explicit time integration. Various algorithms for sorting in  $O(n \log n)$  time and a lower bound showing that  $O(n \log n)$  is optimal. In this study, the bucket sorting algorithm is adopted.

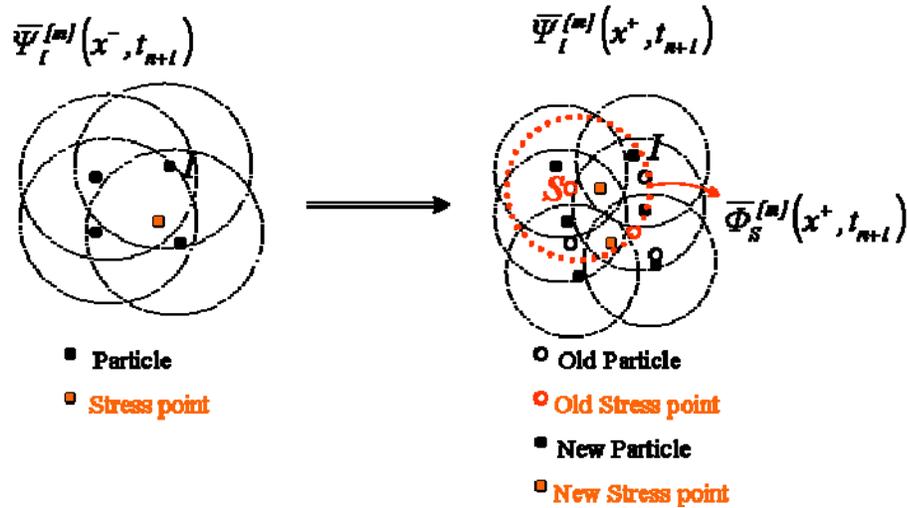


Figure 2: Data mapping in meshfree method

### New Keywords

This section introduces the new keyword that required in the adaptive meshfree method using maximum entropy (ME) approximation. In order to speed up the meshfree computation, a new meshfree formulation ELFORM = 42 was implemented in conjunction with the existing MLS approximation as well as the ME approximation.

\*SECTION\_SOLID\_EFG

Card 1

Variable	SECID	ELFORM	
Type	I	I	

ELFORM. 42: Adaptive EFG using 4-noded integration cell.

Card 2

Variable	DX	DY	DZ	ISPLINE	IDILA	IEBT	IDIM	TOLDEF
Type	F	F	F	I	I	I	I	F
Default	1.01	1.01	1.01	0	0	-1	2	0.01

IEBT EQ. 7: Maximum Entropy approximation

### Numerical Examples

Two forging and extrusion examples considering the thermal effect are analyzed by the improved 3D adaptive EFG method using LS-DYNA. The adaptive procedure is triggered by a

constant time interval parameter defined by user, and the new nodal distribution is controlled by the specified maximum and minimum nodal distance.

### 1. Upsetting Simulation

A cylindrical billet is compressed between two perfectly rough, insulated plates. The initial temperature is 20 degree Centre high, and the initial height of the billet is 36mm with a radius of 9 mm. It is assumed that there is no heat transfer to the environment. The total imposed displacement is 15.84mm in the vertical direction. Due to symmetry, only a quarter of the billet is modeled as shown in Figure 3 (a). During the simulation, the adaptive procedure is used to control the global accuracy due to topology change. The final deformation is plotted in Figure 3 (b). The final distributions of effective plastic strain and the temperature are given in Figure 4 (a), and (b), respectively. The final temperature at the middle of the outer surface is recorded as: 98.9, 98.9, 98.8, 98.8, 98.7, 98.7, which is compared with the experimental result as 100.5 degree Centre high. The reaction force is plotted in Figure (5).

