# ON TECHNIQUES FOR SIMULATING EFFECTS OF CAVITATION ASSOCIATED WITH THE INTERACTION BETWEEN STRUCTURES AND UNDERWATER EXPLOSIONS USING LS-DYNA

*Q.W. Ma and D.J. Andrews*
*Department of Mechanical Engineering*
*University College London*
*Torrington Place, London WC1E 7JE, UK*

## Abstract

In this paper, a two-step technique is suggested to reduce the reflection of shock waves from a truncated boundary without significantly smearing the incident shock wave and to suppress spurious oscillations without significantly affecting accuracy. By comparing the results for a one-dimensional case with a well-known analytical solution, it is shown that the technique works very well.

## Introduction

This work is motivated by the interests of ship designers to avoid the cost of expensive physical tests by using advanced commercial finite element codes to demonstrate an acceptable response of structures to shock waves created by underwater explosions. Many investigators (see for example, Felippa, C.A. & Deruntz, J.A., 1984) have shown that cavitation in the fluid domain may significantly affect the response of a structure immersed within it or at its surface. LS-DYNA is a hydrocode, which has the potential to simulate the problem.

The purpose of this work is to investigate techniques used for simulating the effects of cavitation associated with the interaction between surface structures and underwater explosions by LS-DYNA. The explosions are modelled by shock waves applied on a part (or possibly the whole) of a truncated boundary of a fluid domain. Three issues have been considered for solving the problem: (1) the boundary condition on the truncated boundary; (2) the need to suppress spurious oscillations arising from the modelling and (3) implementation of material laws in the fluid. For a truncated boundary without an incident wave emerging from it, a non-reflecting condition can be used directly for reducing the reflection of shock waves. However, for a truncated boundary where the incident wave is applied, a difficulty arises. The reason is that applying the non-reflecting condition, together with the incident wave to the boundary, results in the incident wave being incorrectly modified; on the other hand, if the non-reflecting condition is not employed, the shock wave scattered due to the motion of the structure is inevitably reflected back, disturbing the physical state of the fluid. This difficulty is solved by an approach that can deal with the conflict whilst maintaining accuracy to a level that is acceptable from an engineering standpoint. As is well known (see, for example, Felippa and DeRuntz, 1984), spurious oscillations may occur when cavitation exists in a fluid domain. A common method of suppressing these oscillations is to introduce artificial damping, as done for instance, by Felippa and DeRuntz (1984). However, this damping may also reduce physical pressure and therefore alter the response of the structure, which is clearly not desirable. A technique is suggested here, which can effectively suppress spurious oscillations without significantly reducing the pressure. Several material laws that may be used for the problem are available in LS-DYNA. An investigation of the implementation and effects of these laws is presented

## Problem Description

The problem investigated is a flat plate initially resting on the surface of half space of fluid. In numerical simulation, the fluid domain is truncated to 150 inch (3.81 m) down below the plate, and a shock wave, represented by a step exponential decay function $p = p_0 e^{-t/\tau}$ ($\tau$: decay time; $p_0$: peak pressure), is applied to the truncated boundary, as shown in Figure 1. This is effectively a one-dimensional problem, and so a column with four elements in each layer is considered with symmetrical conditions imposed on all four vertical surfaces of the column. All the parameters are the same as those used by Felippa and DeRuntz (1984), as given in Table 1, which were presented in Imperial unit there and are also presented in SI equivalent unit here.

**Table 1**

| | Imperial unit | SI equivalent |
|---|---|---|
| Density of fluid | 9.3455E-05 lb sec$^2$ in$^{-4}$ | 1000 kg/m$^3$ |
| Density of plate | 5.3297E-04 lb sec$^2$ in$^{-4}$ | 5700 kg/m$^3$ |
| Sound speed in fluid | 57120 in/sec | 1450 m/sec |
| Atmospheric pressure | 14.7 psi | 1 MPa |
| Peak pressure of shock wave | 103 psi | 7.1 MPa |
| Decay time of the shock wave | 0.9958E-03 sec | 0.9958E-03 sec |

The following considerations led to this example being chosen:

1) An analytical solution for it has been obtained by Bleich and Sandler (1970), which can be used for validating the numerical results.
2) The techniques suggested through this problem are also valid for two- and three-dimensional problems.
3) It is simple and suitable for a large number of numerical investigations on PC machines.

4) The structure may be considered as the central part of a very large three-dimensional structure. Therefore, the results obtained from it can shed some light on the response of such a large three-dimensional structure.

The cavitation is assumed to occur when $p_t = p_w + p_a \leq 0$ or $p_w \leq -p_a$ where $\boldsymbol{p}_t$ and $\boldsymbol{p}_a$ are the total and atmospheric pressure, respectively and $\boldsymbol{p}_w$ is the difference between them, including the incident, scattered and hydrostatic pressures due to water density and plate weight. In the cavitation region, the total pressure is made to go to zero.

