ROBUST PARAMETER DESIGN IN LS-OPT

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ABSTRACT

Robust parameter design creates designs insensitive to the variation of specific inputs. The paper discusses the robust parameter design capability within the context of its implementation in LS-OPT version 3.2. The paper provides the following details: the fundamentals of robust parameter design, the design of experiments based methodology used in LS-OPT, and an example problem.

KEYWORDS:

Robust parameter design, LS-OPT

INTRODUCTION

Robust parameter design creates designs insensitive to the variation of specific inputs.

The field of robust design relies heavily on the work of Taguchi. Taguchi's insight was that it cost more to control the sources of variation than to make the process insensitive to these variations [1]. An alternate view of Taguchi [2] is that building quality into a product is preferable to inspecting for quality. Also, in simulation, the actual results of a robust system are more likely to conform to the anticipated results [1].

The robust design problem definition requires considering two sets of variables: (i) the noise variables causing the variation of the response and (ii) the control variables which are adjusted to minimize the effect of the noise variables. The method adjusts the control variables to find a location in design space with reduced gradients so that variation of the noise variable causes the minimum variation of the responses.

FUNDAMENTALS

The robustness of a structure depends on the gradient of the response function as shown in Figure 1. A flat gradient will transmit little of the variability of the variable to the response, while a steep gradient will amplify the variability of the variable. Robust design is therefore a search for reduced gradients resulting in less variability of the response.

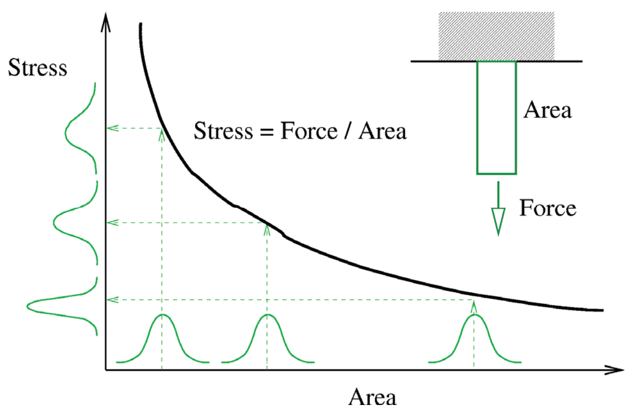


Figure 1: Robustness considering a single variable. Larger mean values of the area result in a smaller dispersion of the stress values. Note that the dispersion of the stress depends on the gradient of the stress-area relationship.

The variation of the response is caused by a number of variables, some which are not under the control of the designer. The variables are split in two sets of variables:

- *Control variables.* The variables (design parameters) under the control of the designer are called control variables,
- *Noise variables.* The parameter not under the control of the designer are called noise variables.

The relationship between the noise and control variables as shown in Figure 2 is considered in the selecting of a robust design. The control variables are adjusted to find a design with a low derivative with respect to the noise variable.

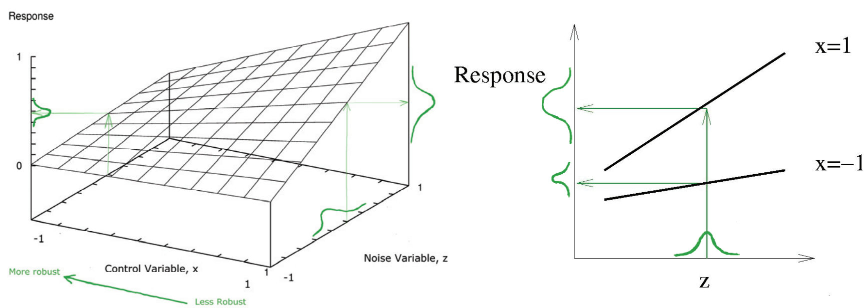


Figure 2: Robustness of a problem with both control and noise variables. The effect of the noise variable *z* on the response variation can be constrained using the control variable *x*. For robustness, the important property is the gradient of the response with respect to the noise variable. This gradient prescribes the noise in the response and can be controlled using the control variables. The gradient, as shown in the figure, is large for large values of the control variable. Smaller values of the control variable will therefore result in a more robust design, because of the lower gradient and accordingly less scatter in the response.

