JANUARY
2005

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VIRTUAL PRODUCT DEVELOPMENT

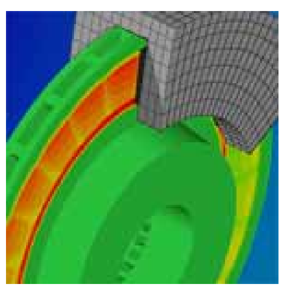
MILITARY-STYLE

Page 3

FROM THE ARCHIVES

DISC BRAKE ANALYSIS

Page 21

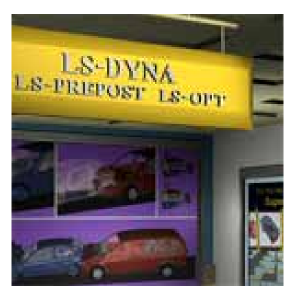


PLUS:

PRODUCT SPOTLIGHT

LSTC'S LS-DYNA

Page 17



FEA INFORMATION RESOURCE MAGAZINE

FEA Information Worldwide Participant's

Contents

FEA Information Announcements

Welcome to the New Year. Entering our 5th year of publishing we have a new format. We would like to thank Stephen Cameron, of Cameron Design, for his cover design and new format. We've introduced new features such as our Cover Story featured on our front cover. Additionally we've expanded our Asia News Area and added an LS-DYNA Resource Page.

Cover Featured Story:

FORENSICS PROJECT LEADS MILITARY TO GREATER USE OF VIRTUAL PRODUCT DEVELOPMENT, by MSC.Software

Featured Article

The Total and Updated Langrangian Fomulations by David J. Benson, Dept. of Mechanical and Aerospace Engineering, University of California, San Diego

Publication reminder

OOP-Simulation – A Tool to Design Airbags? Current capabilities in Numerical Simulation, is archived on www.feapublications.com. The publication is by Andreas Hirth (DaimlerChrysler, Sindelfingen);Benno Beesten, Robert Reilink (Volkswagon, Wolfsburg); Rolf Remensperger, (Porsche, Weissach); Doris Rieger (BMW, Munich) Gunther Seer (Audi, Ingolstadt)

Welcome New Participant:

Beijing Yuntong Forever CPC. Co. Ltd of China, sole distributor of LINUX NETWORX, INC (U.S.A.) in China

Cover Design:

Cameron Design
www.cameron-design.com

Sincerely,
Trent Eggleston & Marsha Victory

Cover Story:

FORENSICS PROJECT LEADS MILITARY TO GREATER USE OF VIRTUAL PRODUCT DEVELOPMENT

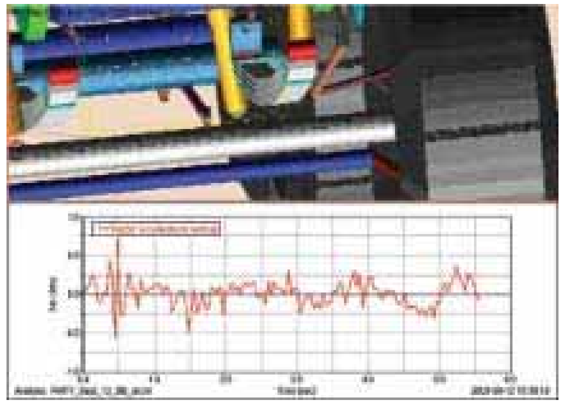
Radian Identifies Military Truck Field Failure Cause with MSC.Adams, Resulting in Solid Model Technical Data Package of Military Truck System for Extended Collaboration

Fort Stuart's parachute drop course

When the M1088/M871A2 truck trailer combination was put into field service at Fort Stuart's parachute drop course, the unusually hilly terrain caused a severe problem with the clearance of the tractor rear end and trailer strike plate. Field sustainment firm, Radian Inc. was contracted to find the cause and develop a field kit to correct the problem. Although the design information was in the form of 2D prints, the information was converted to digital format allowing the use of MSC.Adams Virtual Product Development tools for dynamic analysis. This allowed a much faster determination of the cause and the ability to test the solution virtually to gain a better understanding of the problem before building a prototype.

"We started in the middle of the acquisition cycle, the vehicle had already made it through testing and was in field service when the problem came up," said Victor Kowachek, senior engineer, Radian Inc. "We didn't have the luxury of starting with digital data, just a Technical Data Package, which included 2D paper drawings."

When the problem first occurred, a Radian engineering team was sent to Fort Stuart. The team used instruments to measure the interference and forces that resulted in the frame bending just forward of the fifth wheel location. From this information, finite element (FE) models were created.



Using static FEA, critical stresses were identified, but predictions were required of the vehicle in dynamic mode. Mr. Kowachek said, "We chose MSC.Adams for dynamic kinematic analysis. Using a system level dynamic model created from the FE model, the terrain was run underneath the truck and trailer to determine the forces. From the dynamic analysis, we generated a tremendous amount of information. We created system level dynamic models that we could use later for extended collaboration. Within six weeks, kits for fixing the problem were available in the field. Eventually the kit was incorporated into the next generation truck

The vehicle system will be in the U.S. Army inventory for the next 20 to 30 years.

The vehicle system will be in the U.S. Army inventory for the next 20 to 30 years and during this time there will be many changes and improvements. Continuing to use 2D paper drawings would increase the time and cost of making any changes. As a result of its forensics and repair kit for the vehicle, Radian was contracted to complete a Solid Model Technical Data Package (TDP) of the truck system. In addition to the TDP, a complete Configuration Management (CM) electronic format was also completed.

"Based upon the TDP and CM, the acquisition cycle can be transformed into the modern virtual world with time efficiencies never before achieved in the military ground vehicle area," said Mr. Kowachek. "We defined form, fit and function in the 3D world. Using our 3D solid models, the military can make quickly and efficiently field fixes for the truck system."

Often, U.S. Army vehicles in service are located in a war zone or other difficult to reach locations, so getting parts is extremely difficult. Using what is called a mobile parts hospital, repairs and parts can be made in the field. "For this truck system, we can send them digital information and they can manufacture parts right on the spot," said Mr. Kowachek.

Improvements to the truck system are continuing to be made, including an armor pack, currently being working on by Radian. Because the armor pack completely changes the vehicle characteristics, MSC.Adams kinematic analysis of dynamic models had to be run all over again. Since the digital data was already available, the time and cost of testing the change was reduced.

"The ideal methodology toward *zero prototype product development* is to start with a clean computer screen," said Mr. Kowachek. "However, sometimes the only way is to simply stop, dig into the virtual world anywhere in the product cycle, and expand over a period of time in an efficient manner until zero prototype product development is achieved. The ultimate goal may never be absolutely obtained, but engineering/collaborative product lifecycle management will accelerate to multifold efficiency levels before unknown."

LIGHTS! CAMERA! COMPUTE!

Animators put HP Labs' utility computing technologies to the test

By Simon Firth © Copyright
http://www.hpl.hp.com/news/2004/oct_dec/3Danimation.html

What's exciting is that we are using artists and animators to shape the technology

HP researcher Steve Hinde is getting to be something of a movie mogul.

Back in 2003, when Hinde was looking to test the limits of utility computing, he commissioned a four-minute animated film, "The Painter," to be made using a prototype Utility Rendering Service created by Hinde and his colleagues at HP's Bristol, UK Labs.

Now Hinde is looking to push utility computing further by commissioning 10 short films to be rendered, simultaneously, with the aid of a new version of the HP service.

Rendering is the addition of fine detail into each frame of a 3D animation. It takes a huge amount of computer processing power to produce the high-quality images audiences have come to expect from movies like "Shrek" and "Shark Tale." That makes it ideal for probing the future of utility computing – where users "rent" processing power on an as-needed basis from a remote location rather than owning it themselves.

The new project, named SE3D (pronounced 'seed'), was officially launched this week (17-21 November) at the Brief Encounters Short Film Festival in Bristol, UK. It will be managed by the Watershed media arts center and is co-sponsored by HP and the 3D animation software company Alias Wavefront, whose Maya 3D software the filmmakers will use.

Still from* Two Fellas *by Dan Lane

Dynamic utility computing environment

The 10 animators, selected by a panel of industry heavyweights, now have four months to use Maya and HP's Utility Rendering Service to create their 3D shorts. All 10 films will be showcased in April 2005 at Animated Encounters, an international film festival that Matt Groening, creator of "The Simpsons," has described as "the best place to be if you want to see the best new cartoons and meet the geniuses behind them."

