

Validation of Material Models for the Numerical Simulation of Aluminum Foams

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Abstract

In the last years a lot of studies dealt with the material modeling of metallic foams, especially for the Aluminum ones. All these activities were performed especially for automotive field applications because the high energy-absorbing property of such foams fits very well the requirement to carry impacting loads efficiently.

In spite of this, the industrial applications are not yet so widespread both for manufacturing costs and for a lack of knowledge regarding a whole mechanical characterization. The anisotropic properties of the foams induced mainly by the manufacturing processes, like the continuous casting procedure [1,2] is the reason due until now it has not been possible to assess in a well-known way the foams mechanical performances. In function of the wide spectrum of loading configurations, foaming direction, open and closed cells typology, cells morphology, density, thickness, etc., the output data regarding the mechanical tests are largely scattered (e.g. the stress/strain curves change according to the direction along the experimental test is performed), so a numerical model designed to reproduce accurately the foam behavior needs to take into account the parameters affecting the foam response.

Different FE approaches are being developed at this aim: apart from the micro-structural approach and the macro-structural one [3, 4], from an engineering point of view, the material models available in the explicit, non-linear finite element code LS-DYNA[®] represent a more efficient way to handle and to investigate the foam behavior. The efficiency feature is strictly connected with the CPU time required to perform the numerical analyses for calibration and/or validation purpose: its reduction can be achieved by selecting a simple material model within reasonable accuracy. The easy utilization of material model can be expressed in terms of number of material parameters, or in terms to exploit directly the data coming from material testing, like for example the stress/strain curves.

Several material models for foams are available in the LS-DYNA database and further in the last years different enhancements have been performed with the aim to include the physical phenomenon able to increase the accuracy of the models [5,6].

At the aim to evaluate the behavior of suitable LS-DYNA material models when applied to reproduce in numerical way the experimental behavior of Aluminum foams, an extended study has been set.

The current activity represents the first step of such study because it is focused on the assessment of a procedure able to calibrate and validate in a sharp way the constitutive parameters of advanced material models, especially for the most challenging ones. Therefore, the procedure must accomplish an efficient set-up of numerical models, integrate in a unique workflow all the different experimental tests (so that the same material model could be managed contemporaneously in respect of them), point out which data analysis tools are more suitable to elaborate multivariate data.

The above requirements imply, first of all, to develop as accurate as possible numerical models vs. the corresponding experimental ones, keeping low at the same time the computational effort. A preliminary calibration of the numerical models representing just the experimental equipments has been performed to be sure about the goodness of such stand-alone equipment set-ups (i.e. the equipment set up has been assessed for known material properties). Starting from this baseline, the whole FE models (i.e. with the unknown material properties) have been tested under several FE configurations.

Following, every material model taken into account requires to be calibrated, that is the free parameters embedded in its constitutive relations have to be tuned so that a good fitting between the numerical and experimental data can be reached. As a general rule, to get a good and robust numerical-experimental correlation, we need to handle at the same time the objective functions representing the performance we are interested. For example, if a material model has to be arranged for tension and compression load conditions, but it is adjusted against only uniaxial tests, then a mismatch will come out since Aluminum foam exhibits quite different behavior in tension than in compression: elastic, plastic and compaction phases in compression, while elastic deformation phase followed by fracture in uniaxial tests [5]. For these reasons, the calibration process requires to optimize the free parameters according to different goals, which are usually in conflicting between them. At this purpose, the code LS-DYNA has to be coupled with modeFRONTIER, Process Integration and Design Optimization software. Once all the numerical models related to the corresponding experimental tests have been integrated, an efficient optimization strategy can explore the design space (i.e. the free parameters dominions) in order to get the configurations satisfying the different goals (i.e. specimen response in different loading conditions).

Eventually, efficient and intuitive post-processing tools have been selected to establish some good practices to apply for the data analysis.

The experimental tests supporting the planned investigations are focused to characterize the compression, flexural, shear and dynamic behavior of open and closed cells Aluminum foams, different in terms of morphology (open and closed kind), density and panel thickness. Moreover, loadings in the foaming direction and in the perpendicular one were taken into account.

1. Introduction

The main objective of this activity is to assess a well-established procedure aimed to calibrate and validate the constitutive parameters (i.e. material constants) of LS-DYNA advanced material models for Aluminum foams. All the values related to these parameters, indeed, cannot be derived from simple experimental tests (often these ones do not exist at all), and consequently they must be gained by fitting the numerical material behavior to the experimental one.

With the aim to get a good and robust numerical-experimental correlation, the parameters calibration has to be performed taking into account the material structural response due to different load conditions, so the different objective functions representing the material performances have to be achieved contemporaneously. For these reasons, the calibration process requires to optimize the constitutive parameters (free parameters) according to different goals (objective functions), which are usually in conflicting between them. Numerically, the code LS-DYNA has been coupled with modeFRONTIER, multi-objective optimization and design environment, able to explore the design space (i.e. the free parameters dominions) in order to get the configurations satisfying the different objective functions (i.e. the specimen response in different loading conditions). In addition, this kind of approach allows pointing out how the experimental tests are correlated with the static and dynamic mechanical characterization.

The added value to build up such a well-established procedure, is the possibility to face the problem of the material models calibration by using an approach able to evaluate which are the performances provided by a material model working under different loading conditions, and, especially, to highlight how such performances are correlated each other (since the material model set up is unique). At this aim, a further outcome of the procedure is the designation of efficient methods for the elaboration of multivariate data (i.e. data depending on multiple variables).

The Aluminium foams modeling is a matter of intensive investigation because a satisfying mechanical characterization has still to be achieved, so the definition of a preliminary procedure can be very effective to speed up its accomplishment, especially if, keeping the experimental tests set-up constant, the only foam type changes.

The on-going investigation has been focused on closed and open cells Aluminum foams sketched in Fig. 1, respectively ALPORAS (manufactured by GLEICH GmbH, Kaltenkirchen, Germany) and M-PORE (manufactured by M-PORE GmbH, Dresden, Germany).