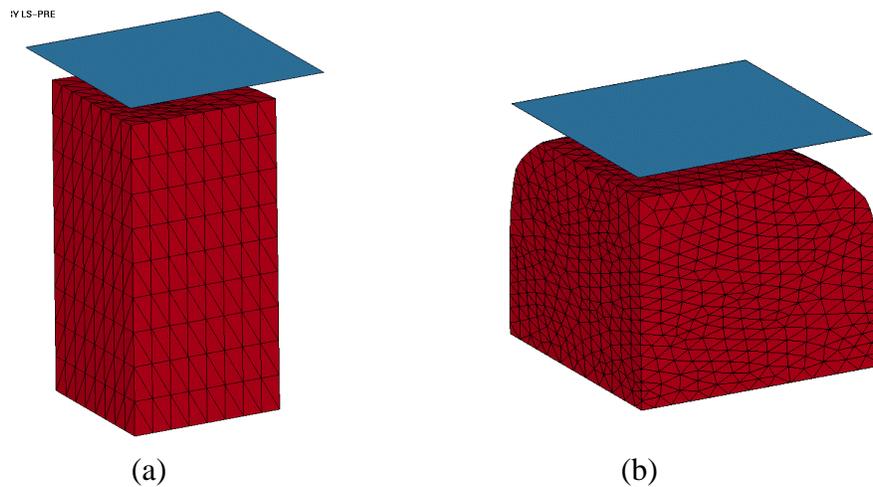


Figure 3: Initial and final configuration

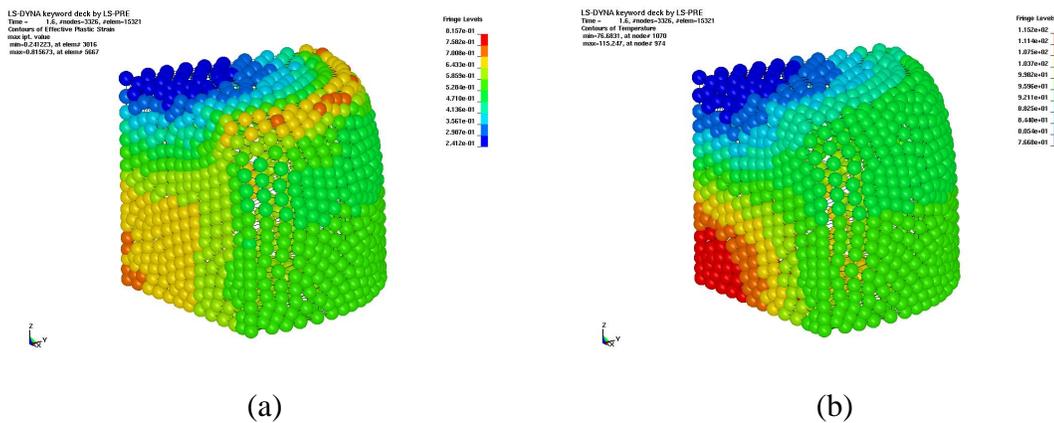


Figure 4: Final distribution of effective plastic strain and temperature

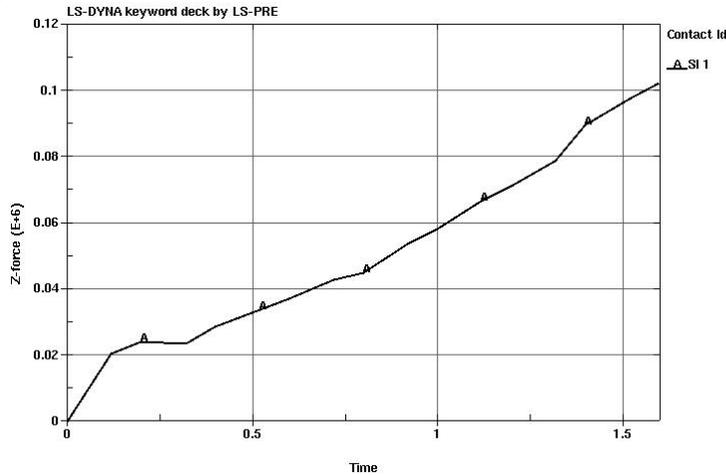


Figure 5: Reaction force

## 2. Extrusion Simulation

An extrusion problem with thermal coupling is analyzed in this example. Material \*MAT\_ELASTIC\_VISCOPLASTIC\_THERMAL is used to model the bulk behavior and a curve is given to describe the relationship of temperature and Young’s modulus. The initial temperature of the workpiece is 1200 degree Centre high. There is a heat transfer between the workpiece and the dies. The problem setting is shown in Figure 6 (a), and the deformed plot at the end of the extrusion is given in Figure 7 (b). Adaptive procedure is employed in the simulation in order to capture the high gradient near the sharp corner. The final distributions of effective plastic strain and temperature are given in Figure 8 (a) and (b), respectively. The deflection-force curve is plotted in Figure 9. It is noted that the oscillation of the force responses observed in the post-extrusion is due to the non-mapping of the contact variables. A local adaptive procedure as well as the implicit analysis maybe considered to improve the oscillation.