While "The Painter" involved a single user on the rendering service, the 10 filmmakers in the SE3D project will be making more complex demands on the technology.

"To really investigate the next stage of research in terms of utility service provisioning you need lots of users," explains Steve Hinde, "so we found this scenario that has people using the service in a sustained and realistic way over a period of months."

It's an idea that offers HP an umbrella under which to test an unusually wide range of new technologies associated with utility computing.

Still from* Peanut Pete - Getting Out *by The Box

But Hinde – as befits his new status as a movie producer – sees additional promise in the SE3D project. Thanks to the attention it is receiving and to the quality of the work likely to be produced, SE3D "promises to help creativity and technology to come together in a new and exciting way," he says.

Potential to transform 3D animation

One person with high hopes for the SE3D project is Bristol-based producer and directer Ben Lock, whose surreal 'mini-epic' set in Depression-era America – "Elroy III, the Potato Head Boy" – will be one of the 10 films made. He says SE3D offers animators like him a rare opportunity.

Typically, says Lock, "you have in your head what you want on the screen, but you have constant limitations. And the biggest limitation is how much you can render."

And not even the big guns always have enough rendering capacity – in early 2004 HP worked with DreamWorks, for example, to create a 1,000-server farm at its Palo Alto, California, lab to act as an extension of DreamWorks' own data center. That gave the studio a vital 50 percent more capacity for the final stages in the making of its recent hit, "Shrek 2."

Lock sees a service such as that offered by the SE3D project as changing that equation.

"Suddenly," he predicts, "you'll unleash all of this creativity that's been held back because people haven't been able to realize their vision through the rendering."

Connect from anywhere

Not only could utility computing open up 3D animation to individual animators, but since the service is provided at a remote location, it allows those animators to be based anywhere they want.

"Now, with a decent Internet connection, there's a whole leveling of the playing field," says Mike Kirwin, another film maker chosen to be a part of SE3D, who is based in the city of Manchester, North West England.

Because Manchester is a small media market, most of Kirwin's work has been commercial. His SE3D film, though, is very much a creative labor of love that he has spent the last four years trying to get made.

Thanks to SE3D, he says, "who knows where it might lead?" noting that plenty of animation classics, from "The Simpsons" to "South Park" to "Toy Story" first appeared as one-off shorts.

Managing the economics of IT

If some of the SE3D animators are hoping the project might offer bigger opportunities down the road, HP researchers are looking for a more immediate payoff.

Most significantly, SE3D offers the chance to test a broad range of HP Labs technologies for utility computing in a real-world setting, and all at the same time.

"We frequently do small-scale integrations of our technologies in Labs," says Peter Toft, the program manager behind SE3D at HP Labs. "But we rarely have the chance to bring together this large a set of technologies.

"We've created what we call the Service Utility," says Toft. "This is a platform that allows us to run many instances of utility services, and allows them to share the available resources."

A key feature of the Service Utility, he notes, is a set of market-based approaches for determining which services get which resources. The market-based mechanisms, called Sumatra and Tycoon, allow users to purchase resources for immediate use as well as purchase resources in advance for use at a future time.

Numerous technologies working together

Coupled with this is a technology called Management by Business Objectives (MBO), which helps make optimized choices about resource allocation when unforeseen circumstances result in resource scarcity.

"Imagine you have promised resources to a team of animators and five machines fail, so you are unable to meet that promise," Toft says. "How do you then rebalance the promises across the set of different services running on the utility?"

MBO, he explains, helps the system efficiently rebalance resources based on what promises have been made, how much each service is paying for resources, and any penalties that the Service Utility might incur as a result of failing to meet promises.

The Utility Rendering Services that animators interact with runs on top of the Service Utility. The rendering service is a fully featured remote service – including a technology called Elephant Store, for efficient storage of multiple versions of animation input data, as well as efficient use of network bandwidth when communicating with the remote service.

Underpinning all of these software components, is a technology called SmartFrog that automatically deploys and manages both the Service Utility and Utility Rendering Service.

"SmartFrog is at the heart of the system's ability to allocate resources and to cope with failure," Toft explains.

Artists shaping new technologies

The unusual SE3D experiment has attracted widespread media interest, including The Discovery Channel, which is following two animators as they make their film.

That attention, along with a high-powered advisory board made up of some of the biggest names in the UK animation industry (including David Sproxton, Executive Chairman of Aardman Animations, Shelley Page, European Representative for DreamWorks and Paul Appleby of the BBC) has researchers hoping that SE3D will both push the cutting edge of utility computing and offer a new model of how creativity and technology can come together.

"What's exciting is that we are using artists and animators to shape the technology that will go on to become industry standard," says Clare Reddington, SE3D coordinator. Artists will have the opportunity to use the technology creatively and at the same time, they'll provide feedback to programmers about how well it works and how they think it should work.

With the launch of SE3D, animators are now getting down to making their films. Even with a huge render farm on tap, digital animation remains a painstaking business, and no one is under any illusion about how much work there is to be done.

"Even though we're only doing about four minutes of animation," says Manchester's Mike Kirwin, "it's going to be a long four months to try and complete it."

BAE SYSTEMS gets a flying start with CSC, an IBM Premier Business Partner

"With the pSeries solution, we have also seen improvements in the speed of reporting from the Baan system, and our recurring support costs are significantly lower."

Steve Heywood, BAE SYSTEMS

The Challenge	Improve performance and capacity of key business systems, reduce recurring hardware and software support costs, and create a scalable solution capable of supporting a growing business
The Solution	Worked with IBM Premier Business Partner CSC to replace more than ten application servers with two partitioned IBM @server pSeries 670 servers running IBM AIX 5.1
The Benefits	Immediate reduction of recurring support costs; significant improvement in speed of batch processing; server consolidation brings simplified hardware management and support; improved scalability, and ability to respond to peaks in demand by switching on additional processors

BAE SYSTEMS is an international company engaged in the development, delivery and support of advanced defense and aerospace systems in the air, on land, at sea and in space. The company designs, manufactures and supports military aircraft, surface ships, submarines, radar, avionics, communications, electronics and guided weapon systems. It is a pioneer in technology with a heritage stretching back hundreds of years. It is also at the forefront of innovation, working to develop the next generation of intelligent defence systems.

BAE SYSTEMS has major operations across five continents and customers in some 130 countries. The company employs more than 90,000 people and generates annual sales of approximately £12 billion through its wholly-owned and joint venture operations. The Air Systems Business Unit of BAE SYSTEMS is based in the UK, and covers four programmes: Nimrod, Hawk, Typhoon (Eurofighter) and the new Joint Strike Fighter (JSF). For each of these aircraft, Air Systems is responsible for design, engineering, component manufacture and assembly.

Says Steve Heywood, Head of Business Systems Development for Air Systems, "Air Systems uses Baan ERP software to handle extremely complex projects, covering the whole lifecycle from procurement and logistics to manufacturing and finance, and the software is integrated with our engineering and project management applications. Baan is a core piece of our business architecture, so it's vital that it works well."

Air Systems faced three key problems: the approaching obsolescence of its existing hardware platform and operating system release; the lack of capacity to sustain and improve the performance of its Baan application; and the high recurring support costs for a platform that could no longer deliver what the business needed.

Steve Heywood says, "With our increasing need to reduce IT costs it was no longer cost-effective to add capacity to the old solution, and we couldn't maintain the performance we needed on the existing hardware. It was becoming increasingly difficult to complete the weekend batch jobs, and that was having an impact on operations during the following business week."

The winning solution

The company asked its long-term IT partner, CSC, to scope out a new solution that would offer increased performance and scalability at reduced costs. CSC looked at options from the incumbent vendor and two others: "The three vendors had broadly similar technical offerings," says Simon Bradley, Account Executive, "but where the longevity of the solution and the price-performance were concerned, IBM was the clear winner." After using an IBM @server pSeries 610 to prove that it would be feasible to migrate the Baan application from the previous vendor's variant of UNIX to IBM AIX, CSC worked with IBM Business Partner SCC to implement a full-scale solution. SCC has extensive experience of the pSeries platform, and helped with pre-sales support and configuration – as well as working closely with IBM to fine-tune the performance of the new servers.

The new solution is based on two eight-way IBM @server pSeries 670 servers, one acting as the production server and one as the backup server, clustered using IBM HACMP (High Availability Cluster Multi-Processing). The production p670 has an additional eight processors that can be activated on demand.

