# STRUCTURAL OPTIMIZATION USING SPACE MAPPING AND SURROGATE MODELS

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**Abbreviations**

RSM Response Surface Methodology
SM Space Mapping
FE Finite Element

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## ABSTRACT

The aim of this paper is to determine if Space Mapping technique using Surrogate Models in combination with the Response Surfaces Methodology (RSM) is useful in optimization of crashworthiness applications. In addition, the efficiency of optimization using Space Mapping will be compared to conventional structural optimization using the Response Surface Methodology (RSM).

To determine the response surfaces, several evaluations must be performed and each simulation can be computationally demanding. Space Mapping technique uses surrogate models, i.e. less costly models, to determine these surfaces and their associated gradients with respect to the object and constraint functions. The original full model is used to correct the gradients from the surrogate model for the next iteration. Thus, the Space Mapping technique makes it possible to reduce the total computing time, needed to find the optimal solution.

Two application problems are used to illustrate the algorithm. All examples are constrained optimization problems with one or two design variables.
In all applications, the algorithm converged to the optimum solution. For the crashworthiness design problems the total computing time for convergence was reduced with 53% using Space Mapping compared to the conventional RSM.

The conclusions are that optimization using Space Mapping and Surrogate Models can be used for optimization in crashworthiness design with maintained accuracy but with a significant reduction in computing time compared to traditional RSM.

## INTRODUCTION

By varying selected design parameters in a model and rerunning a simulation of its behavior, the designer can evaluate new objective and constraint values. Eventually, the influence of the different design parameters will be understood and a better design can be developed. However, a mathematical optimization algorithm would be more efficient and helpful.

The use of structural optimization has increased rapidly during recent years, mainly due to faster computers, better algorithms and more frequent use of Finite Element (FE) simulations. Optimization is a useful tool to improve the design in a well-structured manner.

Structural optimization often uses gradients of the objective and constraints to find a search direction of the optimal solution. For dynamic problems, like impact problems, the solution functions are often noisy and it is hard to find these gradients.

In the Response Surface Methodology (RSM) polynomial surfaces are fit to objective and constraint values in the design space. Due to the construction of these surfaces noisy or unphysical components of the response will be smoothened out. The optimal solution is then searched on these smoothed surfaces, rather than on the real response surfaces. For further reading about RSM, see Myers and Montgomery (1995).
Until recently, only a few attempts have been made to use optimization methods in crashworthiness design problems. Application examples can be found in e.g. Yamazaki and Han (1998), Marklund (1999), Schramm and Thomas (1999), Sobieszczanski-Sobieski et al. (2000), Schramm (2001).

Even if the number of object and constraint evaluations is low, the computing time to evaluate each design can be distressingly long. There is a need for methods where simplified models can be used for most evaluations such that the number of full model evaluations is minimized. The simplified models can be constructed using a coarse mesh, simplified theoretical models, approximate analytic solutions etc.
One method which makes this possible is called Space Mapping where a surrogate model complements the full model. The surrogate model (coarse model) determines the search direction and the full model (fine model) will determine the design point for the next iteration. The use of the coarse model makes it possible to reduce the total computing time and the fine model assures an accurate solution.

The first Space Mapping paper was written by Bander et al. (1995) where the basic theory was stated. In Bandler et al. (1995) the problem was formulated for using Broyden's method for non-linear equations and Bakr et al. (1998) introduced a trust-region methodology. A mathematical viewpoint of space mapping can be found in Madsen and Sondergaard (2000).

Space Mapping and surrogate models have until recently been utilized in electromagnetic and circuit optimization, see e.g. Bandler et al. (1994) and Bakr et al. (2000). Leary et al. (2000) used Space Mapping in structural optimization of a simple cantilever beam.

The aim of this work is illustrate how Space Mapping technique using surrogate models together with response surfaces can be used for structural optimization of crashworthiness problems.

## THEORY

In this paper a combination of RSM and Space Mapping technique have been used. Relevant details of the two methods are presented in the following sections.

*Response Surface Methodology*
The Response Surface Methodology is a method for constructing global approximations of the objective and constraint functions based on functional evaluations at various points in the design space. The strength of the

method is in applications where gradient based methods fails, i.e. when design sensitivities are difficult or impossible to evaluate.

