Study on Optimal Design of Automotive Body Structure Crashworthiness

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ABSTRACT

In this paper the optimal model of thin-walled sections of automotive body for structural crashworthiness is built. With computer design of experiment (DOE), the response surface model (RSM) of design can be obtained by carefully choosing a small quantity of samples in the design space. Pareto genetic algorithm (GA) is used in subsequently optimal design. With optimal design of thin-walled sections, the effects of the section parameters such as dimension and thickness on crashworthiness property are researched.

INTRODUCTION

Improving the safety of automotive is one of main content researched by world automotive industry now. Body structure has played a significant role in automotive passive safety. Improving body structure design by impact test and computer numerical simulation technology has an active action on advancing the body crashworthiness. In recent years there has been a close attention on optimal design of body crashworthiness.

Optimal design of impact structure is a difficult problem due to the nature of numerical crashworthiness analysis. The instability and uncertainty of impact analysis make the simulation process having to go through several iterations before obtaining one satisfactory result. At same time, because of the cost of explicit FEA, the fully integrated optimization process becomes impossible.

During the nonlinear dynamic analysis such as impact analysis, the derivatives of response functions are mostly extraordinary discontinuous. With the assistance of global approximation method such as response surface methodology, the design response can be smoothed and obtaining a global optimal result becomes relatively easy.

In this paper the optimal model of typical part with thin-walled sections in automotive body is built. With HyperMesh as pre- and post process tools, LS-DYNA as calculating core, the structural crashworthiness is analyzed. With computer DOE, the response property of design can be obtained by carefully choosing a small quantity of samples in the design space. The response surface models of the optimal objects are built with the basis of these samples and used in subsequently optimal design. Then the functions of response surface models are analyzed by Pareto GA to obtain the multi-objective optimal results.

The crashworthiness index which indicate the deformed energy absorbed by unit mass structure, the maximal impact force, the mean impact force etc. are the basic indexes to evaluate the crashworthiness optimal design. With optimal design of the part with thin-walled sections, the effects of the section parameters such as dimension and thickness and connection type on crashworthiness property are researched.

OPTIMAL DESIGN MODEL OF THIN-WALLED SECTIONS FOR AUTOMOTIVE BODY STRUCTURAL CRASHWORTHINESS

Design Objective

There are several indexes to measure the crashworthiness, which indicate the property of structure to endure the impact:

Crashworthiness Index η_c. The definition of crashworthiness index is: under some limit conditions, the number of energy by unit structure mass to absorb, that are

$$\eta_c = E_d / M_s \tag{1}$$

where, E_d is the absorbing energy of structure; M_s is the mass of structure.

For the thin-walled sections having same section shape, the crashworthiness index can be calculated used by the following equation (OHKUBO, 1974):

$$\eta_c = P_m / (A_s l) \tag{2}$$

where, P_m is the mean crash force; A_s is the area of section; l is the length of the thin-walled sections.

Maximum Crash Force P_{max} (or Maximum Acceleration). The maximum crash force acquired from the experiment may be occurred at two positions: one is at the beginning of bulking which is critical state determined by the structure elastic-plastic bulking. The other is at the end of the collapse when the whole structure is collapsed and the crash force rose quickly as radiation. In the structure impact study, the former peak value is mainly considered, which has important significance to structural failure and respectively less effect to the energy absorbing ability.

Mean Crash Force P_m. The mean crash force is the mean value of crash force curve vs. collapse displacement, which indicates the whole energy of the thin-walled sections absorbed.