Fig. 1: foam panels – M-PORE and ALPORAS [7]

2. Procedure Assessment

According to the description referred above, the properties driving the developing procedure can be collected within the following LOR (List Of Requirements):

- ✓ Selection of a set of experimental tests able to characterize the static and dynamic behavior of the Aluminium foam samples;
- ✓ Selection of a suitable LS-DYNA material model;
- ✓ Accurate and robust set up of the Finite Element Models (FEM) representing the experimental tests;
- ✓ Material model calibration and validation taking all the different tests into account;
- ✓ Assessment of good practices for multivariate data analysis (i.e. data depending on multiple variables).

Experimental Tests

The experimental tests to be executed, on the one hand have to describe in an accurate way the different material behaviors under different loading conditions, on the other hand they have to provide not redundant informations, i.e. they have to be uncorrelated. To these purposes, a set of experimental tests has been selected taking into account the phenomena to be reproduced.

According to the main application fields of the Aluminum foams (i.e. energy absorbers in structures potentially involved in impact phenomena), quasi-static and dynamic tests have been chosen, so that the calibration of the constitutive parameters could be able to wrap both dynamic and static behavior of the material model. In particular, the following tests have been executed:

- ✓ uniaxial compression
- ✓ 3 points bending
- ✓ Charpy pendulum
- ✓ shear

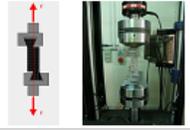
Where standard methods for the tests execution were not available or well established (shear test with eccentric clamps is referred to ASTM C 273 standard, 3 points bending test comes from ASTM D 790, while for the remaining ones a standard method is missing), the approach was to design equipments able to mine one by one the different specimens mechanical properties. The driving requirement for the selection of the specimens' dimensions (if feasible) was to avoid border effects, for at least 7 cells are needed along every direction of the specimen [7]. The mechanical behavior of the specimens has been investigated not only according to their geometry size, but for the M-PORE specimens also according to their density (expressed in terms of ppi - pores per inch).

Since the manufacturing process involves different cells morphology, the foams are no isotropic and show mechanical properties depending upon the direction. In the current work, tests along the foaming direction and in the perpendicular one to it have been carried out.

In Table 1 the set up for the experimental tests and the specimens geometry are sketched. For the uniaxial compression tests, loading and loading-unloading conditions have been evaluated with the aim to identify correctly the foam stiffness [8]. Further the eccentric clamps equipment, for the shear pure test has been exploited an apparatus called "Articulated Quadrilateral" [9]. The idea behind the Articulated Quadrilateral deals with the pure shear theory related to the Mohr's circles. According to the kinematism of the device, if a tensile loading is applied along the vertical direction by using a simple hydraulic machine, a corresponding compression load rise along the horizontal direction. A shaped like a rhombus specimen has been exploited, so that it could provide, according to Mohr theory, a pure shear stress [9].

The Figs. 2-5 provide some typical experimental curves for the tests carried out.

Table 1: Set up for the experimental tests and the specimens geometry

Experimental Tests				Aluminium Foams Specimens	
Test ID	Test Name	Test Set-Up	Test Equipment	ALPORAS	M-PORE
1a	uni-axial compression - parallel to foaming direction	test rate = 0.60-3.00 mm/min = 0.01-0.05 mm/sec		40x40 mm base @thickness (x3): 20 mm 30 mm	40x40 mm base @density, thickness (6 types x7): 30 ppi, 20 mm 30 ppi, 30 mm 45 ppi, 10 mm
	1b	uni-axial compression - perpendicular to foaming direction			test rate = 0.60-3.00 mm/min = 0.01-0.05 mm/sec
2a	shear - eccentric clamps	test rate = 0.5 mm/min = 0.0083 mm/sec		2 types @thickness (x3): 20 mm 30 mm	
3a	3 points bending - parallel to foaming direction	striker_vel = 10 mm/min = 0.166 mm/sec span = 60 mm		30x30x100 mm (x3)	30x30x100 mm (x3) @ density 30 ppi 10x10x100 mm (x3) @ density 45 ppi
3b	3 points bending - perpendicular to foaming direction	striker_vel = 10 mm/min = 0.166 mm/sec span = 60 mm		30x30x100 mm (x3)	30x30x100 mm (x3) @ density 30 ppi 10x10x100 mm (x3) @ density 45 ppi
4a	Charpy pendulum - parallel to foaming direction	impact energy = 8.33 J impact velocity = 2.2 m/s span = 60 mm		10x10x100 mm (x10)	10x10x100 mm (x10) @density: 30 ppi(x10) 45 ppi(x10)
4b	Charpy pendulum - perpendicular to foaming direction	impact energy = 8.33 J impact velocity = 2.2 m/s span = 60 mm		10x10x100 mm (x10)	10x10x100 mm (x10) @density: 30 ppi(x10) 45 ppi(x10)