The design domain is the space spanned by the design variables, i.e. $\{x_1, x_2, ..., x_i\}$. Introducing limits on the design variables separate from the global limits can further narrow the design domain. This creates a sub-domain called the region of interest, where the approximations are calculated. When the optimum is found, the region of interest is moved in the indicated direction during the next iteration and the optimization continues.

The selection of approximation functions to represent the actual behavior is essential. These functions can be polynomials of any order but can also be the sum of different basis functions, e.g. sine and cosine functions. For a general quadratic surface approximation the function will be,

$$y^i = \beta_0 + \sum_{j=1}^{n} \beta_j x_j^i + \sum_{j=1}^{n}\sum_{k=1}^{n} \beta_{jk} x_j^i x_k^i + \varepsilon^i \tag{1}$$

where $\beta_j$ are the constants to be determined, $x^i_j$ are the design points in the region of interest, $\varepsilon^i$ includes both bias errors and random errors and N the number of evaluations. Obviously, the minimum number of function evaluations $N_{min}$ is equal to the number of unknown constants $\beta_j$ To determine the unknown coefficients $\beta_j$, Eq (1) is transformed to matrix notation

$$\mathbf{y} = \mathbf{X} \cdot \boldsymbol{\beta} + \varepsilon \tag{2}$$

In order to minimize the error $\varepsilon^i$, a least square approach is used to find the estimates of $\beta_j$. These values of $\beta_j$ are denoted **b**. Thus,

$$\boldsymbol{\beta} = (\mathbf{X}^t\mathbf{X})^{-1}\mathbf{X}^t\mathbf{y} \tag{3}$$

This approximation is applicable for all types of objectives and constraints in the optimization problem.

*Space Mapping*

The idea of Space Mapping is to use two models for optimization. One fine model that has a high accuracy but is computationally expensive to solve and one coarse model that is fast to solve but has less accuracy. The Space Mapping algorithm takes advantage of the short solution time of the coarse model and the accuracy of the fine model.

Therefore the vast amount of function evaluations is performed on the coarse model and a just few corrections are made with the fine model. The following theory for Space Mapping mainly follows Leary et al. (2000).

The design parameters in the fine model is denoted **x** and the objective function value $f_e$. In the coarse model these parameters are denoted **z** and $f_a$ respectively. The optimum solution of the fine model is denoted,

$$x^* = \arg\min_x f_e(x) \tag{4}$$

and the optimum solution of the coarse model is denoted

$$z^* = \arg\min_z f_a(z) \tag{5}$$

Obviously **z** has the same dimension as **x**. A residual is always present and can be defined as

$$r(x,z) = | f_e(x) - f_a(z) | \tag{6}$$

A mapping function is defined that minimizes the residual,

$$p(x) = \arg\min_{z} r^2(x,z) \tag{7}$$

An illustration of the mapping function is shown in Figure 1. From the definition of the mapping function it follows that

$$f_a(p(x)) \approx f_e(x) \tag{8}$$

The function value of the fine model is approximated with the coarse value when the mapping Eq. (6) is used as argument. Hence the coarse model can be used to minimize the fine model in Eq. (4). A perfect mapping is defined as the case when $\mathbf{z}^*$ satisfies $\mathbf{z}^*=p(x^*)$.

The mapping p is sequentially approximated using linear approximations $p_k$ around the current set of parameters $\mathbf{x}_k$. The approximation is given by

$$p_k(x) = z_k + B_k(x - x_k) \tag{9}$$

Where $B_k$ is an approximation of the Jacobian of the mapping function. Following Madsen and Sondergaard (2000), we use the Broyden's update