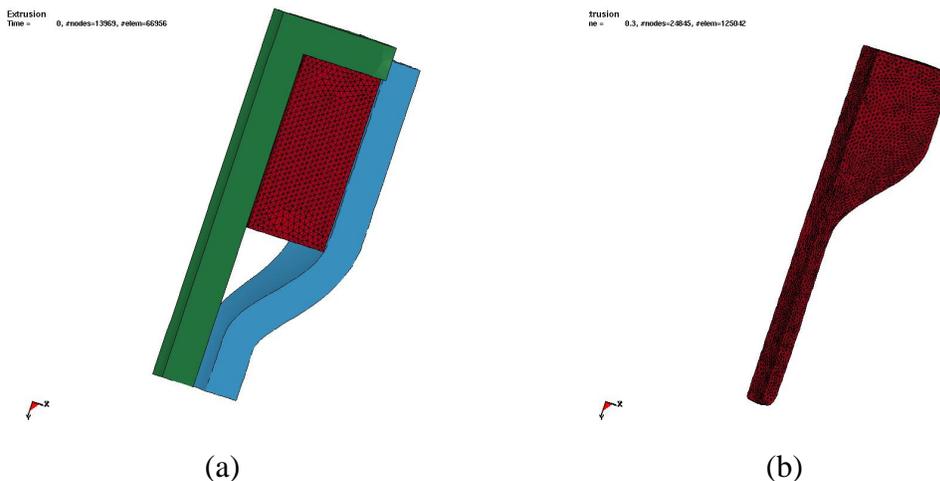


Figure 6: Initial and final configuration

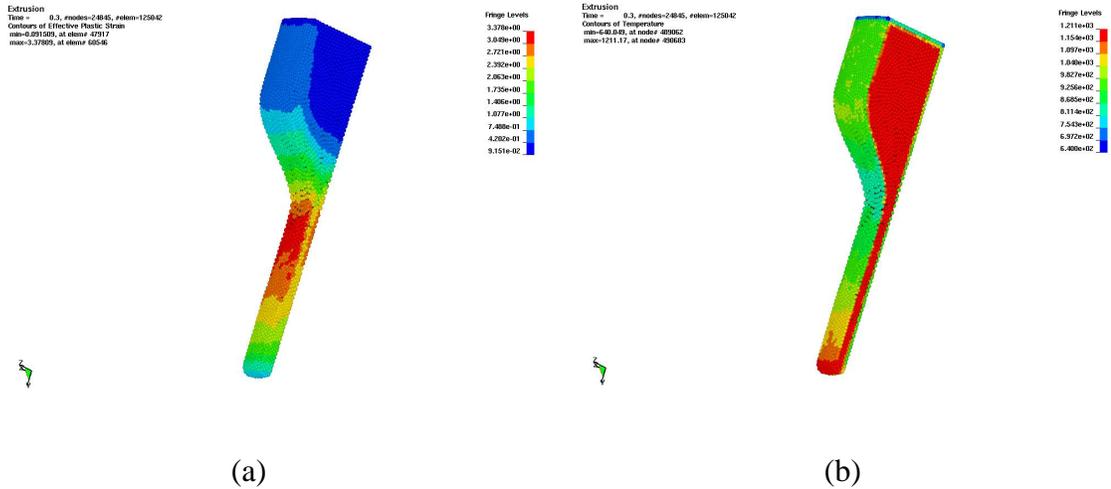


Figure 7: Final distribution of effective plastic strain and temperature

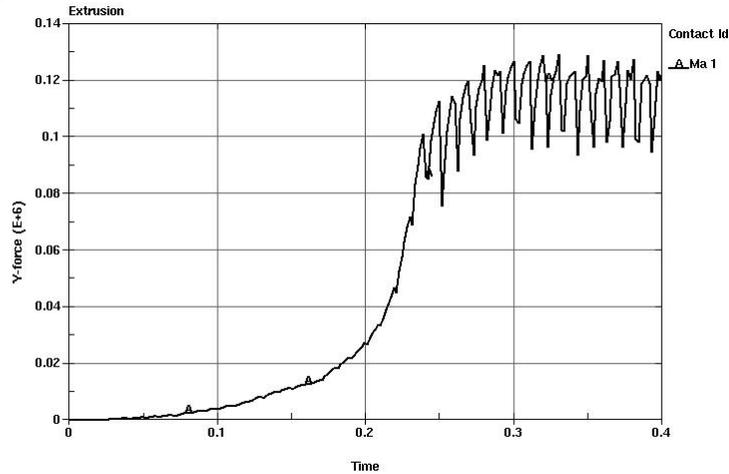


Figure 8: Reaction force

### Conclusion

In this study, a convex approximation is applied to the meshfree formulation in the analysis of metal forming problems using adaptive procedure. The convex approximation assures the positive mass calculation and provides smooth data mapping in the adaptivity using explicit dynamic analysis and therefore improves the accuracy. Two test examples are studied including the thermal effects. In the near future, attention will be paid in the implicit analysis of metal forming problems including the spring-back effect, the residual stress analysis and tool failure study.

## **Acknowledgements**

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## **References**

- [1] Wu, C. T. and Lu, H. S. "Practical fast meshfree analysis", U.S. Patent, 2009.
- [2] Lu, H. S., Wu, C. T., and Xu, J. X. "Forging and extrusion analysis with LS-DYNA using 3D adaptive EFG method", 10th International LS-DYNA Users Conference, 2008.