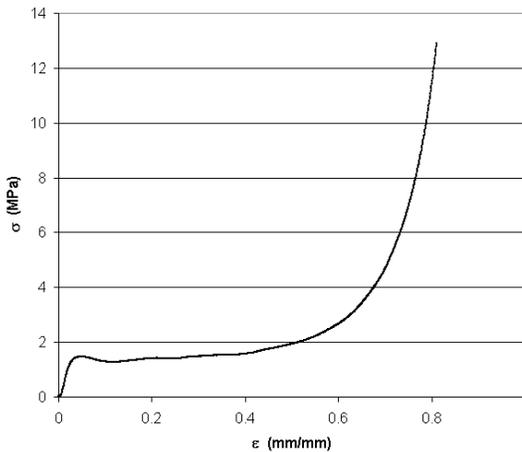


Fig. 2: uniaxial comp. (normal) - ALPORAS 20 mm

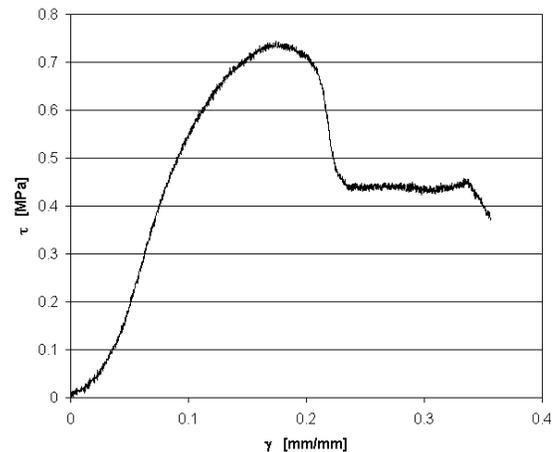


Fig. 3: shear (QA) - ALPORAS 30 mm

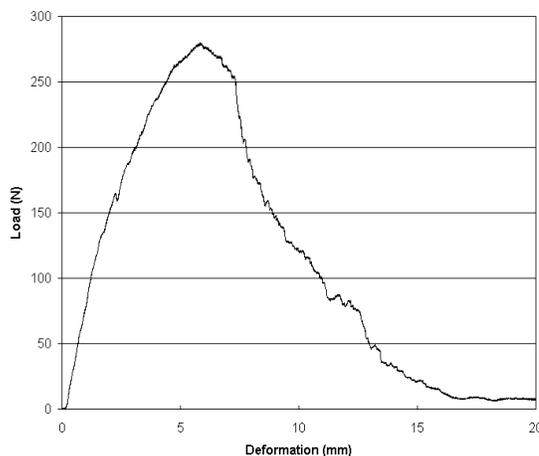


Fig. 4: 3 pts bending test (normal) - M-PORE 30 mm

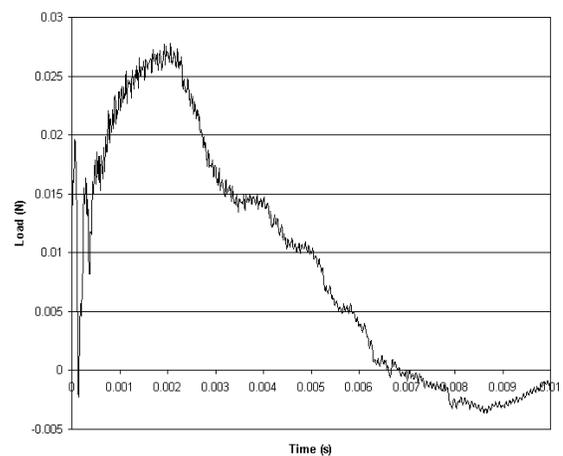


Fig. 5: Charpy (normal) - ALPORAS

Material Model MAT 154

A good number of material models are available for foams in the LS-DYNA database, and further in the last years different enhancements have been performed with the aim to include the physical phenomena able to increase the accuracy of the models. Amongst the LS-DYNA models, the MAT 154 (MAT_DESHPANDE_FLECK_FOAM) has been chosen because it provides satisfactory results compared with the experimental ones available in literature, and further it represents, as explained later, an appropriate model in respect of which the developing procedure can be verified.

The MAT 154 comes from a constitutive model developed by Deshpande and Fleck [10], and after enhanced and implemented in LS-DYNA by Reyes et al. [11].

The Deshpande and Fleck model [10] is isotropic and continuum-based, and its main characteristic is connected with the yield function formulation. It starts from the consideration that, while the volume of a metal keeps to be constant once loaded plastically, the metal foams exhibit a volume variation due to internal buckling and cell walls collapsing, and therefore they show yielding under hydrostatic load conditions. Equivalently, classical Von Mises yield criterion cannot be applied anymore since metal foams are affected by plastic flow under pure hydrostatic stress condition, as demonstrated by Hanssen et al. as well [12].

To take such phenomenon into account, the approach followed by Deshpande and Fleck was to relate the plastic flow not only to the elastic shear energy but to the elastic volumetric energy as well, and consequently to build up a generalized Von Mises yield criterion where the equivalent stress includes not only the deviatoric stress component but also the hydrostatic one. Therefore, in MAT 154 yielding occurs when the total elastic energy reaches a critical value that, assuming to be the same for all kinds of stress configurations, can be set equal to the total elastic energy in the uniaxial stress state.

It comes that in the yield surface $\Phi = \hat{\sigma} - \sigma_y$, the equivalent yield stress $\hat{\sigma}$ is given by [13]:

$$\hat{\sigma} = \hat{\sigma}(\sigma, \alpha) = \frac{\sigma_{VM}^2 + \alpha^2 \sigma_m^2}{1 + \left(\frac{\alpha}{3}\right)^2} \quad \text{|||}$$

where:

σ_{VM} = Von Mises stress

σ_m = mean stress (hydrostatic pressure)

α = shape factor. It defines the shape of yield surface and can be expressed in terms of the plastic Poisson ratio ν_p [12]:

$$\alpha^2 = \frac{9(1 - 2\nu_p)}{2(1 + \nu_p)}$$

The values of α^2 needs to belong to the range [0,4.5], otherwise α is physical meaningless. The 0 value corresponds to the Von Mises criterion, while 4.5 means that lateral plastic deformation does not exist in uniaxial compression test [5].

The yield stress function σ_y takes the evolution of yield surface into account because it is the sum of the initial compressive yield stress and the strain hardening:

$$\sigma_y = \sigma_p + R(\hat{\epsilon}) = \sigma_p + \gamma \frac{\hat{\epsilon}}{\epsilon_D} + \alpha_2 \ln \left[\frac{1}{1 - \left(\frac{\hat{\epsilon}}{\epsilon_D}\right)^\beta} \right] \quad \text{|||}$$

where:

σ_p = foam plateau stress (initial compressive yield stress)

$R(\hat{\epsilon})$ = strain hardening

$\hat{\epsilon}$ = equivalent true strain

ϵ_D = densification strain (true compaction strain). It can be expressed in function of the shape factor α , current foam density ρ_f and base material density ρ_{f0} :

$$\varepsilon_D = -\ln\left(\frac{\rho_f}{\rho_{f0}}\right)$$

α_2, γ, β = material parameters.