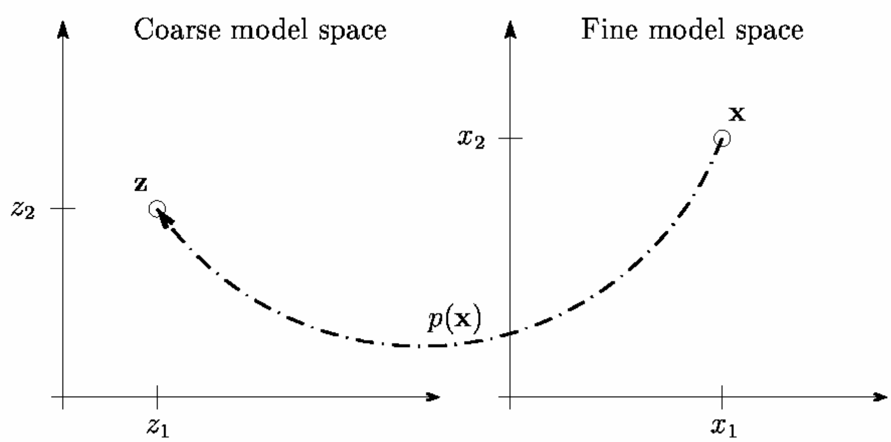


**Figure 1 Illustration of Space Mapping**

$$B_{k+1} = B_k + \frac{z_{k+1} - z_k - B_k h_k}{h_k^t h_k} h_k^t \tag{10}$$

where $h_k = x_{k+1} - x_k$. The parameters $z_k$ come from Eq. (7) and hence they satisfy $z_k=p(x_k)$. Since the linear mapping only is valid in a neighborhood of $x_k$, a trust region is introduced. Hence linearization is only accepted for

$$\{x : |x - x_k| < \delta_k\} \tag{11}$$

where $\delta_k$ is the size of the trust region in step k. The trust region is updated according to

$$\delta_{k+1} = \begin{cases} \gamma \cdot \delta_k \text{ if } \gamma > 0.5 \\ \gamma \cdot 0.5 \text{ if } \gamma < 0.5 \end{cases} \qquad (12)$$

where $\gamma = (|x_{k+1} - x_k|)/\delta_k$

## WORKING SCHEDULE

Our implementation of the Space Mapping algorithm is based on a constrained optimization. Further implementation details are given below and the working schedule is shown in Figure 3. The algorithm starts by evaluating the coarse model with mapped parameters at the D-optimal points. The RSM optimization code LS-OPT, see (Stander 1999), is used to produce the design of experiments and to extract the results. These results are then used to approximate the response surfaces according to Eq. (3) for the objective $f_a^{RSM}(z)$ and for each constraint $\sigma_a^{J,RSM}(z)$. To find the next set of fine parameters $x_{k+1}$ an optimization is done with the approximated surfaces from the coarse model. The mapping function $p_k(x)$ is used as parameter in the optimization. Next the fine model is evaluated in $x_{k+1}$ to get the objective value $f_e(x_{k+1})$ and the constraint values $\sigma_e^{conJ}(x_{k+1})$. The residuals between the extracted values and the values from the approximated surfaces of the coarse model are minimized to find the next set of coarse parameters $z_{k+1}^{obj}$ and $z_{k+1}^{conJ}$. Finally the matrices $B_{k+1}^{obj}, B_{k+1}^{conJ}$ and the trust regions for the fine and coarse parameters are updated. All optimization is performed in LS-OPT and all function evaluations in our applications have been done with the FE code LS-DYNA, see (Hallquist 1999).