Says Simon Bradley, "It was important to be able to deal with workload peaks without having to go through the whole process of securing finance, purchasing an upgrade, followed by installation and test. The Capacity Upgrade on Demand option acts like an insurance policy – we now know that if we need the extra capacity, we can get it quickly."

Increased performance, reduced costs

The migration to AIX on pSeries has gone very smoothly, and Steve Heywood comments that the business was hardly aware of any disruption. "What was immediately noticeable was increased speed of the weekend batch suites," he comments. "With the pSeries solution, we have also seen improvements in the speed of reporting from the Baan system, and our recurring support costs are now significantly lower."

Simon Bradley adds, "The pSeries offers far more power than the previous hardware, so we were able to consolidate numerous application and database servers to logical partitions [LPARs] on the two p670s. The p610s that were originally used for the proof of concept are now employed as test, development, quality assurance and pre-production environments, again using LPARs to keep the number of physical servers low.

"Supportability is key for us – we need to reduce our costs year-on-year so that we can pass the savings on to our customers. The pSeries was significantly less expensive to provide as a solution than the competing offerings, so BAE SYSTEMS has made considerable annual cost savings."

A balanced team

The close relationship between CSC and BAE SYSTEMS was a key factor in the success of the new solution. Simon Bradley explains: "Our teams have worked together for several years, during which time there has been a two-way flow of information and skills. We've practically become a single team, and there is a great deal of trust on both sides."

With the new IBM solution, BAE SYSTEMS has a scalable platform that will enable the Air Systems business to grow without incurring unmanageable IT costs. There are currently 1,500 users for the Baan application, and the company is considering extending the system to cover another site. The pSeries set-up also facilitates the simultaneous support of several different releases of the Baan software in different LPARs.

Steve Heywood concludes, "The move to pSeries has resulted in improved performance and scalability, at reduced cost of ownership – and CSC were instrumental in the success of the project."

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Industry News Brief

ANSYS Acquires Century Dynamics

(excerpt from News Release)

ANSYS, Inc. announced that it has acquired Century Dynamics, Inc., a leading provider of simulation software for solving linear, nonlinear, explicit and multi-body hydro-dynamics problems.

Century Dynamics' main product, AUTODYN(R), can solve many types of problems with its extensive solver library. Solvers include computational structural dynamics finite element solvers (FE), finite volume solvers for fluid dynamics (CFD), mesh-free particle solvers for high velocity, large deformation and fragmentation problems (SPH), and multi-solver coupling for multi-physics solutions including coupling between FE, CFD and SPH methods.

Mike Wheeler, vice president and general manager of the Mechanical Business Unit at ANSYS, "Our immediate goal with Century Dynamics is to begin technology-sharing projects and have products embedded into the ANSYS(R) Workbench(TM) later this year."

Century Dynamics will continue to operate as a separate subsidiary, maintaining its products, sales channels and other partner relationships.

"Century Dynamics offers strong capabilities in structural and fluid dynamics problems and is a leading innovator in techniques for fluid-structure interaction which will enhance current ANSYS technology," said Naury Birnbaum, a founder of Century Dynamics, Inc. "Plus, our strengths in the government/defense and offshore fields are a natural complement to the ANSYS product line. Customers and partners of Century Dynamics will directly benefit from this acquisition by working with an innovative company like ANSYS that is truly leading the advancement in simulation technology."

About Century Dynamics, Inc.: Century Dynamics designs, develops, globally markets and supports advanced engineering and scientific simulation software. It provides solutions for understanding how designs and processes behave in real world applications. Century Dynamics services a wide spectrum of industries including aerospace, automotive, defense, education, marine, offshore, petrochemical, power and security…

About ANSYS, Inc.: ANSYS, Inc., founded in 1970, develops and globally markets engineering simulation software and technologies widely used by engineers and designers across a broad spectrum of industries…

Asia Pacific News

Korea - Event Announcement:

LS-DYNA and Optimization Korea Conference
Korea - March 18, 2005 hosted by KOSTECH
www.kostech.com

China - Company Review:

Beijing Yuntong Forever CPC. Co. Ltd

http://cpc.ytforever.com

Beijing Yuntong Forever CPC. Co. Ltd. is a leading high-tech company certified by Beijing Scientific and Technological Commission. Beijing Yuntong is the sole distributor of LINUX NETWORX, INC (U.S.A.) in China, responsible for the sale and service support in China of Evolocity cluster serial products mainly.

Evolocity is a world famous Linux Cluster and widely used in CAE high performance parallel computing. It supports all commercial CFD and FEA software, very good platform for LS-DYNA MPP computing.

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China - Seminar Review - by Gill Ding, ANSYS-China, LTD

ANSYS Vehicle Simulation Technology Seminar – January News
Chinese Voice coming from Ansys-China
ANSYS-CHINA, LTD. - e-mail: china@ansys.com.cn
www.ansys.com.cn

Closely following the Chongquing VPG Seminar, Ansys-China held two additional seminars on the same topic. The seminars were held consecutively in Beijing and Shanghai on the 14th and 16th of Dec.

Over 200 attended the seminars for the purpose of 'Free Full Vehicle Simulation'.

The two seminars were emceed by Professor Huangshilin, from Tsinghua University.

Professor Huangshilin is a famous authority in the automotive vehicle field.

Mr. Arthur Tang, the VP of ETA, gave an informative speech on VPG and its use in the automotive vehicle field.

During the conference the attendees discussed the future of CAE technology in the automotive vehicle field with experts and engineers of Ansys-China.

Dr. Gulei - Ford presented the explicit and implicit technology of LS-DYNA

Special focus was the presentations by Dr. Gulei, the Senior Security Technology Engineer of Ford, as well as the professors from Tongji University and Ji Lin University.

Delegates in both seminars were from international, well-known auto companies or research institutes, such as Foton, BJC, Volkswagen, Great Wall Motors, and IVECO etc. There was a general consensus that VPG would play an important role in China's vehicle development.

During the meeting, representatives from China Automotive News, China Manufacture Information, Science and Technology Daily, and Auto Industry interviewed Mr. Arthur Tang and Dr. Huang about various issues surrounding VPG. Over 20 media references were reported as a result of the ANSYS-China conference.

China – Press Release **Excerpt from www.amd.com**

AMD Powers Hebei Province, China Long-Distance Education Project
Over 26,000 AMD processor-based systems implemented in one of the largest education projects of its kind

Beijing -- January 10, 2005 --AMD (NYSE: AMD) today announced that Lenovo, Dawning and AMD have jointly won one of China's most ambitious long-distance education projects – the Hebei Long-Distance Education Project. As part of the win, AMD, Lenovo and Dawning will provide over 26,000 AMD processor-based systems, including AMD Sempron™ processor-based Lenovo Qitian M 6200M PCs and AMD Opteron™ processor-based Dawning Tiankuo A Series i200-A servers, to Hebei Education Bureau.

As a significant part of China's "informatilization" initiative, which is designed to facilitate more effective communication across the entire country, long-distance education was selected by China's Central Government as a way to improve nationwide education. Hebei Province was chosen to be the demo site for the launch of the long-distance education program, which began in October 2004.

"AMD is very pleased to contribute to China's education development, together with Lenovo and Dawning, by supporting long-distance education in Hebei's rural area. This is a breakthrough for AMD China that highlights overall momentum for AMD as well as our technically advanced products," said Karen Guo, corporate vice president of AMD and president of AMD Greater China. "AMD will help hundreds of thousands of students improve their education by using our products to enhance their learning experience. This win is a testament to our commitment to develop the China market, and highlights our vision of succeeding with local partners."

According to the project's managers, hundreds of networked computer classrooms will be built. Every computer classroom will deploy one AMD Opteron processor-based server and 30 desktop PCs. At least 200,000 students will benefit from this project.

"The remote education project is a very important project for both the Hebei province and the whole country, as the Central Government chooses Hebei to be the demo site for nationwide remote education. After thorough-testing and evaluation, we believe AMD processor-based PCs and servers will provide tremendous benefits to our teachers and students," said Mr. Yan Chunlai, assistant supervisor of the Hebei Education

Bureau.

By implementing AMD Sempron processor-based Lenovo Qitian M 6200 systems in the Hebei long-distance education project, Lenovo demonstrates its commitment to China's education. Lenovo is one of the first PC manufacturers to launch AMD Sempron processor-based systems. The well-received Lenovo "Yuan Meng" PCs, based on AMD Sempron processors, offer users in China an opportunity to purchase a PC with performance and features designed to meet their daily home and business computing needs.