METHODOLOGY

The dual response surface method as proposed by Myers and Montgomery [3] using separate models for process mean and variance is considered. Consider the control variables **x** and noise variables **z** with $Var(\mathbf{z}) = \sigma_z^2 \mathbf{I}_{r_z}$. The response surface for the mean is $E_z[y(x,z)] = \beta + x'\beta + x'\beta x$ considering that the noise variables have a constant mean. Response surface for variance considering only the variance of the noise variables is $Var_z[y(x,z)] = \sigma_z^2 l'(x) l(x) + \sigma^2$ with $Var(\mathbf{z}) = \sigma_z^2 \mathbf{I}_{r_z}$, σ^2 the model error variance, and l the vector of partial derivatives $l(x) = \dfrac{\partial y(x,z)}{\partial z}$.

The search direction required to find a more robust design is requires the investigation of the interaction terms $x_i z_j$. For finding an improved design, the interaction terms are therefore required. Finding the optimum in a large design space or a design space with a lot of curvature requires either an iterative strategy or higher order terms in the response surface.

For robust design, it is required to minimize the variance, but the process mean cannot be ignored. Doing this using the dual response surface approach is much simpler than using the Taguchi approach because multicriteria optimization can be used. Taguchi identified three targets: smaller is better, larger is better, and target is best. Under the Taguchi approach the process variance and mean is combined into a single objective using a signal-to-noise ratio (SNR). The dual response surface method as used in LS-OPT does not require the use of a SNR objective. Fortunately so, because there is wealth of literature in which SNRs are criticized [3]. With the dual response surface approach both the variance and mean can be used, together or separately, as objective or constraints. Multicriteria optimization can be used to resolve a conflict between process variance and mean as for any other optimization problem.

Visualization is an important of investigating and increasing robustness. As Myers and Montgomery state: "The more emphasis that is placed on learning about the process, the less important *absolute optimization* becomes."

EXPERIMENTAL DESIGNS

One extra consideration is required to select an experimental design for robust analysis: provision must be made to study the interaction between the noise and control variables.

Finding a more robust design requires that the experimental design considers the $x_i z_j$ cross-terms, while the x_i^2 and z_i^2 terms can be included for a more accurate computation of the variance.

The crossed arrays of the Taguchi approach are not required in this response surface approach where both the mean value and variance are computed using a single model. Instead combined arrays are used which use a single array considering x and z combined.

LS-OPT IMPLEMENTATION

The implementation of robust design in LS-OPT only required that the variation of a response be available as a composite. The standard deviation of a response is therefore available for use in a constraint or objective, or in another composite.

The LS-OPT command defining the standard deviation of another response or composite to be a composite is:

composite 'var x11' noise 'x11'

The variation of response approximated using response surfaces is computed analytically as documented for the LS-OPT stochastic contribution analysis [4]. For neural nets and composites a quadratic response surface approximation is created locally around the design, and this response surface is used to compute the robustness. Note that the recursion of composites (the standard deviation of a composite of a composite) may result in long computational times especially when combined with the use of neural networks. *If the computational times are excessive, then the problem formulation must be changed to consider the standard deviations of response surfaces.*

EXAMPLE

Consider the two-bar truss problem as shown in Figure 3. Variable x1, the area, is a noise variable described using a normal distribution with a mean of 2.0 and a standard deviation of 0.1. The distance between the legs, x2, is a control variable which will be adjusted to control the variance of the responses. The maximum stress is considered as the objective for the robust design process.

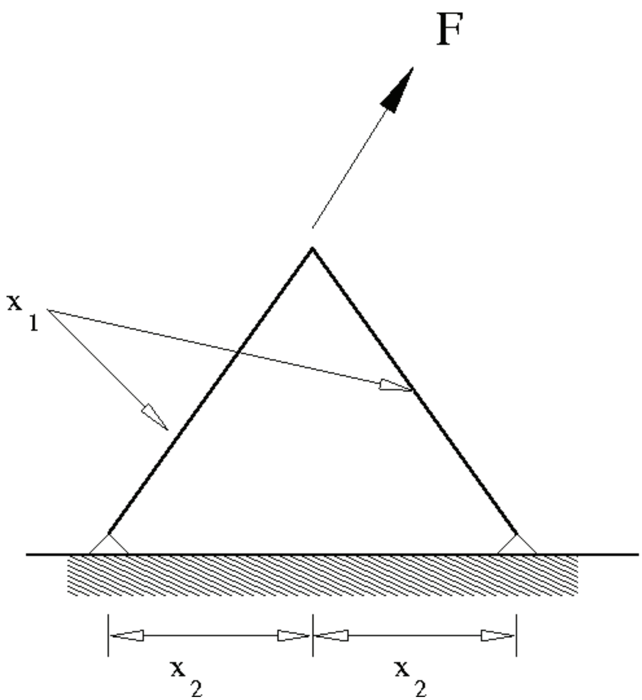


Figure 3: The two-bar truss problem. The problem has two variables: the thickness of the bars and the leg widths as shown. The bar thicknesses are noise variables while the leg widths are adjusted (control variables) to minimize the effect of the variation of the bar thicknesses. The maximum stress in the structure is monitored.