Design Parameters

The energy absorbed and the mass of structure are determined by the section dimension and thickness of the thin-walled sections. So the design parameters are the following (Figure 1):

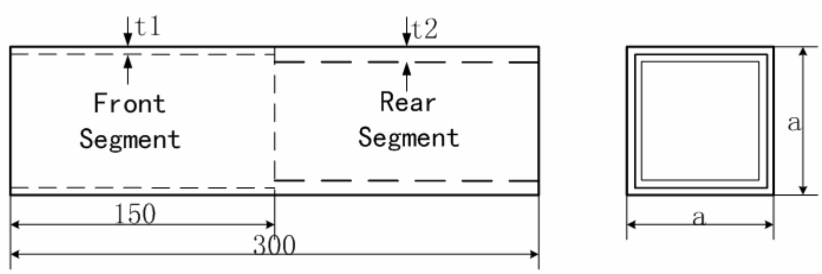

Figure 1. Design Parameters of the Thin-walled Sections

Section dimensions: $x_2 < x_3$

Thickness of the front segment: t_1

Thickness of the rear segment: t_2

Design Constraints

To assure the rear segment not collapsing before the front segment of the thin-walled sections, the thickness of rear segment must be larger than the front segment. That is

$$x_2 < x_3 \tag{3}$$

The maximum crash force can not be above 100KN, that is

$$P_{max} < 100\ KN \tag{4}$$

DESIGN OF EXPERIMENT (DOE) FOR STRUCTURAL CRASHWORTHINESS

When constructing RSM for the thin-walled sections crashworthiness, the appropriate DOE are adopted to calculate the coefficients of RSM, acquiring enough observation value of response parameters and making RSM constructed easily.

From the experiment scheme with minimum experiment points, the number of experiment points should be equal to the number of coefficients in the RSM. This experiment program is named saturation experiment program, which has fewer freedoms. So the experiment program with residual freedoms is adopted generally, which is the number of experiments N larger to the number of coefficients in the RSM. For example to the quadratic RSM (see equation

6), the number of the coefficients is $q = 1 + n + n + C_n^2 = (n+1)(n+2)/2$. To construct the quadratic RSM, the number of experiments is not less than q. In the RSM design, even if the design parameters are fewer, the full-factors experiment is not adopted generally for the residual freedoms are larger. So the experiment design such as the orthogonal design, central combination design and equal square design can be adopted generally.

Code Transformation

In the RSM design, the variety range of every design parameters may be different, even for some parameters with large variety. To deal with them easily, the linear transformation, named code transformation is adopted properly to the value of design parameters and the corresponding relationships between the parameters level and codes are built. The code transformation makes the range of factors transforming to the cube with its center at origin and overcome the difficult of different dimensions in the design and analysis.

Three design parameters are a, t_1, t_2 and every parameter has two levels. The levels of design parameters are showed in table 1. Three design parameters are transformed as the following formulation correspondingly：

$$x_1 = \frac{a-90}{10} \quad x_2 = \frac{t_1 - 1.5}{0.5} \quad x_3 = \frac{t_2 - 1.5}{0.5} \tag{5}$$

Central Combination Design

The central combination design is consists of 2^n factor design points with two levels, $2n$ axial points and n_c central points. Among them the axial points are distributed symmetrically on n coordinate axles with the origin point as central. The distance between the axial points and the central points is named axial arm γ and γ is a coefficient determined by the orthogonality of DOE. The central points are points when every design parameters are taken as 0 level and the experiments of the central points can be done once or more repeatedly (TIEMAO, 1990).

For code variable, the coordinates of experiment points are showed in table 2. The numeric experiments are processing by the central combination design matrix $\boldsymbol{D}$. The results of DOE are showed in table 3.

$$\boldsymbol{D} = \begin{bmatrix} a & t_1 & t_2 \\ -1 & -1 & -1 \\ -1 & -1 & 1 \\ -1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \\ -\alpha & 0 & 0 \\ \alpha & 0 & 0 \\ 0 & -\alpha & 0 \\ 0 & -\alpha & 0 \\ 0 & 0 & -\alpha \\ 0 & 0 & -\alpha \end{bmatrix}$$

Factor design points

Central points

Axial points

Table 1. 2 Level Values of Design Parameters

Design Parameters	a	t_1	t_2
Level 1	80	1.0	1.0
Level 2	100	2.0	2.0

Table 2. Coordinate of Experiment Points

Code Variable	Design Variable		
	a	t_1	t_2
$-\alpha = -1.215$	77.85	0.89	0.89
1	80	1.0	1.0
0	90	1.5	1.5
1	100	2.0	2.0
$+\alpha = +1.215$	102.15	2.11	2.11

Figure 2 is the FE model of the thin-walled sections (No.1 numerical experiment). Figure 3 and figure 4 are deformation of the thin-walled sections in No.1 and No. 10 numerical experiment.