"Dawning and AMD jointly launched an extensive 64-bit computing marketing campaign in August 2004, and today Dawning's AMD Opteron processor-based 32- and 64-bit computing server platform has penetrated several industries including education, government, meteorology, electricity and commerce," said Mr. Li Jun, president of Dawning. "We believe we were chosen to support the Hebei long-distance education project because of our project operation experience, the advanced technology in our server systems combined with the AMD Opteron processor and our strong service team."

About Hebei Long-Distance Education Project

The long-distance education initiative for elementary and middle schools in rural areas is an endeavor of tremendous strategic importance that focuses on driving the policy of enhancing national strength through technical and educational advancements and further improving education and people's standard of living for rural areas. As a pilot province for the initiative, Hebei will put in place e-education programs in pilot municipalities including Zhangjiakou, Chengde and Baoding to improve quality and efficiency of education in the countryside. These programs are designed to introduce high-quality education resource-based advanced long-distance education model to drive education balance, bridging the gap between eastern and western China and enhancing the social and economic development in rural areas.

About the AMD Sempron™ Processor
The AMD Sempron™ processor family is redefining everyday computing for today's value-conscious buyers of desktop and notebook PCs…This full-featured product line enables performance at the top of its class on home and business applications.

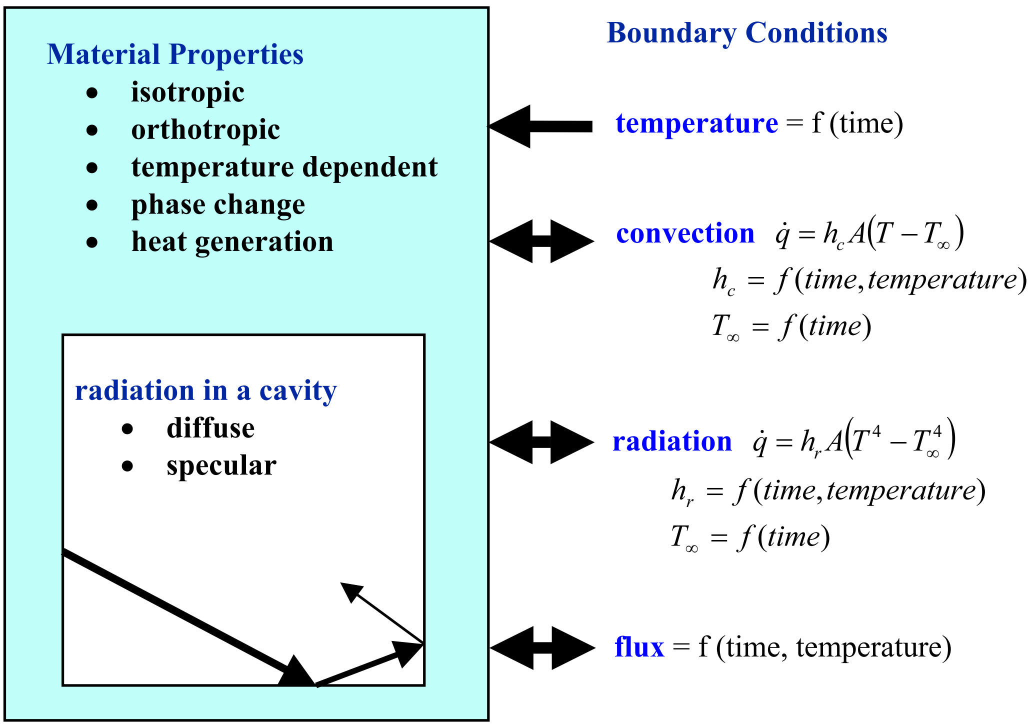
About the AMD Opteron™ Processor
The world's first 32-bit and 64-bit processor compatible with the x86 architecture, the AMD Opteron processor is based on AMD64 technology with Direct Connect Architecture. Direct Connect Architecture helps eliminate the bottlenecks inherent in a front-side bus by directly connecting the processors, the memory controller and the I/O to the central processor unit to enable improved overall system performance and efficiency…

About AMD
AMD (NYSE:AMD) designs and produces innovative microprocessors, Flash memory devices and low-power processor solutions for the computer, communications and consumer electronics industries. AMD is dedicated to delivering standards-based, customer-focused solutions for technology users, ranging from enterprises and governments to individual consumers.

Product Spotlight – LS-DYNA

Application – Heat Transfer Problems

LS-Dyna can solve steady state and transient heat transfer problems on 2-dimensional parts, cylindrical symmetric parts (axisymmetric), and 3-dimensional parts. Heat transfer can be coupled with other features in LS-DYNA to provide modeling capabilities for thermal-stress (e.g., upsetting, extrusion, forging) and thermal-fluid (e.g., casting) coupling. Heat transfer modeling options in LS-DYNA are summarized in the following figure.



Boundary conditions include temperature, flux, convection, and radiation. Material properties can be isotropic or orthotropic. Thermal conductivity and heat capacity can be functions of temperature. The material can undergo solid-liquid phase change and be defined with a heat generation rate that can be a function of time and temperature. Enclosure radiation heat transfer can be modeled using diffuse or specular surfaces.

Several example problems are shown in the AVI Library on the www.feainformation.com website.

Two classes are being offered to introduce users to the heat transfer, thermal stress, and thermal fluid capabilities in LS-Dyna

1. LSTC, Livermore, California on April 7 and 8, www.lstc.com link classes
2. CADFEM, Stuttgart, Germany on April 21 and 22, http://www.cadfem.de/Schulungen.1189.0.html?do=showevent&id=292

LS-DYNA Resource Page

Interface - Hardware - OS And General Information

LS-DYNA General Information- www.lstc.com sales@lstc.com

Version: 970

Classes:
www.lstc.com classes

30-day demonstration licenses available – no fee

Sales
sales@lstc.com

Participant Hardware and OS that run LS-DYNA (alpha order)

All Hardware and OS listed have been fully QA'd by Livermore Software Technology Corporation

AMD Opteron Linux	HP PA8000 HPUX	INTEL IA32 Linux, Windows	SGI Mips IRIX6.5
CRAY XD1 Linux	HPIA64 HPUX or Linux	INTEL IA64 Linux	
FUJITSU Prime Power SUN OS 5.8	HP Alpha True 64	INTEL Xeon EMT64 Linux	SGI IA64 Altix
FUJITSU VPP Unix_System_V	IBM Power 4/5 AIX 5.1	NEC SX6 Super-UX	

LS-DYNA Resource Page

Participant Software Interfacing or embedding LS-DYNA

Each software program can interface to all, or a very specific and limited segment of the other software program. The following list are software programs interfacing to or having the LS-DYNA solver embedded within their product. For complete information on the software products visit the corporate website.

ANSYS - ANSYS/LS-DYNA
www.ansys.com/products/environment.asp

ANSYS/LS-DYNA - Built upon the successful ANSYS interface, ANSYS/LS-DYNA is an integrated pre and postprocessor for the worlds most respected explicit dynamics solver, LS-DYNA. The combination makes it possible to solve combined explicit/implicit simulations in a very efficient manner, as well as perform extensive coupled simulations in Robust Design by using mature structural, thermal, electromagnetic and CFD technologies.

AI*Environment: A high end pre and post processor for LS-DYNA, AI*Environment is a powerful tool for advanced modeling of complex structures found in automotive, aerospace, electronic and medical fields. Solid, Shell, Beam, Fluid and Electromagnetic meshing and mesh editing tools are included under a single interface, making AI*Environement highly capable, yet easy to use for advanced modeling needs.

ETA – DYNAFORM
www.eta.com

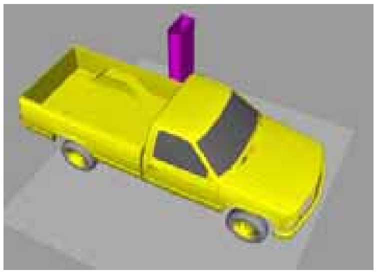
Includes a complete CAD interface capable of importing, modeling and analyzing, any die design. Available for PC, LINUX and UNIX, DYNAFORM couples affordable software with today's high-end, low-cost hardware for a complete and affordable metal forming solution.