Even if Reyes et al. [11] implemented the Deshpande and Fleck foam model in LS-DYNA adding two fracture criteria and the statistical variation of the foam density, just one fracture criterion and no density statistical variation option are available in version 971 of LS-DYNA yet. As demonstrated, indeed, by Hanssen et al. [12], a crushable foam model missing in a fracture criterion is not enough accurate in results prediction, especially in uniaxial tension and shear tests for which fracture do occur. The fracture criteria still now available in LS-DYNA assumes that elements are removed when the critical value of volumetric strain CFAIL is reached. Being only the hydrostatic deformation involved, the deviatoric one is skipped in such criterion.

Further, because its porous structure, foam is characterized by a variation of the material properties. Since foam properties are strictly connected with its density, without modeling the pores geometry, Reyes et al. [11] inserted into the model the properties variation by a statistical variation of the foam density (i.e. the mesh elements density was given by a suitable probability distribution).

The equations representing the model can be calibrated from uniaxial compression tests [5, 12].

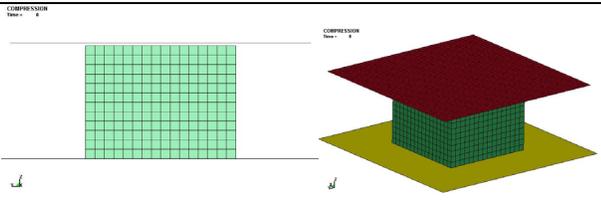
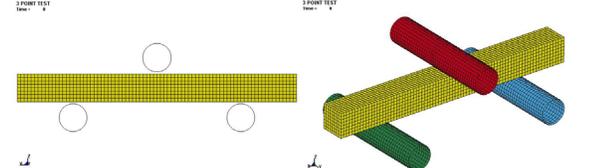
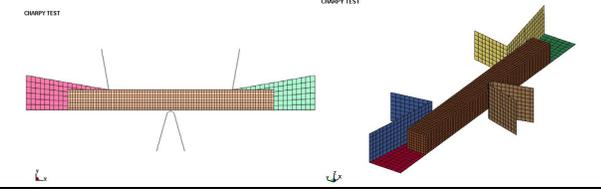
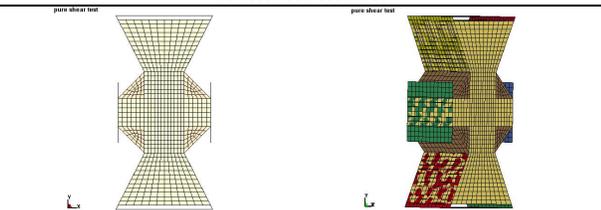
Numerical Set Up

The build up of the finite element models has been addressed to get accurate results within reasonable computation times, especially in the perspective that a calibration process has to be carried out. For these reasons, a preliminary FEM robustness investigation has been performed for each one of the four numerical models corresponding to the experimental tests, with the aim to assure that the model behavior gets independent from the mesh element size and loading rate, and to realize how contacts policy affect the numerical performances. In Table 2 are summarized the numerical set-ups for the different FE models (eccentric shear modeling still needs some more investigation).

Regarding to the element meshes, since there are different geometry sizes for the same experimental test, numerous configurations have been investigated and the elements number referred in Table 2 is aimed to suggest a good compromise between the required CPU time and the numerical accuracy.

The loading in the “boundary prescribed motion” card has been applied by using a sinusoidal function so that no impulsive force rose. At the same time, to avoid CPU times very demanding for the quasi-static tests, a suitable scale factor has been inserted.

Table 2: Numerical set-ups of FE models

Uniaxial compression test			
element mesh	sample elements	contact	loading
	<ul style="list-style-type: none"> • about # 3000 • fully integrated S/R solid 	ASTS	BOUNDARY_PRESCRIBED_MOTION_RIGID (velocity)
3 point bending test			
element mesh	sample elements	contact	loading
	30x30x100: <ul style="list-style-type: none"> • # 3000 • default element 10x10x100: <ul style="list-style-type: none"> • # 5100 • default element 	ASTS	BOUNDARY_PRESCRIBED_MOTION_RIGID (velocity)
Charpy test			
element mesh	sample elements	contact	loading
	<ul style="list-style-type: none"> • # 5100 • default element 	ASTS	PART_INERTIA (velocity)
Shear test			
element mesh	sample elements	contact	loading
	<ul style="list-style-type: none"> • about # 8000 • fully integrated S/R solid 	ASTS	BOUNDARY_PRESCRIBED_MOTION_RIGID (displacement)

Calibration and Validation

The whole calibration and validation procedure can be subdivided into the following steps (the first two are functional to last one):

- ✓ Policy of the numerical-experimental data comparison
- ✓ Process integration and optimization strategy
- ✓ Calibration and validation procedure

Policy of the numerical-experimental data comparison

The calibration process of a material model can be described as an appropriate optimization challenge, with material constitutive parameters as input variables to be optimized in respect of the different material performances. More in details, each one of the experimental measurements represent a target to be reached by the corresponding numerical model simulation. Hence, an independent objective function has been firstly assigned to every physical quantity to be fitted, and then minimized by using the relative error formulation:

$$\frac{|F_{FEM}^j - F_{exp}^j|}{F_{exp}^j} \times 100$$

where F_{FEM}^j is the numerical value for the j th physical data, and F_{exp}^j is the experimental one. The numerical-experimental data fitting has been carried out in terms of the quantities able to characterize the foam behavior at the different experimental test phases. That means, for example, managing individually the stiffness in elastic, plastic and compaction phases in uniaxial compression tests, or just the peak value of a stress-strain shear curve, and so on. In the current investigation the following 10 objective functions have been arranged for the experimental-numerical comparison:

Uniaxial compression test

Stress-strain curves @ 3 main phases → 3 objective functions

Shear tests

Eccentric clamps - stiffness → 1 objective function

Quadrilateral articulated – stiffness → 1 objective function

3 points bending test

Force-displacement curves @ 2 main phases → 2 objective functions

Peak values → 1 objective function

Charpy pendulum test

Absorbed energy → 1 objective function

Force-displacement curve → 1 objective functions

All the above comparisons deal with every foam type (ALPORAS and M-PORE) and every different density (different ppi for M-PORE foam).