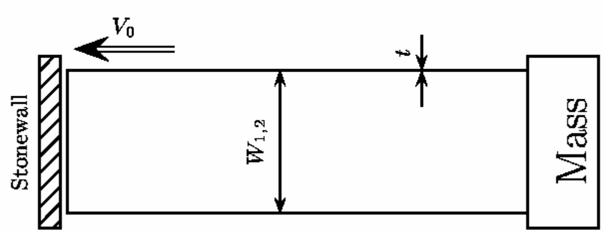


Figure 2 Geometry of square tube

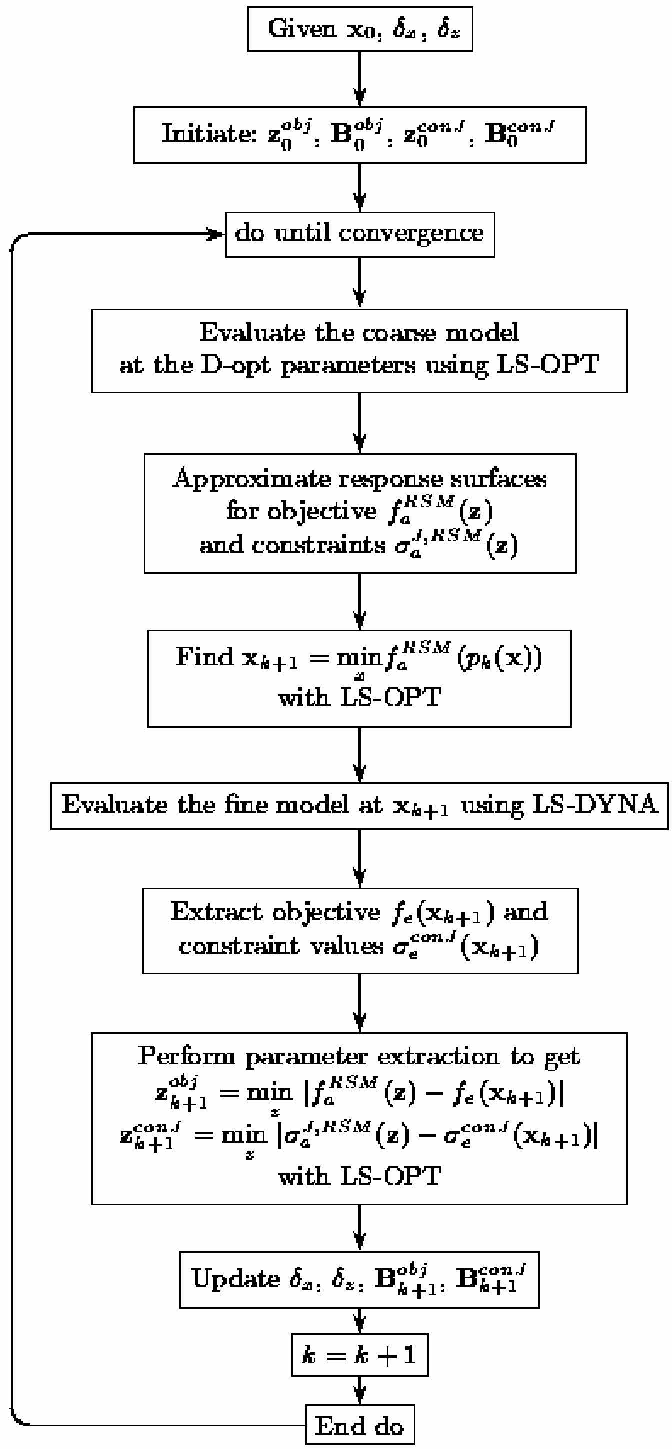


**Figure 3 Working schedule for Space Mapping with constraints.**
**Here J=1,..., number of constraints**

## EXAMPLES AND RESULTS

In this section the optimization algorithm using Space Mapping is tested and compared to traditional RSM. Two application examples were developed, out of which one was optimized using both the present Space Mapping algorithm and the traditional RSM. LS-OPT was used for RSM with linear surfaces (RSM-L) and quadratic surfaces (RSM-Q). For all examples the same initial conditions (i.e. starting point and design space) were chosen for Space Mapping, RSM-L and RSM-Q. All problems are described below and the design variable history is given for the results using Space Mapping. The optimum solution and the total computing time are given for results both from Space Mapping and from traditional RSM. The functional evaluations have been done with LS-DYNA.

*Minimum mass of a square tube subjected to impact*
A square tube has a rigid mass attached to its rear end. The tube and the rigid mass have an initial velocity and impacts into a rigid wall, see Figure 2.

*Objective:* The objective is to minimize the mass of the tube excluding the rigid mass with a constraint on the total displacement at the rear end. The design variables are the sheet thickness (th) and the width (w) of the square tube, see Figure 2. The optimization problem is formulated as,

$$
\begin{aligned}
&\min_{tH,W} \quad \text{mass} \\
&\text{s.t.} \quad -\infty < \text{disp} < 110 \text{ mm} \\
&\qquad\quad 1.0 < \text{t1} < 2.0 \text{ mm} \\
&\qquad\quad 55 < \text{w} < 65 \text{ mm}
\end{aligned}
$$

*Fine model:* The responses of the tube are evaluated using LS-DYNA. The FE model consists of 2976 shell elements and the computing time was approximately 70 minutes.