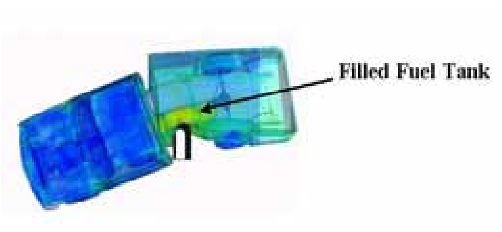
ETA – VPG
www.eta.com

Streamlined CAE software package provides an event-based simulation solution of nonlinear, dynamic problems. eta/VPG's single software package overcomes the limitations of existing CAE analysis methods. It is designed to analyze the behavior of mechanical and structural systems as simple as linkages, and as complex as full vehicles

MSC.Software
"MSC.Dytran LS-DYNA"
www.msc.software.com

Tightly-integrated solution that combines MSC.Dytran's advanced fluid-structure interaction capabilities with LS-DYNA's high-performance structural DMP within a common simulation environment. Innovative explicit nonlinear technology enables extreme, short-duration dynamic events to be simulated for a variety of industrial and commercial applications on UNIX, Linux, and Windows platforms. Joint solution can also be used in conjunction with a full suite of Virtual Product Development tools via a flexible, cost-effective MSC.MasterKey License System.





Side Impact With Fuel Oil Inside

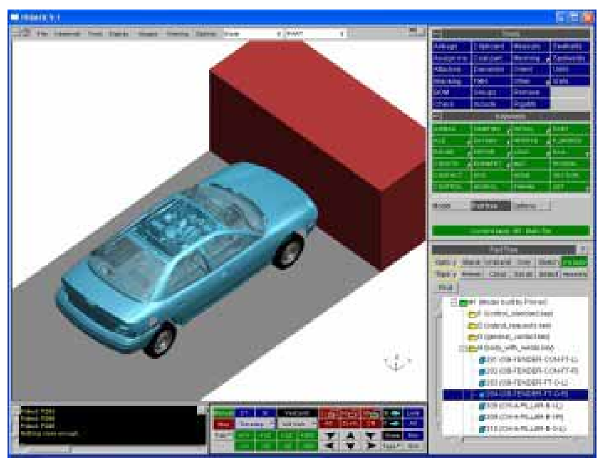
MSC.Software - MSC.Nastran/SOL 700

The MSC.Nastran™ Explicit Nonlinear product module (SOL 700) provides MSC.Nastran users the ability access the explicit nonlinear structural simulation capabilities of the MSC.Dytran LS-DYNA solver using the MSC.Nastran Bulk Data input format. This product module offers unprecedented capabilities to analyze a variety of problems involving short duration, highly dynamic events with severe geometric and material nonlinearities.

MSC.Nastran Explicit Nonlinear will allow users to work within one common modeling environment using the same Bulk Data interface. NVH, linear, and nonlinear models can be used for explicit applications such as crash, crush, and drop test simulations. This reduces the time required to build additional models for another analysis programs, lowers risk due to information transfer or translation issues, and eliminates the need for additional software training.
The MSC.Nastran Sol 700 will be released in November 2005. Beta release is available now !

Oasys software for LS-DYNA
www.arup.com/dyna

Oasys software is custom-written for 100% compatibility with LS-DYNA. Oasys PRIMER offers model creation, editing and error removal, together with many specialist functions for rapid generation of error-free models. Oasys also offer post-processing software for in-depth analysis of results and automatic report generation.



AVI Library Feature: AVI #609 Disc Brake Analysis

Dr. Arthur B. Shapiro

HOW TO USE LS-DYNA TO CALCUALTE FRICTIONAL HEATING

LS-DYNA can solve for the frictional energy due to sliding at the interface between 2 parts. The input file friction_to_heat.k for the example problem described below can be downloaded from:

ftp://ftp.lstc.com/outgoing/HeatTransfer/

KEYWORD INPUT

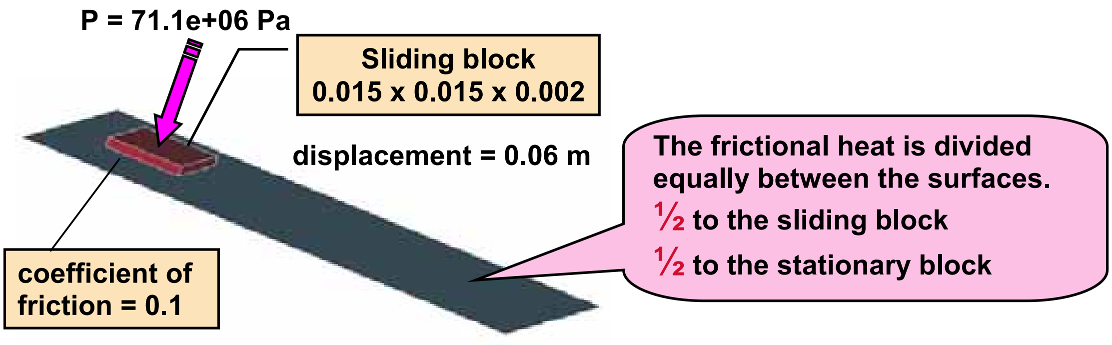
The parameter FRCENG on the *CONTROL_CONTACT keyword is used to activate the calculation of frictional sliding energy

```
*CONTROL_CONTACT
  slsfac    rwpnal    islchk    shlthk
  usrstr    usrfrc     nsbcs .........
   sfric     dfric       edc .........
  ignore    frceng   skiprwg .........
```

Set frceng = 1 to calculate contact frictional energy

EXAMPLE PROBLEM

Consider a block sliding on a plane as shown in the following figure.



The normal force is $F_N = PA = (71.1e+06)(0.015 \times 0.015) = 16{,}000 \ N$

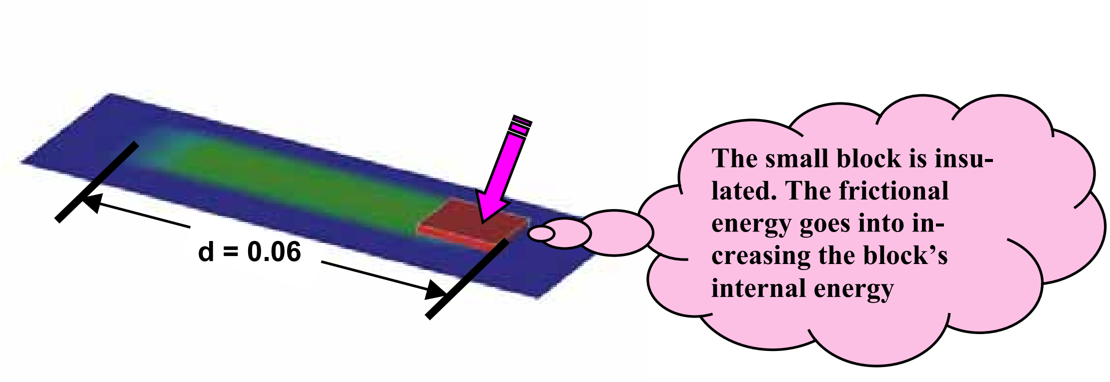
The frictional work is $W_f = \mu F_N d = (0.1)(16{,}000)(0.06) = 96 \ J$

The frictional work is divided equally between the surfaces. One half goes into the sliding block and the other half into the stationary block. The frictional work goes into increasing the block's internal energy. The increase in temperature for the small sliding block is:

$$Q = \frac{W_f}{2} = \frac{96}{2} = 48 \ J$$

$$\Delta T = \frac{Q}{\rho c V} = \frac{48}{(7900)(460)(0.015 \times 0.015 \times 0.002)} = 29.4 \ C$$

The following figure shows the final position of the moving block and temperature fringes for the example problem.



The LS-DYNA calculated frictional energy is printed to the GLSTAT file shown below.

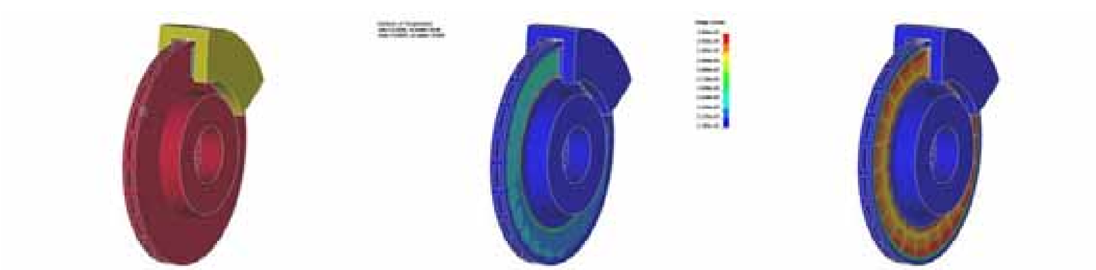
```
dt of cycle    1358 is controlled by shell      element     161

 time...........................   9.99990E-04
 time step......................   7.37916E-07
 kinetic energy.................   4.08254E+00
 internal energy................   9.94541E-03
 spring and damper energy.......   1.00000E-20
 system damping energy..........   0.00000E+00
 sliding interface energy.......   9.50279E+01
 external work..................   1.02005E+02
 eroded kinetic energy..........   0.00000E+00
 eroded internal energy.........   0.00000E+00
 total energy...................   9.91204E+01
 total energy / initial energy..   9.71725E-01
 energy ratio w/o eroded energy.   9.71725E-01
 global x velocity..............   5.21575E-01
 global y velocity.............. -3.95185E-03
 global z velocity..............   2.00433E-02
 time per zone cycle.(nanosec)..             0
```

The LS-DYNA calculated temperature for the sliding block (~28 C) can be found in either the TPRINT file or displayed using LS-PrePost. The discrepancy between the calculated and analytical answer is due to the coarseness of the mesh being used. Only one element is used to model the sliding block.