- [3] Wu, C. T. and Koish, M. "A meshfree procedure for the microscopic analysis of particle-reinforced rubber compounds" *International Journal of Interaction and Multiscale Mechanics*, Vol. 2, No. 2, pp. 147-169, 2009.
- [4] Wu, C. T., Park, C. K. and Chen, J. S. "A generalized approximation for the meshfree analysis of solids", *International Journal for Numerical Methods in Engineering*, Submitted, 2010.
- [5] Park CK, The development of a generalized meshfree approximation for solid and fracture analysis, Dissertation, 2009, The George Washington University, DC, USA.
- [6] Belytschko T, Lu YY, Gu L, "Element-free Galerkin methods", *International Journal for Numerical Methods in Engineering* 1994, 37(2) pp. 229-256.
- [7] Liu WK, Jun S, Zhang YF, "Reproducing kernel particle methods", *International Journal for Numerical Methods in Fluids* 1995, 20 pp. 1081-1106.
- [8] Lu, H. and Chen, J. S. "Adaptive meshfree particle method", *Lecture Notes in Computational Science and Engineering*, Vol. 26, pp. 251-267, 2002.
- [9] Lu, H. S., Wu, C. T., and Xu, J. X. "Forging and extrusion analysis with LS-DYNA using 3D adaptive EFG method", 10th International LS-DYNA Users Conference, 17-(21-24), 2008.