*Coarse model:* In this case the coarse model uses the same FE mesh as the fine model, but an un-physically larger density is used for the tube material. The critical time step for the time integration scheme in explicit FE methods is inversely proportional to the square root of the mass of the elements in the model. By increasing the density of the material the critical time step is increased and the total computing time is lowered. However, mass scaling introduces errors and the coarse model will not yield a correct solution. The computing time for the coarse model was approximately 7 minutes.

The tube mass used as objective in the optimization is the unscaled mass. It will be identical both in the fine and the coarse model. This is not the case for the constraint. Typical displacements of the attached mass for the fine and the coarse models are given in Figure 4.

To be able to compare the different optimization algorithms the same starting design and region of interest were chosen. All three methods converged to the optimum point for the square tube problem. The total cost were 32.6, 18.7 and 8.87 hours computing time for optimization with RSM-Q, RSM-L and Space Mapping, respectively. The optimization algorithm using Space Mapping reduces the computing time compared to RSM-L with 53 %, see Table 1. The design variable history using Space Mapping is given Figure 5 and Table 1.

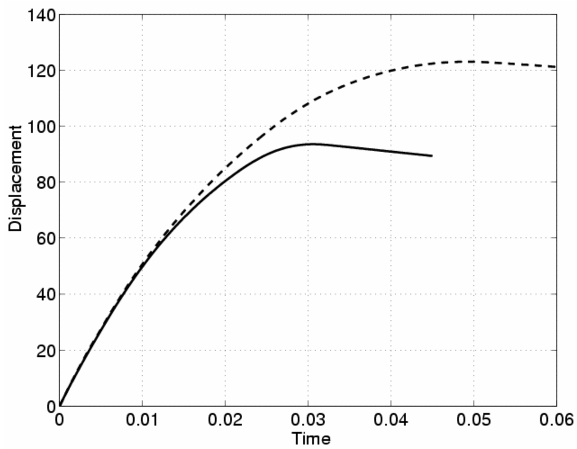


**Figure 4 Typical displacements of the mass of the square tube as function of time. The maximum value is used as constraint (solid=fine model, dashed=coarse model)**

**Table 1 Parameter (x*), objective (f*) and constraint (σ*) results for optimization using traditional RSM and Space Mapping. Cost in hours computing time**

| | $x^*$ | $f^*$ | $\sigma^*$ | iter | fine | coar | cost |
|---|---|---|---|---|---|---|---|
| **RMS-Q** | 1.347;55 | 0.777 | 110.0 | 3 | 28 | 0 | 32.6 |
| **RMS-L** | 1.349;55 | 0.778 | 109.7 | 3 | 16 | 0 | 18.7 |
| **SM** | 1.395;55 | 0.775 | 110.4 | 4 | 4 | 36 | 8.87 |

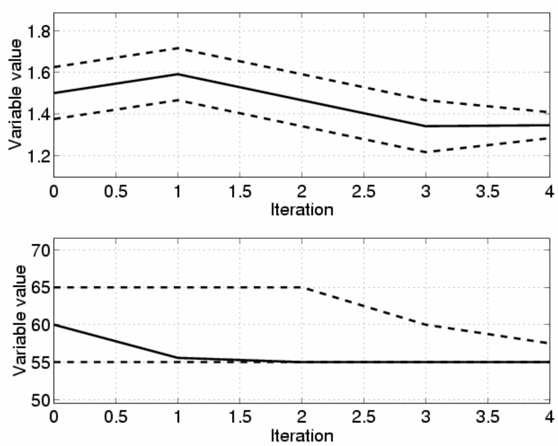


**Figure 5 Design variable history for the square tube problem using Space Mapping (solid=design variable history, dashed=limits)**

*Minimum mass of a vehicle front structure*
Saab Automobile AB provided a complete FE model of a vehicle. From this full model two submodels were developed one fine and one coarse model.

*Objective:* The sheet thickness (th) in the front rail, see Figure 7, is the design variable in this example. The objective is to minimize the mass while the maximum section force in the rail must be larger than 42 kN. A typical behavior of the section force for the fine and coarse models can be found in Figure 8. The following optimization problem can be formulated,

$$\begin{aligned} &\min_{th} \quad \text{mass} \\ &\text{s.t.} \quad 42 < \text{secforce} < \infty \text{ kN} \\ &\qquad\; 1.2 < \text{t1} < 2.0 \text{ mm} \end{aligned}$$

*Fine model:* The fine model is a symmetry part of the vehicle front, where the rear end is fixed and a stonewall impacts into the front, see Figure 6. The model consists of 51213 elements and 99 parts and the computing time is approximately 12 h.