AUTOMOBILE DISC BRAKE

LS-DYNA is used to better understand the interaction between the brake pad and rotor interface. Modeling the conversion of sliding frictional energy to heat is important in the design. As the rotor temperature increases, the temperature gradient can cause rotor deformation due to thermal expansion. This can lead to banded hot-spotting, tapered wear, and noise.

The following three figures show the disc brake at three different times during the simulation. The first figure on the left shows the geometry at the initial time. The rotor is given an initial rotational velocity and a force is applied to the pads. All parts start with an initial temperature of 25C. The middle figure shows temperature fringes after a few rotations of the rotor. The rotor is heating up due to frictional heating. In this problem, the pads are making greater contact with the rotor near the rotor hub. This is indicated by the higher temperature pattern (i.e., red). The third figure on the right shows the temperature pattern when the rotor has come to rest. The cooler (i.e., yellow) radial lines on the rotor are due to the rotor ribs acting as a heat sink. A movie showing the results can be found on www.feainformation.com. Click the "AVI Library" link in the left hand menu and then go to AVI movie 609 under the heading "Contact".



Hardware & Computing and Communication Products

www.amd.com

www.fujitsu.com

www.hp.com

www-1.ibm.com/servers/deepcomputing

www.intel.com

www.nec.com

www.sgi.com

www.cray.com

Software Distributors

Alphabetical order by Country

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Software Distributors (cont.)

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USA	**Altair Western Region** www.altair.com
USA	**Engineering Technology Associates** www.eta.com
USA	**Dynamax** www.dynamax-inc.com
USA	**Livermore Software Technology Corp.** www.lstc.com
USA	**ANSYS Inc.** www.ansys.com
UK	**Oasys, LTC** www.arup.com/dyna/

Consulting And Engineering Services

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Italy Firenze www.numerica-srl.it	**Numerica SRL** info@numerica-srl.it 39 055 432010
UK Solihull, West Midlands www.arup.com	**ARUP** Brian Walker brian.walker@arup.com 44 (0) 121 213 3317
USA Irvine, CA www.altair.com	**Altair Engineering Inc.Western Region** Harold Thomas info-ca@altair.com
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USA	Dr. Bhavin V. Mehta	Ohio University
USA	Dr. Taylan Altan	The Ohio State U – ERC/NSM
USA	Prof. Ala Tabiei	University of Cincinati
USA	Tony Taylor	Irvin Aerospace Inc.

Informational Websites

FEA Informationwebsites	www.feainformation.com
TopCrunch – Benchmarks	www.topcrunch.org
LS-DYNA Examples (more than 100 Examples)	www.dynaexamples.com
LS-DYNA Conference Site	www.ls-dynaconferences.com
LS-DYNA Publications to Download On Line	www.dynalook.com
LS-DYNA Publications	www.feapublications.com
LS-DYNA Forum	http://portal.ecadfem.com/Forum.1372.0.html
LS-DYNA CADFEM Portal	http://www.lsdyna-portal.com

FEA Information Site News & Events

Archived on the Weekly News Page

Dec. 06	Fujitsu	Bringing together the world's fastest bus technology and the latest in SPARC64® processor development, PRIMEPOWER.
	AMD	Based on AMD64 technology, the AMD Athlon™ 64 processor is the newest addition to the award-winning AMD Athlon processor family
	ERAB	Distributor headquartered in Sweden
Dec 13	NEC	NEC Express5800 140HD - A high-end pedestal-type enterprise server that realizes mission-critical computing through its high CPU performance and HDD scalability
	IBM	IBM Deep Computing delivering innovative and powerful breakthrough solutions to address the demands of intense computation, visualization, or manipulation and management of massive amounts of data.
	THEME	Distributor headquartered in Korea
Dec 20	ANSYS	From CAD to mesh generation for analysis, ANSYS ICEM CFD provides sophisticated geometry acquisition, mesh generation, post-processing, and mesh optimization tools.
	MSC.Software	Virtual Product Development is the coordinated, efficient application of technology to enhance a firm's established New Product Development process by enabling people to make better product development decisions.
	Altair West	Altair Distributor located in California

Events

2005	
March 18	**LS-DYNA and Optimization Korea Conference,** Korea
May 17-20	**NAFEMS World Congress** - Malta
May 25-26	**5th European LS-DYNA Conference** - The ICC, Birmingham UK
June 25-27	**8th U.S. National Congress on Computational Mechanics,** Austin, TX
2006	
June	**9th International LS-DYNA Users Conference** - Deerborn, MI
July	**Seventh WCCM** - Los Angeles, CA

The Total and Updated Lagrangian Formulations

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Lagrangian finite element formulations use a mesh that moves with the material, and the material points are labeled with their coordinates in a *reference configuration.* The total Lagrangian formulation uses the initial configuration at $t = 0$ for the reference configuration, where the coordinates are labeled $\boldsymbol{X}$ in most continuum mechanics textbooks. An updated Lagrangian formulation uses the current coordinates, $\boldsymbol{x}$, evaluated at the end of the current time step $n + 1$, as its reference configuration. The current coordinates are also commonly referred to as the *spatial* or *Eulerian* coordinates. Since $\boldsymbol{X}$ and $\boldsymbol{x}$ are linked through the motion of the material, transformations that convert one formulation to the other are possible, and if the transformations are performed correctly, identical solutions will be obtained regardless of which formulation is used.

The choice of formulation is based on convenience, and to a lesser extent, personal aesthetic judgments. The plasticity of metals is most often described in terms of the Cauchy stress, which refers to the current coordinates. Hyperelastic materials are often described in terms of the second Piola-Kirchhoff stress (PK2 for brevity) or a strain energy functional defined in terms of the Green-Lagrange strain. The most convenient formulation to use to solve a nonlinear problem is the one that requires the fewest transformations between the discrete FE equations and the stress and strain used to define the material response. Most commercial codes use the updated formulation because most of nonlinear calculations involve the plasticity of metals. When material models formulated in terms of PK2 are used in commercial codes, the PK2 stress is transformed to the Cauchy stress before the element forces are calculated.

1 Virtual Displacements and Velocities

The calculus of variations is a tool commonly used in mechanics. Central is the concept of a *virtual displacement*, $\delta \boldsymbol{u}$. Virtual displacements are classically defined as an arbitrary displacement from the current configuration that satisfies all the displacement boundary conditions, which means that if $\boldsymbol{u}(\boldsymbol{x}) = g(t)$ is specified at a particular point $\boldsymbol{x}$, then $\delta \boldsymbol{u}(\boldsymbol{x}) = 0$ since $\boldsymbol{u}(\boldsymbol{x}) + \delta \boldsymbol{u}(\boldsymbol{x}) = g(t)$.

In most cases, an equation that is true for the velocity is also true when the velocity is replaced with the virtual displacement. For example, if the displacement is a function of a set of generalized coordinates, $\boldsymbol{q}$,

$$\boldsymbol{u} = \mathcal{F}(\boldsymbol{q}) \tag{1}$$

then

$$\dot{\boldsymbol{u}} = \boldsymbol{v} = \frac{\partial \mathcal{F}}{\partial \boldsymbol{q}} \dot{\boldsymbol{q}} \quad \text{and} \quad \delta \boldsymbol{u} = \frac{\partial \mathcal{F}}{\partial \boldsymbol{q}} \delta \boldsymbol{q} \tag{2}$$

The differentiation of the displacement relation is, in fact, the usual way in which the relationship between virtual displacements are derived.

In some texts, the virtual displacements are replaced with the velocities, and instead of *virtual work*, the relations give *virtual power*.