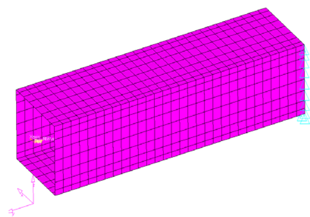

Figure 2. FE Model

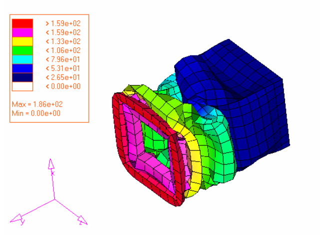

Figure 3. Deformation of No.1 Numerical Experiment

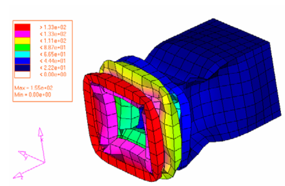

Figure 4. Deformation of No.10 Numerical Experiment

Table 3. Results of Central Combination DOE of Thin-walled Sections Crashworthiness

Column / No. / Experiment No.	a	t_1	t_2	Calculation results		
				η_c	P_{max}	P_m
	1	2	3	J/kg	kN	kN
1	80	1.0	1.0	9169.93	43.00	31.20
2	80	1.0	2.0	10145.52	113.05	61.77
3	80	2.0	1.0	7528.42	61.22	37.76
4	80	2.0	2.0	11283.97	143.82	112.83
5	100	1.0	1.0	8252.45	74.78	38.51
6	100	1.0	2.0	8890.45	144.94	65.59
7	100	2.0	1.0	6614.18	74.17	41.49
8	100	2.0	2.0	9787.47	164.67	133.06
9	90	1.5	1.5	9911.46	95.78	70.15
10	77.85	1.5	1.5	12204.27	112.28	74.86
11	102.15	1.5	1.5	9175.70	102.95	72.83
12	90	0.89	1.5	7629.88	85.70	35.56
13	90	2.11	1.5	7804.81	85.33	62.13
14	90	1.5	0.89	5423.61	68.51	34.57
15	90	1.5	2.11	9514.41	112.21	84.22

OPTIMAL DESIGN OF RSM FOR STRUCTURAL CRASHWORTHINESS

Modeling of RSM

y_j are gained by DOE with different level combinations of design parameters x_i. Quadratic polynomial model are adopted to approximate the actual function $f(\boldsymbol{x})$ by fitting of data obtained. For quadratic function is quadratic curved face in the design space, the response surface is used to approach the actual function. Assume the quadratic polynomial function is:

$$y = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} \beta_{ii} x_i^2 + \sum_{p<i}^{n} \beta_{pi} x_p x_i + \varepsilon \tag{6}$$

The regression coefficients $\hat{\boldsymbol{\beta}}$ of quadratic response surface are as following (DEHUI, 1996):

$$\begin{aligned} \hat{\beta}_0 &= \frac{1}{k}\sum_j y_j = \bar{y} & \hat{\beta}_i &= \sum_j x_{ij} y_j \Big/ \sum_j x_{ij}^2 \\ \hat{\beta}_{ii} &= \sum_j x_{ij}^2 y_j \Big/ \sum_j \left(x_{ij}^2\right)^2 & \hat{\beta}_{pi} &= \sum_j x_{pj} x_{ij} y_j \Big/ \sum_j \left(x_{pj} x_{ij}\right)^2 \end{aligned} \tag{7}$$

According to equation 7, the following functions are obtained.