*Coarse model:* In the coarse model only the most important parts are included and the number of elements was 15955 and the number of parts 20, see Figure 7. The computing time for the coarse model was approximately 2 h.

In the coarse model many load-carrying structures are eliminated and the front rail may not collapse in a correct way. Therefore boundary conditions are extracted from certain interfaces of the fine model. These boundary conditions are then used as prescribed displacements and velocities in the coarse model during the whole simulation. These boundary conditions are updated every iteration when the fine model is simulated. This method of local refinements is often called reanalysis.

*Results:* Due to the excessive computing time of the fine model, this problem is not evaluated with traditional RSM optimization. Only the objective, constraint and design variable history from the optimization are given as results from the Space Mapping optimization.

**As shown in Figure 9 the Space Mapping algorithm converges to the optimum solution within five iterations.**

Table 2 shows that the mass in the front rail is reduced from 1.65 kg to 1.45 kg and the cross section force is 42.7 kN which is close to the constraint value of 42 kN.

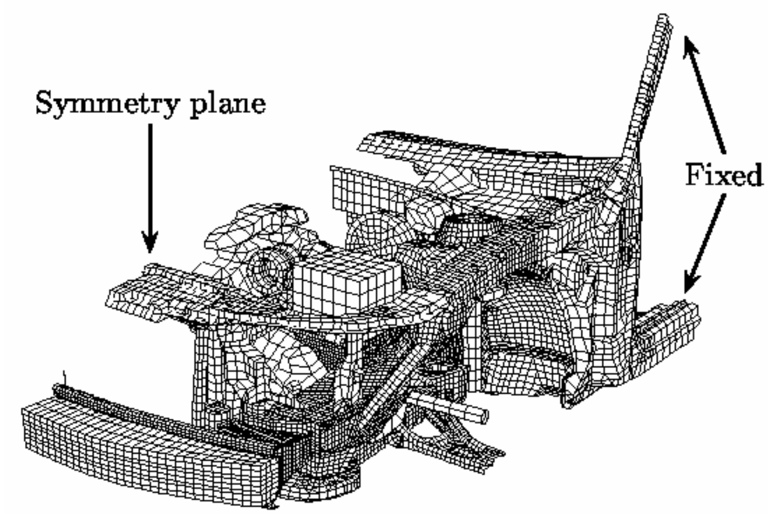


**Figure 6 Vehicle frontal structure. Fine model**

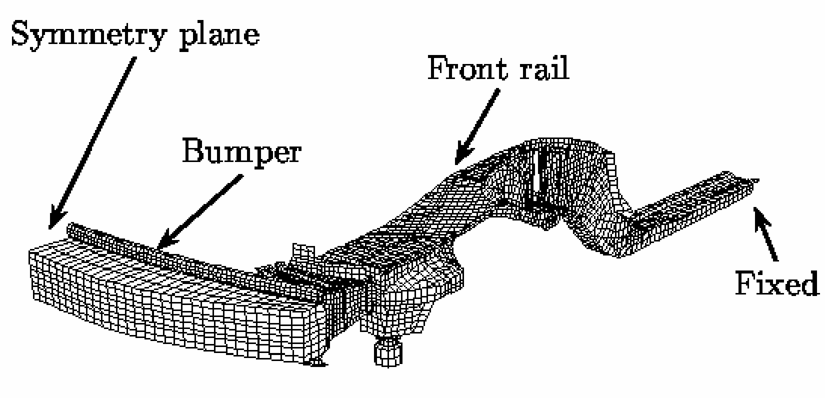


**Figure 7 Vehicle frontal structure. Coarse model**

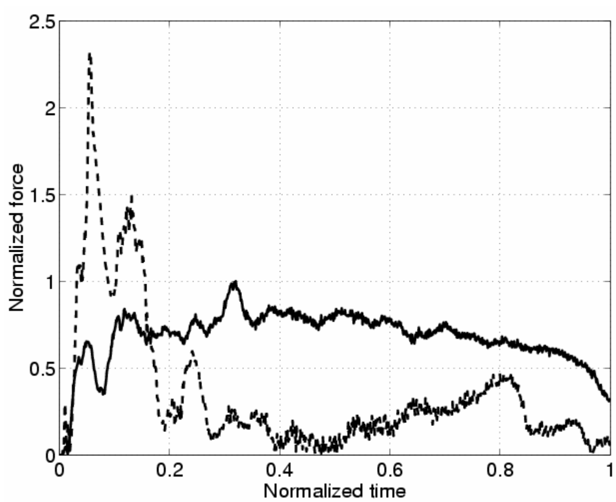


**Figure 8 Typical behavior of the section force as function of time.**
**The maximum value is used as constraint (solid=fine, dashed=coarse model)**

**Table 2 Design variable, objective and constraint history for the vehicle front structure optimization using Space Mapping**

| Iteration | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $x_k$ | 1.650 | 1.568 | 1.490 | 1.475 | 1.436 | 1.450 |
| $f_k$ | | 2.295 | 2.846 | 2.819 | 2.742 | 2.751 |
| $\sigma_k$ | | 44508 | 43425 | 43578 | 42318 | 42702 |

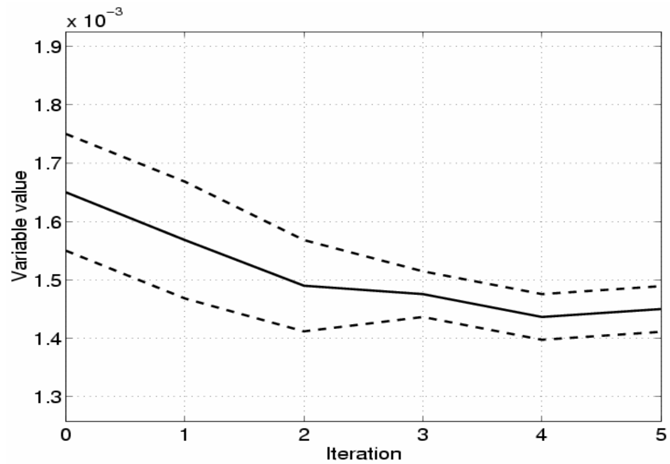


**Figure 9 Design variable history for the vehicle front structure optimization using Space Mapping (solid=design variable history, dashed=limits)**

## CONCLUSIONS

The optimization algorithm using Space Mapping has shown to be well suited for problems in crashworthiness design and sheet metal forming. All examples converged to the correct optimum and the computing time has been decreased with a maximum of 53% relative to traditional RSM optimization.