The purpose to get a flexible, all-round material model set-up, requires to handle these objective without any a priori assumption on their relative priorities. This should be done by avoiding the classical approach of building an overall “fitness function” to be minimized, such as their weighted sum. Instead, by relying on the modeFRONTIER Multi-Objective optimization algorithms, such ten objectives (relative errors) can be considered as independent and minimized simultaneously, using the Pareto Dominance criteria [14] without the need to specify any weighting factor.

The material parameters to be tuned (i.e. the free parameters) are the same referred in Material Model paragraph (10 input variables in general case).

Process integration and optimization strategy

Being the number of free parameters and objective functions too high, a simple trial-and-error procedure cannot be applied for the calibration-validation process. The efficient method proposed here, instead, takes advantage of the modeFRONTIER Process Integration and Design Optimization capabilities.

The process integration of the numerical models related to the experimental tests can be implemented and described within modeFRONTIER by means of the workflow depicted in Fig. 6. This chart represents a sort of “high-level” programming language to describe a generic design

process. It is made up by pre-defined blocks (called “nodes”), which represent the fundamental components of any process. In modeFRONTIER such nodes can be drag-and-dropped from a library, and linked, to build up a network following simple rules: it is the same procedure anyone uses when combining alphabet letters to build up words and phrases. Hence, Fig. 6 simply describes the most general workflow both for calibration and for validation phase: as explained further later, according to the on-going phase it is enough to “switch off” the numerical simulations to be not included. Such workflow holds on both for ALPORAS foams and for the M-PORE ones.

From top to bottom, following the blue links, there is the representation of the so-called “Data Flow”. The green block at the top left define the input variables (the constitutive parameters) for which a suitable range of variations can be set. Each time a new combination of their values is proposed by the modeFRONTIER internal optimization strategy, the MAT 154 card file is updated (node “mat_inp”) and transferred to the five LS-DYNA models. Such computations produce outputs that are post-processed and finally give numerical forecasts of the ten physical responses, whose values are rearranged in the ten relative errors with the aim to minimize the discrepancy (yellow blocks).

From left to right of Fig. 6, following the red link, the so-called “Logic Flow” is represented: it is the sequence of operations that modeFRONTIER will automate, and the logic driving them. The “DOE” node, the first block at the left side, means “Design Of Experiments”. Such node allows the user to design a suitable initial population (combinations of input variable values) in respect of an efficient exploration of the design phase. Looking at the performances provided by such configurations, the “Scheduler” node realizes how the investigated phenomena behave and, according to its internal search strategy, starts to generate completely new designs. The new configurations flow sequentially into the five LS-DYNA models, whose numerical simulation is run in batch modality by modeFRONTIER, so that eventually a new evaluation of the objective functions is performed.

Therefore, the workflow describes in a graphical and a very intuitive way, how the whole process is carried out. Being the optimization strategy simply enclosed in the DOE and Scheduler blocks, the setting of a new optimization strategy to solve the same design process more efficiently does not require any change on the workflow structure.

The “Design Optimization” logic can be any combination of the optimization algorithms and the advanced tools (such as Response Surface Models –RSM–) available in modeFRONTIER, and aims to improve all the assigned objectives simultaneously.

The approach now described guarantees a strong reduction of the needed engineering-time, thanks to the automation of all the repetitive operations. High-qualified human resources can instead concentrate into data analyses and decision-making. Also, the pure computational time needed to solve most of the today’s typical engineering challenges (with typical hardware/software resources) can be strongly reduced, thanks to the efficiency of the optimization strategies. In fact, within modeFRONTIER there are the state-of-the-art mechanisms to generate DOEs, more than eighteen optimization algorithms, as well as RSMs to speed up optimum search when computational time is high [15].

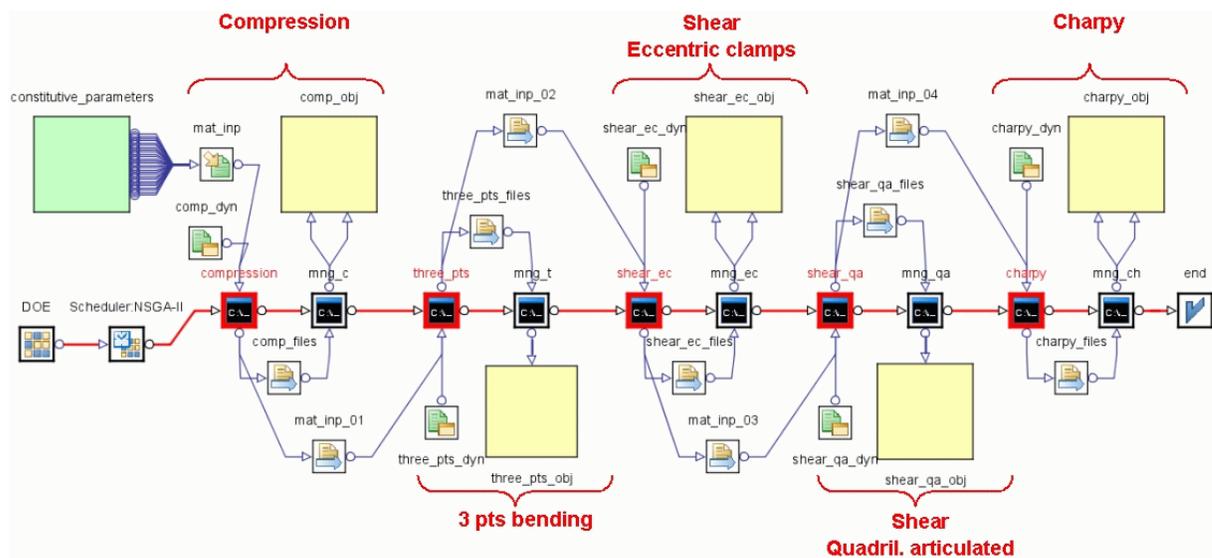


Fig. 6: modeFRONTIER workflow

Calibration and validation procedure

The calibration and validation procedure can be explained as in the following:

- ✓ Output Correlations and Sensitivity Analysis – In a first step, a numerical-experimental data comparison is performed in respect of all experimental tests. In such phase, no optimization (and consequently calibration) is carried out, since just the designs belonging to the initial DOE are evaluated. By selecting a suitable DOE (full or reduced factorial, latin square, etc.), indeed, it is possible to point out both the correlations between the different objective functions, and the correlations between the constitutive parameters and the objective functions. Getting the outputs-outputs relationships, allows reducing the calibration challenge size because the correlated outputs (i.e. the redundant ones) can be omitted, as well as a whole experimental test. The number of the objective functions, therefore, can there be reduced. At the same time, highlighting the correlations input-output enables to freeze at a constant value the least significant input variables. The number of the input variables, therefore, can there be reduced. The modeFRONTIER workflow can be updated by simply “switching off” the nodes related to the previous steps.
- ✓ Enhanced Calibration – By using a Multi-Objective optimization algorithm, a calibration with the only most sensitive input variables and only not correlated numerical models is carried out.
- ✓ Validation – Check the best solutions coming from the previous calibration, by running the whole set of five models and comparing the 10 outputs with the corresponding experimental results.

The more the challenge is complex, in terms of number of constitutive parameters to tune (inputs) and number of static/dynamic tests to fit (outputs), the higher is the advantage of this approach. In the current investigation, the number of constitutive parameters is relatively small (in comparison to other material models), while several objective functions have been assessed. The idea behind such working conditions is to verify how strong are the correlations between the outputs and inputs.

It should be noted that before to perform any material model calibration, a “test equipment calibration” has been performed. Especially for the three points bending and the Charpy tests, some uncertainties do exist (geometrical dimensions, numerical assumptions, etc.) and affect the corresponding numerical model set-ups. Therefore a preliminary calibration in which the constitutive parameters of a simple material model (i.e. standard Al alloy) were completely known, allowed to verify the goodness of the stand alone numerical test equipment.

Multivariate Data Analysis

The last point of the whole procedure deals with the assessment of good practices for multivariate data analysis (i.e. data depending on multiple variables). Strictly speaking, such point illustrates the instruments to perform the calibration and validation phases described previously, but since such data analysis tools live stand-alone as well, they are reported separately. Different data analysis approaches exist, but here a couple, amongst the more efficient of the classical and exploratory ones, is pointed out.

In order to reduce the number of tests required to calibrate the material, the first idea is to search for correlated objectives. A suitable DOE scheme is required to the purpose to uniformly sample the design space of input variables (i.e. the scalar product of the free parameters dominions), avoiding self-correlation between the variables themselves in the chosen configuration set. A Full Factorial DOE satisfies such properties, and allows detecting main effect and interactions between factors and responses. Its main drawback is the number of design evaluations, which grows with the number of variables with a $2^{n^{Var}}$ law (i.e. with 10 input variables it follows 210 designs). A good compromise between completeness of information and sample size can be achieved by using a Reduced Factorial DOE with saturation option. It is able to evaluate just the main effects of each variable, skipping the higher-order interactions. Reduced Factorial is basically a “2-level Full Factorial” scheme, but with saturation option (see Fig. 7) the number of configurations are further reduced because the values of some input variables is expressed in function of the other ones. In the example of Fig. 7, x_4 depends upon (x_1, x_2) , and some more two variables, i.e. x_5 and x_6 , could be expressed respectively as a function of (x_1, x_3) , and (x_2, x_3) . An alternative way to precede it is relying on a “Latin Square” DOE scheme.

n. design	x1	x2	x3	x4 (=x1*x2)
1	+	+	+	+
2	+	+	-	+
3	+	-	+	-
4	+	-	-	-
5	-	+	+	-
6	-	+	-	-
7	-	-	+	+
8	-	-	-	+

Fig. 7: saturation in Reduced Factorial scheme

Once the DOE has been processed, the correlation between the objectives can be evaluated by using the Correlation Matrix. The Fig. 8 sketched a typical objectives' Correlation Matrix in which the only configurations related to DOE generation are taken into account (no optimization). The first-order correlation between two entities is expressed by means of a normalized index spanning from -1 to +1: a value equal to +1 (-1) denotes a full direct (inverse) correlation, while a low absolute value means low correlation. In such mock-up case, it can be turned out that the objectives 5 and 6 are strongly directly correlated (relative errors were assessed here as well, hence the direct correlation holds on for every coupling) with the objective 1. Moreover, objective 6 is correlated with the 5 one, the 7th with the 4th, and so on for the remaining red boxes, while the yellow ones are mildly correlated.

In a similar way, the correlations between inputs and outputs can be estimated by using a Correlation Matrix in which not only the objectives but also the input variables are inserted. A further efficient tool to do that is the Overall Student Chart. Being based on Student's t-test, it is extremely useful to easily detect simultaneously which input variables are the least sensitive ones over the set of the objectives functions. In particular, as shown in Fig. 9, every objective function is sketched by means of pie chart in which the input variables affecting more than a given threshold the current objective are highlighted, while the least sensitive ones are grey-shaded in the pie chart and hidid from the table at the right side of the picture after moving the "threshold filter" at the bottom.