Function of quadratic RSM on crashworthiness index:

$$\hat{y}_1 = 8988 - 754.4x_1 - 94.2x_2 + 1233.8x_3 + 1352.4x_1^2 - 659.9x_2^2 - 828x_3^2 - 29.8x_1x_2 - 115x_1x_3 + 664.4x_2x_3 \quad (8)$$

Function of quadratic RSM on maximum crash force:

$$\hat{y}_2 = 90.55 + 7.86x_1 + 6.18x_2 + 33.45x_3 + 12.66x_1^2 - 2.3x_2^2 + 0.98x_3^2 - 3.73x_1x_2 - x_1x_3 + 4.11x_2x_3 \quad (9)$$

Function of quadratic RSM on mean crash force:

$$\hat{y}_3 = 62.19 + 2.98x_1 + 14.64x_2 + 25.99x_3 + 9.62x_1^2 - 7.3x_2^2 - 0.16x_3^2 + 1.6x_1x_2 + 1.63x_1x_3 + 13.62x_2x_3 \quad (10)$$

Analysis of Variance (ANOVA) of RSM

The above quadratic RSMs on design objectives can really indicate the statistical regularity between design objectives and design parameters or be the useful quadratic approximate model. It should be concluded by ANOVA and F verification. Tables 4 to 6 are the variance analysis tables of quadratic RSM on crashworthiness index, maximum crash force and mean crash force.

Table 4. ANOVA of Quadratic RSM on Crashworthiness Index

Source	Quadratic Sum	DOF	Mean Square	Statistic F
Regression	3.9514e7	9	4.390e6	**6.4882**
Residual	3.3834e6	5	6.767e5	
Sum	4.2912e7	14		

Table 5. ANOVA of Quadratic RSM on Maximum Crash Force

Source	Quadratic Sum	DOF	Mean Square	Statistic F
Regression	14334	9	1592.7	**4.5857**
Residual	1736.6	5	347.32	
Sum	16066	14		

Table 6. ANOVA of Quadratic RSM on Maximum Crash Force

Source	Quadratic Sum	DOF	Mean Square	Statistic F
Regression	12004	9	1333.7	**16.74**
Residual	398.47	5	79.69	
Sum	12402	14		

F verification is carried out for the quadratic RSM function of crashworthiness index:

F (9, 5, 0.05) = 4.77; F (9, 5, 0.01) = 10.2; F (9, 5, 0.05) <F < F (9, 5, 0.01). It shows that the function is significant.

F verification is carried out for the quadratic RSM function of maximum crash force:

F (9, 5, 0.05) = 4.77; F< F (9, 5, 0.05). It shows that the function is not significant. So the significance levels of each factor regression coefficients are checked. The regression coefficients of interactive term $x_1 x_3$, quadratic terms x_2^2 and x_3^2 are not significant and can be removed directly from the function. The modified quadratic RSM function is (LUQUAN, 1987):

$$\hat{y}_2 = 90.55 + 7.86x_1 + 6.18x_2 + 33.45x_3 + 12.66x_1^2 - 3.73x_1x_2 + 4.11x_2x_3 \tag{11}$$

ANOVA and F verification are carried out again and the results show that the modified quadratic RSM function is significant.

F verification is carried out for the quadratic RSM function of mean crash force:
F (9, 5, 0.01) =10.2; F > F (9, 5, 0.01). It shows that the function is significant.

Analysis on RSM

The fitted quadratic RSM can be changed as the following:

$$\hat{y}(x) = \hat{\beta}_0 + \boldsymbol{x}^T \boldsymbol{b} + \boldsymbol{x}^T \boldsymbol{B}\boldsymbol{x} \tag{12}$$

where

$$\boldsymbol{x} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}, \quad \boldsymbol{b} = \begin{Bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_n \end{Bmatrix} \quad \text{and} \quad \boldsymbol{B} = \begin{Bmatrix} \hat{\beta}_{11} & \hat{\beta}_{12}/2 & \cdots & \hat{\beta}_{1n}/2 \\ \hat{\beta}_{21}/2 & \hat{\beta}_{22} & \cdots & \hat{\beta}_{2n}/2 \\ \vdots & \vdots & \vdots & \vdots \\ \hat{\beta}_{n1}/2 & \hat{\beta}_{n2}/2 & \cdots & \hat{\beta}_{nn} \end{Bmatrix}$$