2 A Review of Virtual Work

The traditional statement of virtual work for a continuous body for small deformations in static equilibrium is

$$\int_\Omega \boldsymbol{\sigma} : \delta \boldsymbol{\epsilon} \mathrm{d}\Omega = \int_\Omega \rho \boldsymbol{b}^T \delta \boldsymbol{u} \mathrm{d}\Omega + \int_\Gamma \boldsymbol{\tau}^T \delta \boldsymbol{u} \mathrm{d}\Gamma \tag{3}$$

where $\boldsymbol{\sigma}$ is the engineering stress, $\boldsymbol{\epsilon}$ is the linear strain, ρ is the density, $\boldsymbol{b}$ is the body force per unit mass, $\boldsymbol{\tau}$ is the surface traction vector, and Ω and Γ are the volume and surface domains of the body. For small strain, small deformation problems, no distinction is made between the initial and current configurations.

If all the variables in Equation 3 are associated with the appropriate variables referenced to the current configuration, Equation 3 is valid for large deformations and strains. Specifically, $\boldsymbol{\sigma}$ is identified with the Cauchy stress, ρ with the current density, Ω and Γ refer to the configuration at the current time, and $\boldsymbol{b}$ and $\boldsymbol{\tau}$ are defined in terms of the current coordinates. The virtual strain, $\delta\boldsymbol{\epsilon}$ is defined as

$$\delta\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial\delta u_i}{\partial x_j} + \frac{\partial\delta u_j}{\partial x_i}\right) \quad \delta\epsilon = \frac{1}{2}\left(\frac{\partial\delta\boldsymbol{u}}{\partial\boldsymbol{x}} + \left[\frac{\partial\delta\boldsymbol{u}}{\partial\boldsymbol{x}}\right]^T\right). \tag{4}$$

For small deformations, the virtual strain corresponds to a legitimate strain measure, the engineering strain, being applied to the virtual displacement. This doesn't hold true for the finite deformation case, i.e.,

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{5}$$

is not a valid large deformation measure of strain, and the corresponding deformation rate,

$$D_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{6}$$

is not integrable into a valid large deformation strain measure. These quantities are, however, *work conjugate* with the Cauchy stress. This means that, for example,

$$\int_0^T \left(\int_\Omega \boldsymbol{\sigma} : \boldsymbol{D}\mathrm{d}\Omega\right) \mathrm{d}t \tag{7}$$

is the total internal work performed by the body from time $t = 0$ to $t = T$. There are other pairs of work conjugate stress-strain pairs, and they are discussed later.

3 A Review of Kinematics

Kinematics is the description of motion, whether we're talking about individual particles or continuous body. Many possible kinematic descriptions are possible; only the most commonly used one is discussed here. Associated with the description of the motion, there is also the description of the how that motion changes locally, namely the *strain*. Just as there are many different descriptions of motion, there are many different strains. For this article, only those that are work conjugate to the most commonly used stresses are discussed. Many material models are described in terms of strains that aren't discussed here, for example, the principal stretches of the material are often used in models for polymers. The strain used in evaluating the material model doesn't have to be the same one used in the virtual work statement.

The current coordinates $\boldsymbol{x}$ are related to the initial coordinates $\boldsymbol{X}$ and the displacement $\boldsymbol{u}$ according to

$$\boldsymbol{x} = \boldsymbol{X} + \boldsymbol{u} \tag{8}$$

and the velocity $\boldsymbol{v}$ is

$$\boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} \tag{9}$$

because the derivative of $\boldsymbol{X}$ with time is zero.

The deformation gradient $\boldsymbol{F}$ is defined as

$$F_{ij} = \frac{\partial x_i}{\partial X_j} = \delta_{ij} + \frac{\partial u_i}{\partial X_j} \quad \boldsymbol{F} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{X}} = \boldsymbol{I} + \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{X}}. \tag{10}$$

and taking the derivative of it with respect to time gives

$$\dot{F}_{ij} = \frac{\partial v_i}{\partial X_j} = \frac{\partial v_i}{\partial x_k}\frac{\partial x_k}{\partial X_j} = L_{ik}F_{kj} \quad \dot{\boldsymbol{F}} = \boldsymbol{L}\boldsymbol{F} \tag{11}$$

$$L_{ij} = \frac{\partial v_i}{\partial x_j} \quad \boldsymbol{L} = \frac{\partial \boldsymbol{v}}{\partial \boldsymbol{x}}. \tag{12}$$

Notice that the velocity gradient $\boldsymbol{L}$ is evaluated with respect to the current coordinates $\boldsymbol{x}$, not the reference configuration, $\boldsymbol{X}$.

The first variation of any function has the same structure as its time derivative, and therefore the first variation of $\boldsymbol{F}$ is

$$\delta\boldsymbol{F} = \delta\boldsymbol{L}\boldsymbol{F} \tag{13}$$

$$\delta\boldsymbol{L} = \frac{\partial \delta\boldsymbol{u}}{\partial \boldsymbol{x}}. \tag{14}$$

Noting that

$$\mathrm{d}x_i = \frac{\partial x_i}{\partial X_j}\mathrm{d}X_j = F_{ij}\mathrm{d}X_j \quad \mathrm{d}\boldsymbol{x} = \boldsymbol{F}\mathrm{d}\boldsymbol{X} \tag{15}$$

then the relationship between the lengths of the differentials is

$$\mathrm{d}\ell = \mathrm{d}\boldsymbol{x}^T\mathrm{d}\boldsymbol{x} = \mathrm{d}\boldsymbol{X}^T\boldsymbol{F}^T\boldsymbol{F}\mathrm{d}\boldsymbol{X}. \tag{16}$$

Therefore, difference between the lengths is

$$\mathrm{d}\boldsymbol{x}^T\mathrm{d}\boldsymbol{x} - \mathrm{d}\boldsymbol{X}^T\mathrm{d}\boldsymbol{X} = \mathrm{d}\boldsymbol{X}^T\boldsymbol{F}^T\boldsymbol{F}\mathrm{d}\boldsymbol{X} - \mathrm{d}\boldsymbol{X}^T\mathrm{d}\boldsymbol{X} = 2\mathrm{d}\boldsymbol{X}^T\boldsymbol{E}\mathrm{d}\boldsymbol{X}. \tag{17}$$

where $\boldsymbol{E}$ is the Green-Lagrange strain tensor,

$$E_{ij} = \frac{1}{2}\left(F_{ki}F_{kj} - \delta_{ij}\right) \quad \boldsymbol{E} = \frac{1}{2}\left(\boldsymbol{F}^T\boldsymbol{F} - \boldsymbol{I}\right) \tag{18}$$

The Green-Lagrange strain is used frequently in material models for rubber.

Taking the derivative of the Green-Lagrange strain with time gives

$$\dot{\boldsymbol{E}} = \frac{1}{2}\left(\{\boldsymbol{F}^T\boldsymbol{L}^T\}\boldsymbol{F} + \boldsymbol{F}^T\{\boldsymbol{L}\boldsymbol{F}\}\right) = \frac{1}{2}\boldsymbol{F}^T\left(\boldsymbol{L} + \boldsymbol{L}^T\right)\boldsymbol{F}. \tag{19}$$

and its first variation is given by

$$\delta\boldsymbol{E} = \frac{1}{2}\boldsymbol{F}^T\left(\delta\boldsymbol{L} + \delta\boldsymbol{L}^T\right)\boldsymbol{F} \tag{20}$$

Equation 20 will be used later to demonstrate that the second Piola-Kirchhoff stress is work conjugate to the Green-Lagrange strain.

Material models for plasticity, which typically use the Cauchy stress, are usually described by differential equations using the deformation rate, $\boldsymbol{D}$, and the spin, $\boldsymbol{W}$,

$$D_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \quad \boldsymbol{D} = \frac{1}{2}\left(\boldsymbol{L} + \boldsymbol{L}^T\right) \tag{21}$$

$$W_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right) \quad \boldsymbol{W} = \frac{1}{2}\left(\boldsymbol{L} - \boldsymbol{L}^T\right) \tag{22}$$

For pure rigid body motion, $\boldsymbol{D} = \boldsymbol{0}$ and the relationship between the angular velocity $\boldsymbol{\omega}$ and the spin is

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{x} = \boldsymbol{W}\boldsymbol{x}. \tag{23}$$

4 Work Conjugate Stress and Strain Pairs

The work conjugate stress-strain pairs are defined as satisfying the relation

$$\delta W^{\text{int}} = \int_\Omega \sigma_{ij}\delta\epsilon_{ij}\mathrm{d}\Omega = \int_\Omega \boldsymbol{\sigma} : \delta\boldsymbol{\epsilon}\mathrm{d}\Omega \tag{24}$$

$$= \int_\Omega \boldsymbol{\sigma} : \delta\boldsymbol{L}\mathrm{d}\Omega \tag{25}$$

$$= \int_V \boldsymbol{\Sigma} : \delta\boldsymbol{\Xi}\mathrm{d}V \tag{26}$$

where $\boldsymbol{\Sigma}$ and $\boldsymbol{\Xi}$ are a work conjugate stress-strain pair, and V is the reference configuration, which is restricted in this section to being either the current configuration, Ω, or the initial configuration, Ω_0. It is important to remember that these pairs are associated with a particular configuration.