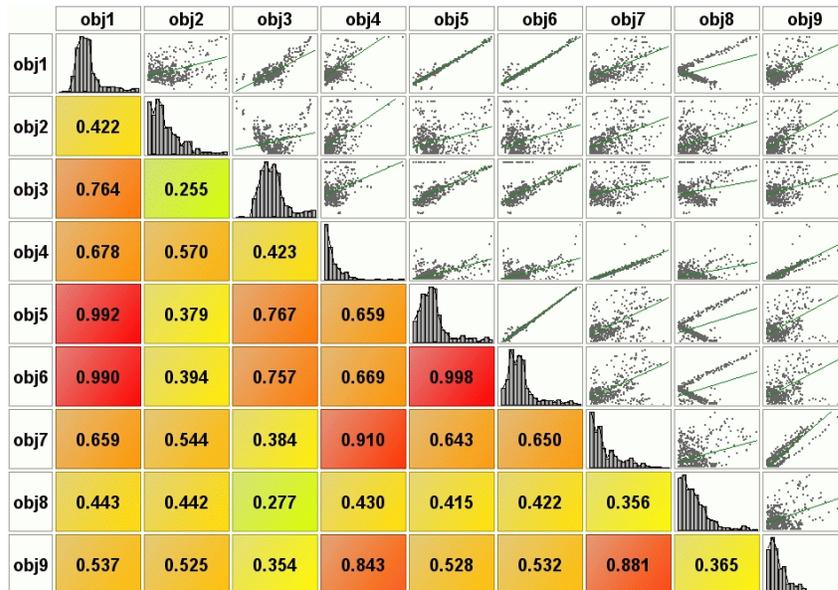


Fig. 8: objectives' Correlation Matrix

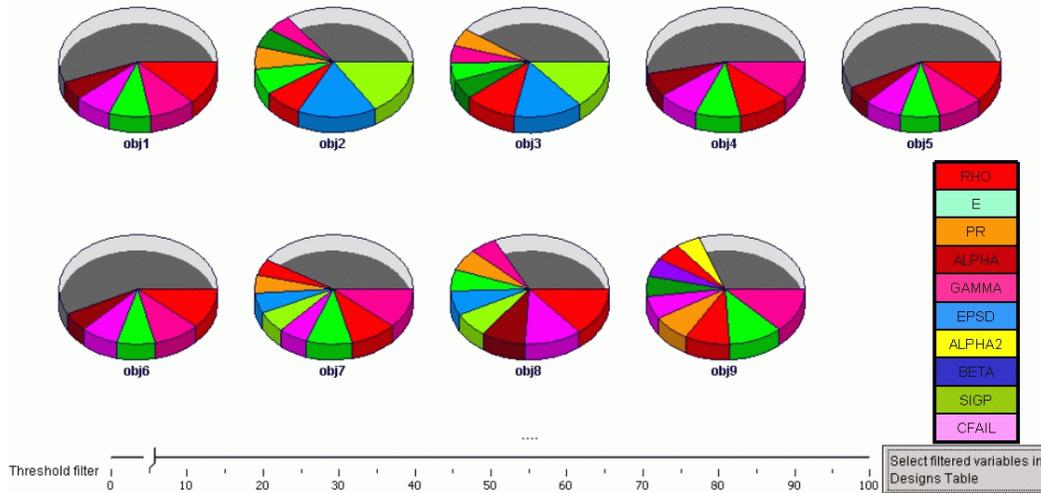


Fig. 9: Overall Student Chart

A third very flexible and intuitive instrument to study multivariate data is the so-called Parallel Chart. In this type of chart, firstly a set of parallel axes is drawn with the aim to represent whichever variables (both input and output), and then a line connecting the values of each variable on each corresponding axis sketches every design (see Fig. 10). Since the Parallel Chart allows to modify in real time the range of every single variable, it can be used to filter the most interesting solutions in the database and to realize, like in this case, which designs “survive” if the lower or upper value of one or more input variables is removed. Applying this method, the Fig.11 vs. Fig.10 and Fig.12 vs. Fig.10 comparisons highlight the correlations already mined between the 5th and 6th objectives with the 1st one (Fig.11), or the not correlation of the 8th objective from the other ones (Fig.12). A similar approach can be followed if the input variables are accounted for.