If want to abtain the optimal point in the model such as equation 12，the first partial derivative at this point equal to zero is the necessary condition of the point existence. That is

$$\frac{\partial \hat{y}}{\partial x} = \begin{bmatrix} \frac{\partial \hat{y}}{\partial x_1} \\ \frac{\partial \hat{y}}{\partial x_2} \\ \vdots \\ \frac{\partial \hat{y}}{\partial x_n} \end{bmatrix} = \frac{\partial}{\partial x_1}\left(\hat{\beta}_0 + \boldsymbol{x}^T \boldsymbol{b} + \boldsymbol{x}^T \boldsymbol{B}\boldsymbol{x}\right) = \boldsymbol{b} + 2\boldsymbol{B}\boldsymbol{x} = 0 \tag{13}$$

Suppose $\boldsymbol{x}_0 = (x_{10}, x_{20}, \cdots, x_{n0})^T$ is stable point, then

$$\boldsymbol{x}_0 = \frac{1}{2}\boldsymbol{B}^{-1}\boldsymbol{b} \tag{14}$$

According quadratic RSM model equations of crashworthiness index, maximum crash force and mean crash force (equation 8, 11, 10), the stable points correspondingly are as following:

Crashworthiness index: $\boldsymbol{x}_0 = (-0.3197, -0.3575, -0.8663)^T$
Maximum crash force: $\boldsymbol{x}_0 = (1.5097, 8.1189, 2.8704)^T$
Mean crash force: $\boldsymbol{x}_0 = (-0.2923, 2.0204, 3.2750)^T$

After typical analysis, the above stable points are not extreme points. It is obvious that exclude the stable pint of crashworthiness index is in design space interval $[-1, 1]$, other stable points of optimal objectives are beyond the design space interval and each stable points of optimal objectives are not equal. So the proper method should be deal with the optimal solution and make the optimal objectives optimum totally.

In the condition of the design constraints satisfied, the crashworthiness index is most important for it directly indicate the crushing energy absorbed.

Figure 5.and figure 6. are the RSMs of crashworthiness index when $x_1 = -1$ and $x_1 = 1$. It is obviously that the value of crashworthiness index when $x_1 = -1$ is larger than the value of $x_1 = 1$ if x_2 and x_3 keep same values. So it can be concluded initiatively that the maximum value of crashworthiness index can be obtained when $x_1 = -1$. It also can be observed in table 7.
Figure 7 and Figure 8 are the RSMs of maximum crash force and mean crash force when $x_1 = -1$.

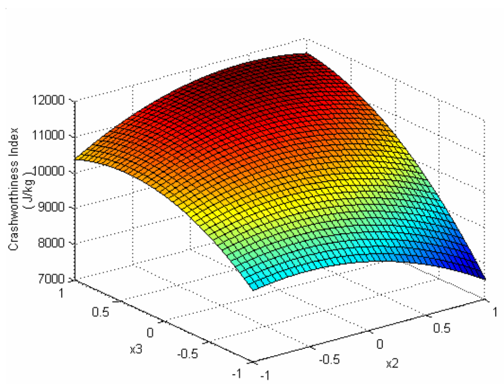

Figure 5. The Response Surface Model of Crashworthiness Index ($x_1 = -1$)

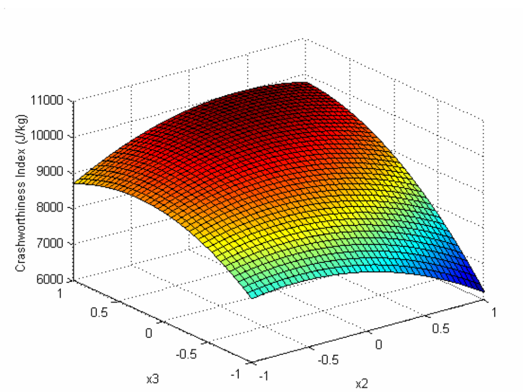

Figure 6. The Response Surface Model of Crashworthiness Index ($x_1 = 1$)