An interaction response surface considering the effect of variables and the interaction between variables is used to approximate the stress response.

The stress response is shown in Figure 4. From the figure it can be seen that the 'base' variable must be set to values of large than 0.4 to obtain a minimum variation of the stress considering that the design will then be in the flattest region of the response. A value of 0.5 is obtained in the optimization results as shown in Figure 5. Also shown in the optimization results is the design history of the stress standard deviation. Note that the standard deviation response stayed fairly insensitive to changes in the control variable after iteration 4 and that the initial subregion size for the 'base' variable was too large, resulting in initial increase in 'base' variable due to an inaccurate initial response surface.

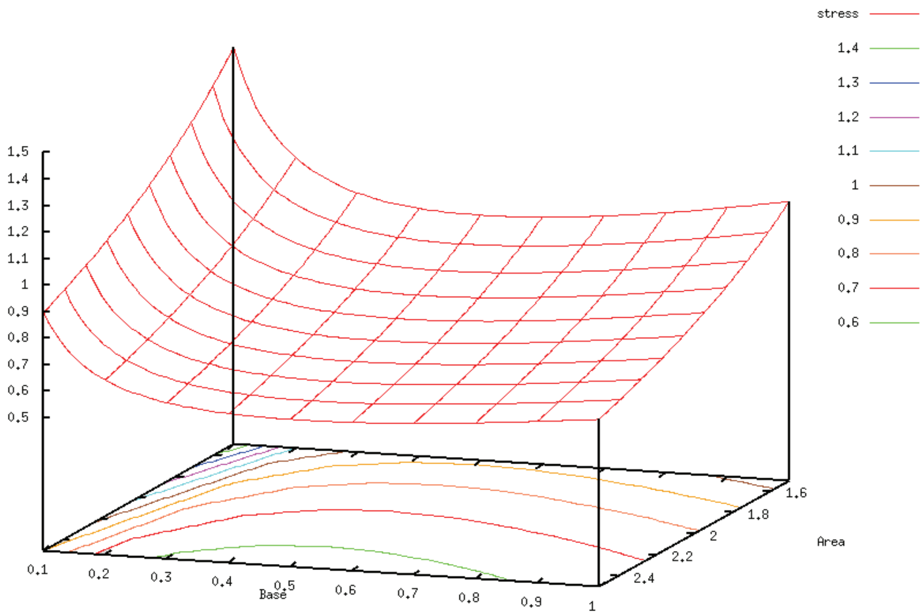
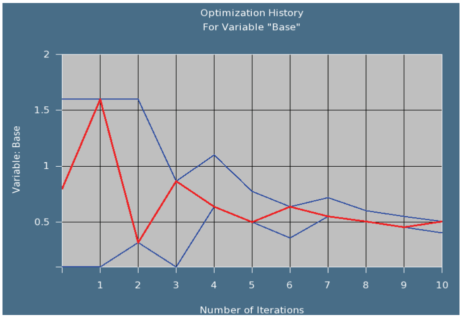


Figure 4 Contours of stress response. The flattest part of the response is when variable 'base' equals 0.5.



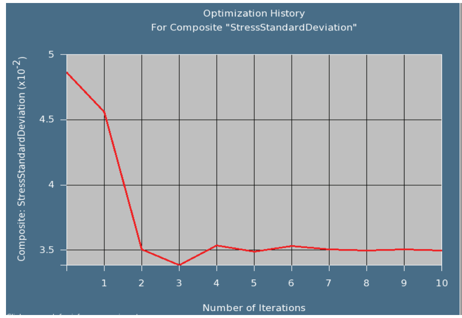


Figure 5 Optimization histories. Design variable 'base' is shown on the left and the standard deviation of the stress response is shown on the right.

SUMMARY

Robust parameter design creates designs insensitive to the variation of specific inputs. The variables are considered in two sets: the noise variables are not under the control of the designer and causes problems, while the control variables can be adjusted by the engineer to minimize the effect of the noise variables. The procedure finds location in the design space with reduced gradients with respect to the noise variables. Such a design has the minimum variation of the responses.

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