The square tube problem with two design variables converged using the Space Mapping optimization in 8.87 hours, which was a 53% reduction in computing time compared to the fastest traditional RSM optimization method. The coarse model was very simple to set-up when the fine model was developed. Only one small modification to the fine model to include mass scaling was introduced. The constraint in this optimization was the displacement of the rear end. The differences in the results from the fine and coarse evaluations were rather large, but still the algorithm converged.

The Space Mapping algorithm also converged for the vehicle impact problem. The computing time for each evaluation was decreased from 12 h to 2 h, when the coarse model was used. If a complete vehicle model should be optimized the time saving can be much larger than in these examples, since the simulation time for a full vehicle model today can be more than 100 h. The algorithm using Space Mapping can then significantly reduce the total simulation time.

Generally the optimization algorithm using Space Mapping worked well for all problems and the CPU time was often significantly reduced. This time saving must also be related to the time it takes to produce a coarse model. If the model building is time consuming, perhaps it is most time efficient to use a traditional RSM optimization. This must be taken into account when an optimization method is chosen
.
The algorithm using Space Mapping seems to be less stable compared to the traditional RSM method, e.g. if a bad starting design is chosen Space Mapping might not converge due to a too bad mapping between the fine and coarse models. The starting design must be 'intelligently' chosen based on knowledge on how the Space Mapping technique works and how the model behaves for parameter changes.

The Space Mapping method needs to be tested on larger examples with more design variables and more constraints. In addition a recommendation must be established on how much the results from the models can differ and how to choose the starting design. Other coarse models e.g. analytic solutions or other less computing time demanding solvers can also be used.

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