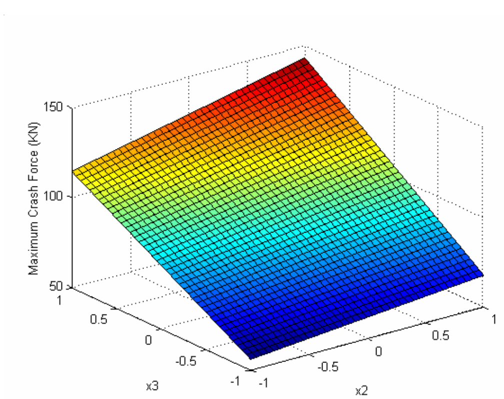

Figure 7. The Response Surface Model of Maximum Crash Force ($x_1 = -1$)

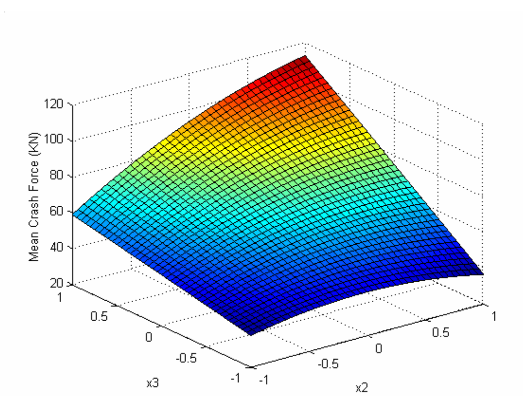

Figure 8. The Response Surface Model of Mean Crash Force ($x_1 = -1$)

Figure 9~11 are the contours of RSM of optimal objectives when $x_1 = -1$. It can be concluded initiatively that the maximum value of crashworthiness index is concentrated in the shadow in figure 9. Figure 10 shows that x_3 must be smaller than 0.4 to meet the constraint (equation 4). Figure 10~11 also indicate that the values of maximum and mean crash force are larger with improving the thickness of thin-walled sections (x_2, x_3).

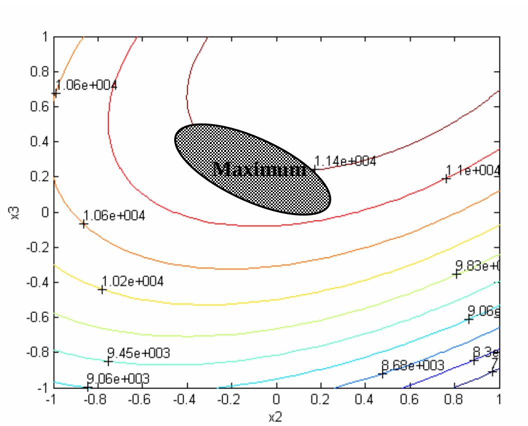

Figure 9. The Contour of RSM on Crashworthiness Index ($x_1 = -1$)

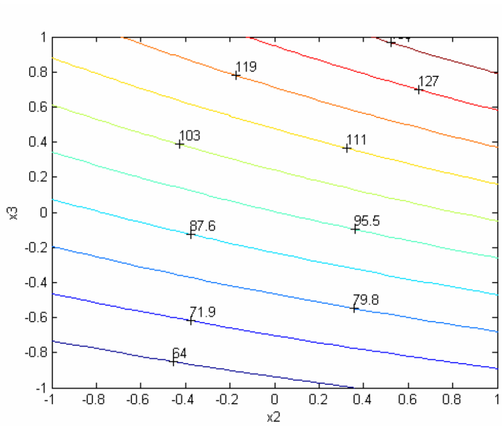

Figure 10. The Contour of RSMon Maximum Crash Force ($x_1 = -1$)

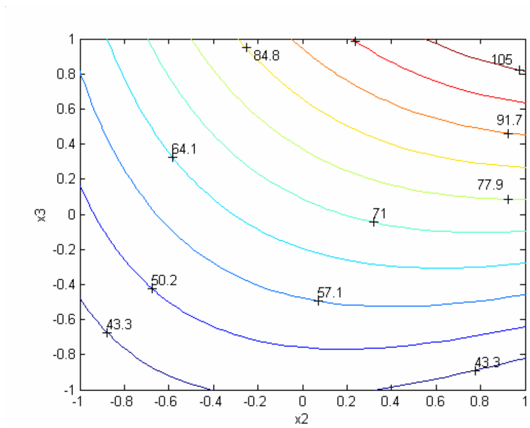

Figure 11. The Contour of RSM on Mean Crash Force ($x_1 = -1$)

Obtaining the Global Optimal Solution

Pareto GA is adopted to calculate the global optimal solution of RSM function. The initial population is 40; the solution set is 60; the evolution steps are 200; the crossover probability is 0.5 and the mutation probability is 0.008.