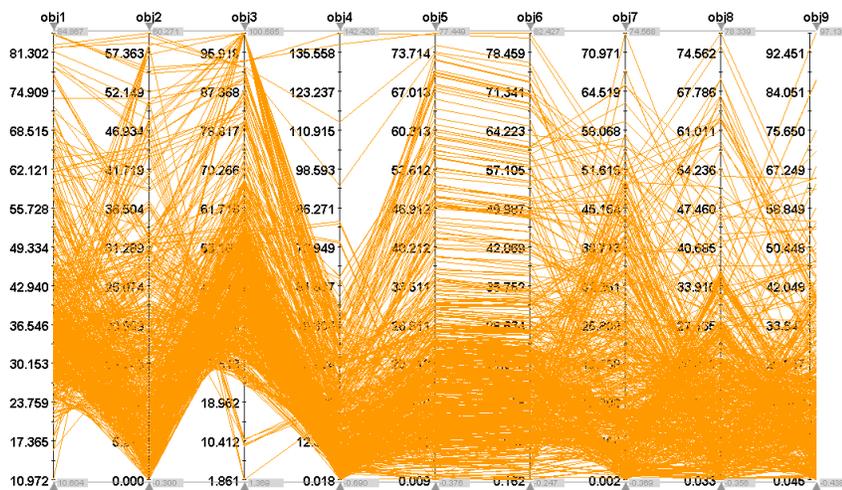


Fig. 10: objectives' Parallel Chart

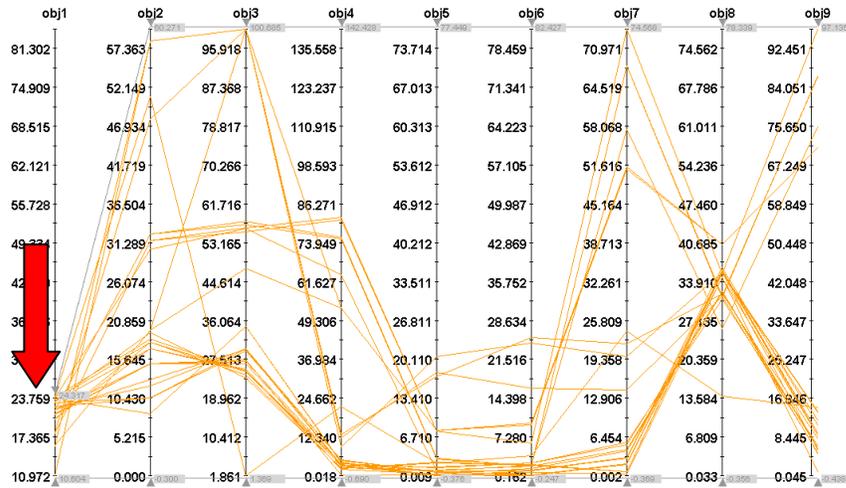


Fig. 11: objectives' Parallel Chart – filtering 1st objective

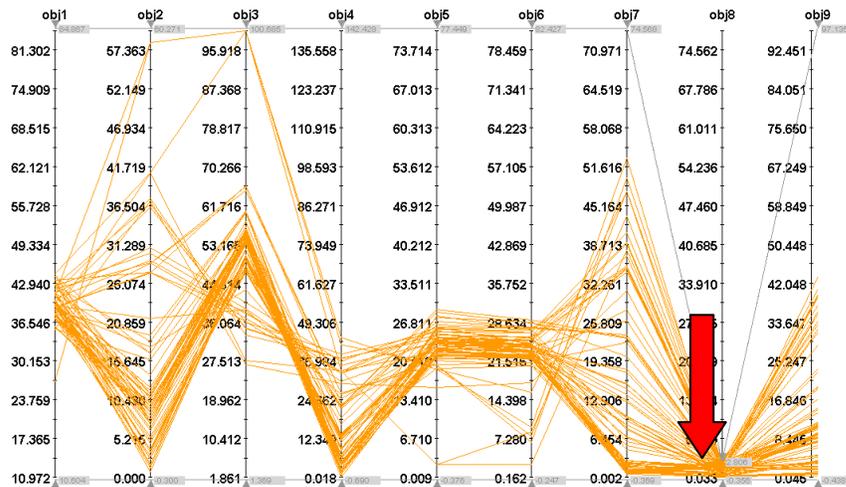


Fig. 12: objectives' Parallel Chart – filtering 8th objective

Conclusions

A well-established procedure aimed to calibrate and validate the constitutive parameters of advanced material models has been proposed. The method relies on integration of the explicit, non-linear finite element code LS-DYNA into the Process Integration and Design Optimization platform modeFRONTIER. The procedure has been addressed to realize how the variables (inputs and outputs), involved in the material model calibration, are correlated between them and consequently to point out how the different experimental tests are physically correlated. By the way, some good practices to perform multivariate data analysis have been suggested.

The LS-DYNA MAT 154 calibration in respect of Aluminium foams has been arranged, with the aim to apply and verify the referred procedure in a challenging real case.

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References

- [1] J. Banhart, *Progr. Mater. Sci.* 46, 2001, pp. 559–632
- [2] C.S. Marchi, A. Mortensen, *Acta Mater.* 49, 2001, pp. 3959–3969
- [3] L.J. Gibson, M.F. Ashby: “Cellular Solids”, Cambridge University Press, Cambridge, UK, 1997
- [4] W. Ehlers, G. Eipper: “Finite Elastic Deformations in Liquid-Saturated and Empty Porous Solids. Transport in Porous Media”, 1999
- [5] A. Reyes, O. S. Hopperstad, T. Berstad, A. G. Hanssen, M. Langseth: “Constitutive Modeling of Aluminium Foam Including Fracture and Statistical Variation of Density”, *European Journal of Mechanics A/Solids* 22 (6), 2003, pp. 815–835
- [6] A. Hirth, P. Du Bois, K. Weimar: “A Material Model For Transversely Anisotropic Crushable Foams in LS-DYNA”
- [7] M. De Giorgi, A. Forte, R. Nobile, F. Palano: “Caratterizzazione a Compressione di Schiume di Alluminio”, AIAS – Associazione Italiana per l’Analisi delle Sollecitazioni – XXXVII Convegno Nazionale, 10-13 Settembre 2008, Università di Roma “La Sapienza”
- [8] E. Andrews, W. Sanders, L.J. Gibson: “Compressive and tensile behaviour of Aluminium foams”, *Materials Science and Engineering A270*, 1999, pp. 113-124
- [9] F. Palma: “Caratterizzazione meccanica a taglio di varie tipologie di schiume in Alluminio a celle chiuse”, Tesi di Laurea A.A. 2007-2008, Università degli Studi del Salento, Facoltà di Ingegneria
- [10] V.S. Deshpande, N.A. Fleck: “Isotropic models for metallic foams”, *J. Mech. Phys. Solids* 48, 2000, pp. 1253-1283
- [11] A. Reyes, O. S. Hopperstad, T. Berstad, M. Langseth: “Implementation of a Constitutive Model for Aluminium Foam Including Fracture and Statistical Variation of Density”, 8th international LS-DYNA User Conference, 2004
- [12] A. G. Hanssen, O. S. Hopperstad, M. Langseth, H. Ilstad: “Validation of constitutive models applicable to aluminium foams”, *International Journal of Mechanical Sciences* 44, 2002, pp. 359-406
- [13] “LS-DYNA Key User’s Manual, Version 971”, 2007
- [14] Rao, S.S.: “Engineering Optimization, 3rd ed.”, Wiley Interscience, 1996.
- [15] “modeFRONTIER User’s Manual, Version 4.1.1”, 2009.