The relationship between the integrals over the initial and current domains is

$$\int_\Omega \phi(\boldsymbol{x})\mathrm{d}\Omega = \int_{\Omega_0} \phi(\boldsymbol{x})J\mathrm{d}\Omega \tag{27}$$

where $J = \det\boldsymbol{F}$, and $\phi(\boldsymbol{x})$ is an arbitrary integrable function. Since $\boldsymbol{x}$ is a function of $\boldsymbol{X}$, the integrand on the right hand side is similarly well defined in terms of $\boldsymbol{X}$.

A direct transformation of the left hand side of Equation 26 using Equation 27 gives

$$\int_\Omega \boldsymbol{\sigma} : \delta\boldsymbol{\epsilon}\mathrm{d}\Omega = \int_{\Omega_0} \boldsymbol{\sigma} : \delta\boldsymbol{\epsilon}J\mathrm{d}\Omega_0. \tag{28}$$

The stress $J\boldsymbol{\sigma}$, commonly called the *Kirchhoff* stress, is therefore work conjugate with $\delta\boldsymbol{\epsilon}$ (and $\boldsymbol{D}$) with respect to the initial configuration Ω_0.

If $\delta\boldsymbol{\Xi}$ is expressed in terms of $\delta\boldsymbol{L}$,

$$\delta\Xi_{ij} = \Psi_{ik}\delta L_{kl}\Phi_{lj} \quad \delta\boldsymbol{\Xi} = \boldsymbol{\Psi}\delta\boldsymbol{L}\boldsymbol{\Phi} \tag{29}$$

then the virtual work is

$$\delta W^{\text{int}} = \int_V \Sigma_{ij}\delta\Xi_{ij}\mathrm{d}V = \int_V \Sigma_{ij}\Psi_{ik}\delta L_{kl}\Phi_{lj}\mathrm{d}V \tag{30}$$

$$= \int_V \boldsymbol{\Sigma} : (\boldsymbol{\Psi}\delta\boldsymbol{L}\boldsymbol{\Phi})\,\mathrm{d}V \tag{31}$$

By comparing the terms in Equations 26 and 30, the work conjugate stress to Ξ_{ij} is

$$\sigma_{kl} = \Psi_{ik}\Sigma_{ij}\Phi_{lj} \quad \boldsymbol{\sigma} = \boldsymbol{\Psi}^T\boldsymbol{\Sigma}\boldsymbol{\Phi}^T \quad \text{if } V = \Omega \tag{32}$$

$$\sigma_{kl} = \frac{1}{J}\Psi_{ik}\Sigma_{ij}\Phi_{lj} \quad \boldsymbol{\sigma} = \frac{1}{J}\boldsymbol{\Psi}^T\boldsymbol{\Sigma}\boldsymbol{\Phi}^T \quad \text{if } V = \Omega_0 \tag{33}$$

$$\tag{34}$$

For the deformation gradient, comparing Equation 13 to Equation 29, gives $\boldsymbol{\Psi} = \boldsymbol{I}$ and $\boldsymbol{\Phi} = \boldsymbol{F}$. The stress conjugate to the deformation gradient, $\boldsymbol{P}$, using the reference configuration, is the first Piola-Kirchhoff stress. Using Equation 33 gives the relation

$$\sigma_{kl} = \frac{1}{J}P_{kj}F_{lj} \quad \boldsymbol{\sigma} = \frac{1}{J}\boldsymbol{P}\boldsymbol{F}^T. \tag{35}$$

The stress that is work conjugate to the Green-Lagrange strain is the second Piola-Kirchhoff stress, $\boldsymbol{S}$. Substituting $\delta\boldsymbol{\epsilon}$ for $\delta\boldsymbol{L}$ in Equation 29 leads again to Equations 32 and 33. Like the first Piola-Kirchhoff stress, it uses the reference configuration. Using Equation 20, $\boldsymbol{\Psi} = \boldsymbol{F}^T$, and $\boldsymbol{\Phi} = \boldsymbol{F}$, and the relation to the Cauchy stress is

$$\sigma_{kl} = \frac{1}{J}F_{ik}S_{ij}F_{lj} \quad \boldsymbol{\sigma} = \frac{1}{J}\boldsymbol{F}\boldsymbol{S}\boldsymbol{F}^T. \tag{36}$$

To summarize, the expressions for the virtual work associated with the stress may be expressed

$$\delta W^{\text{int}} = \int_\Omega \boldsymbol{\sigma} : \delta\boldsymbol{\epsilon}\,\mathrm{d}\Omega = \int_{\Omega_0} \boldsymbol{P} : \delta\boldsymbol{F}\,\mathrm{d}\Omega_0 = \int_{\Omega_0} \boldsymbol{S} : \delta\boldsymbol{E}\,\mathrm{d}\Omega_0 \tag{37}$$

5 The Updated and Total Lagrangian Formulation

The statement of virtual work equates the virtual work performed by the body and the external work. Nothing, per se, requires that the two terms be performed using the some reference configuration, however, traditionally, the updated and total Lagrangian formulations are written in the current and initial configurations respectively. The updated Lagrangian formulation, therefore, takes the form given in Equation 3,

$$\int_\Omega \boldsymbol{\sigma} : \delta\boldsymbol{\epsilon}\mathrm{d}\Omega = \int_\Omega \rho\boldsymbol{b}^T\delta\boldsymbol{u}\mathrm{d}\Omega + \int_\Gamma \boldsymbol{\tau}^T\delta\boldsymbol{u}\mathrm{d}\Gamma. \tag{38}$$

To write down the total Lagrangian equation requires that the external work be written in terms of the initial coordinate system. Noting that the differential mass, $\mathrm{d}m$, is

$$\mathrm{d}m = \rho\mathrm{d}\Omega = \rho_0\mathrm{d}\Omega_0, \tag{39}$$

the body force term, with $\boldsymbol{b}$ defined per unit mass is readily expressed as

$$\int_\Omega \rho\boldsymbol{b}^T\delta\boldsymbol{u}\mathrm{d}\Omega = \int_{\Omega_0} \rho_0\boldsymbol{b}^T\delta\boldsymbol{u}\mathrm{d}\Omega_0. \tag{40}$$

The virtual work associated with the traction force is also simply expressed, however, the symbolic representation gives little insight,

$$\int_\Gamma \tau^T\delta\boldsymbol{u}\mathrm{d}\Gamma = \int_{\Gamma_0} \tau_0^T\delta\boldsymbol{u}\mathrm{d}\Gamma_0. \tag{41}$$

The total Lagrangian formulation is therefore

$$\int_{\Omega_0} \boldsymbol{P} : \delta\boldsymbol{F}\mathrm{d}\Omega_0 = \int_{\Omega_0} \boldsymbol{S} : \delta\boldsymbol{E}\mathrm{d}\Omega_0 = \int_{\Omega_0} \rho_0\boldsymbol{b}^T\delta\boldsymbol{u}\mathrm{d}\Omega_0 \int_{\Gamma_0} \tau_0^T\delta\boldsymbol{u}\mathrm{d}\Gamma_0. \tag{42}$$

Note that in both the updated and total Lagrangian formulations that $\boldsymbol{u}$ is defined in terms of the global coordinate system that remains constant with time. The virtual work ultimately provides us with the nodal forces,

$$\delta W = \boldsymbol{f}^T\delta\boldsymbol{u} = \sum_{A,i} f_{Ai}\delta u_{Ai} \tag{43}$$

where A is the node and i is the component of the force. The nodal force f_{Ai} is therefore independent of whether the updated or total Lagrangian formulation is used, and they will produce the same solution. Since the forces are the same, the stiffness matrix is also independent of the formulation.