For the multi-objectives constraint optimal problem, GA can not be adopted to solve directly. One of methodology is convert the constraint problem to the non-constraint problem by penalty function. Traditional penalty function methodology is constructing evaluation function by adding the value of penalty function to the function value or multiplying with function value. The result obtained can not indicate how far the solved point distant with the region of feasible solution. In GA, the quality of solution is determined by its fitness. So the penalty function constructed should satisfy the following conditions: firstly, the status of point in or out the feasible region should be reflected by fitness function; secondly, the fitness of point is larger with it closer to feasible region; thirdly, the fitness of point is larger with it closer to the Pareto optimal solution set. According above requirements, the fuzzy penalty function is introduced in the Pareto GA as following:

Suppose the degree of the k th point violation to the i th constraint is d_{ki},

$$d_{ki} = \begin{cases} 0 & g_i(x_k) \le 0 \\ g_i(x_k) & otherwise \end{cases} \tag{15}$$

For the points violating the constraints, the penalty function should be add to the objective function corresponding. Suppose the maximum degree of the k th point violating to n constraints is $MaxD = max(d_{k1}, d_{k2}, \cdots, d_{kn})$, then the penalty function is as following:

$$R_k = \begin{cases} 0 & max\,D \le 0.001 \\ 2 & 0.001 < max\,D \le 0.01 \\ 3 & 0.01 < max\,D \le 0.02 \\ 4 & 0.02 < max\,D \le 0.05 \\ 5 & 0.05 < max\,D \le 0.1 \\ 6 & 0.1 < max\,D \le 0.4 \\ 7 & 0.4 < max\,D \le 1.0 \\ 8 & 1.0 < max\,D \le 2.0 \\ 9 & 2.0 < max\,D \le 5.0 \\ 10 & 5.0 < max\,D \le 15.0 \\ 100 & 15.0 < max\,D \end{cases} \tag{16}$$

In Pareto GA, the floating numbers are coded to election, crossover and mutation. It's improving the running speed of GA. In crossover operation, simple crossover, heuristic crossover and arithmetic crossover are all adopted at the same time. In mutation operation, boundary mutation, uniform mutation, nonuniform mutation and multinonuniform mutation are adopted. The effect of crossover and mutation can be realized sufficiently by above treatment and premature convergence can be avoided. The election mode is Roulette (or named fitness ratio method).

In Table 7, ten global optimal solutions are obtained by Pareto GA.

Table 7. Ten Global Optimal Solutions Obtained by Pareto GA

No.	x_1	x_2	x_3	η_c (J/kg)	P_{max} (kN)	P_m (kN)
1	-1.000	-0.074	0.054	11164.18	96.42	69.09
2	-1.000	-0.383	0.112	11134.29	95.11	64.89
3	-1.000	-0.263	0.015	11083.10	93.22	65.21
4	-1.000	-0.355	0.045	11083.02	93.28	64.16
5	-1.000	-0.355	0.015	11050.41	92.29	63.56
6	-1.000	-0.383	0.015	11038.21	92.01	63.04
7	-1.000	-0.390	-0.015	11035.12	91.95	62.91
8	-1.000	-0.336	-0.077	10950.26	89.55	62.09
9	-1.000	-0.671	0.107	10927.93	91.99	58.42
10	-1000	-0.455	-0.084	10892.97	88.17	59.84

CONCLUSIONS

In this research, the response surface method and Pareto Genetic Algorithm is developed to structural crashworthiness optimization of the thin-walled sections in automotive body. The relations between the section parameters and crashworthiness property are observed based on the numerical solutions. As evidenced by the result in the numerical simulation, the above methodologies show promise in implementing crashworthiness optimization.

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