A shock wave represents a jump in the medium's physical quantities, such as pressure, density and velocity, and so the disturbance produced by a shock is theoretically discontinuous. Due to the discontinuity, numerical analysis may not yield reasonable solutions. To resolve this, an artificial viscous term (called artificial bulk viscosity) is usually added to the physical pressure so that the shock discontinuity is changed into a rapidly varying but continuous transition wave. In LS-DYNA, the following expression for bulk viscosity is used (Hallquist, J.O., 1998):

$$q = \begin{cases} Q_1 \rho l^2 \dot{\varepsilon}_{kk}^2 - Q_2 \rho l c \dot{\varepsilon}_{kk} & \dot{\varepsilon}_{kk} < 0 \\ 0 & \dot{\varepsilon}_{kk} \geq 0 \end{cases} \tag{1}$$

where $\dot{\varepsilon}_{kk}$ is the strain rate of deformation; $Q_i$ (i=1,2) are artificial damping coefficients; ρ is the density of fluid; $c$ is sound speed of fluid; $l$ is characteristic length of an element , given by $l = \sqrt[3]{V_e}$ ($V_e$ is the volume of a element). It should be noted that the instantaneous values of $\rho$, $c$ and $l$ may be used, and so $q$ depends not only on $\dot{\varepsilon}_{kk}$ and $\dot{\varepsilon}_{kk}^2$ but also on their higher orders.

## Boundary Condition

As mentioned above, the physical fluid domain is truncated for the purpose of computation, and the incident wave is applied to the truncated boundary. When the incident wave reaches the plate, it will be reflected and scattered. The reflected and scattered waves travel back to the boundary. Physically, the pressure should be transmitted through the boundary. However, if no proper condition is employed in numerical analysis, these waves are inevitably reflected back, contaminating the physical flow. If, on the other hand, a condition is applied to the boundary, together with the incident wave, the incident wave will unrealistically be modified from the beginning. Numerical investigations for the one-dimensional problem have been carried out for these two cases, and the vertical velocities of the plate are shown in Figure 2, together with the velocity obtained by a proper technique discussed below. It can be seen that if a non-reflecting condition is not applied on the boundary, then when the reflected wave from the truncated boundary reaches the plate, the calculated velocity becomes considerably larger in the latter half of the period than the much more reasonable result shown in the Figure 2. If on the other hand the non-reflecting condition is implemented from the very beginning, together with the incident pressure, the velocity is significantly reduced in the first half of the period. Clearly, the results obtained in these two cases would not be considered acceptable.

It should be noted here that this difficulty is not only associated with the one-dimensional problem but also exists for two- or three- dimensional cases. In these latter cases, the boundary where the incident wave is applied may not be the whole of the truncated boundary, but is certainly a part of the boundary. For a truncated boundary (as shown in Figure 1) without an incident wave, a non-reflecting condition can be used from the beginning. However, on the part of the truncated boundary where the incident wave is applied, the same difficulty as discussed in the previous paragraph will arise.

An approximate approach is suggested in this work to resolve this difficulty. In this approach, the whole computational process is divided into two steps. At the first step, the incident shock wave is applied to the truncated boundary (or a part of it) until the pressure becomes a small fraction of its peak value without any imposed condition to deal with the reflection. Then, the restart facility of LS_DYNA is utilised for the calculation of the second step, in which the non-reflection condition is imposed on the boundary. It is necessary that the first step stops before the scattered waves from the structure reach the boundary (see Figure 1). The longest time for this step is given by $T_{1b} \leq 2L/c$ , where $L$ is the shortest distance between the truncated boundary and the structure. In addition, the calculated time for the first step should also satisfy $T_{1b} \geq -\tau \ln \alpha$ for the incident pressure with an exponential decay like the one used in this analysis. Note α is the ratio of instantaneous incident pressure at a time $t$ to the pressure when it is at its peak. α should be a small value appropriately selected, such as 0.04. For given values of τ and α, the computational domain must be large enough to meet these two requirements. For example, if α is set to be 0.04, the length of the computational domain, $L$, should be larger than 91 inch (2.31m) from the above inequalities.

The resultant velocity of the plate obtained by this technique has been plotted in Figure 2 as a dotted line. The result agree well with the discrete values obtained from the analytical solution given by Bleich and Sandler (1970), as shown in Figure 3, which implies that the proposed technique effectively resolves the difficulty discussed above. One may notice that a larger oscillation exists in Figure 2 than in Figure 3. That is because a larger level of damping is used in the second step for the results shown in Figure 3. The reason why different damping is employed is presented below. Although the effectiveness of this two-step technique in the two- or three-dimensional cases is not presented here, it is considered that it should also be applicable in those cases, as long as similar treatment is used for the truncated boundary where the incident wave is applied.

It should be noted that this technique is not suitable for cases where the applied wave is a step function without decay. Fortunately, this is unlikely situation with shock waves in water.

## Suppression of spurious oscillations

As is well known, the occurrence of cavitation can induce spurious pressure oscillations (see, for example, Felippa & DeRuntz, 1984), which can be translated into the spurious oscillations in the response of the structure, as shown in Figure 4. A common method of suppressing these oscillations is to introduce an appropriate damping value. For example, Felippa & DeRuntz (1984) added a term, $\beta hc^2\dot{s}$ ($h$, time step; $\dot{s}$, strain rate and $\beta$, an appropriately selected damping coefficient) to a pressure expression.

In LS-DYNA, two approaches may be employed to introduce the damping term. One of them is through using the bulk viscosity given in Equation 1. As discussed above, the main purpose of using bulk viscosity is to resolve the difficulty associated with the shock induced discontinuity. It mainly plays a role in the region ahead of the shock wave, where $\dot{\varepsilon}_{kk} < 0$ but does not have an effect at the region behind the shock wave where $\dot{\varepsilon}_{kk} > 0$ is usually satisfied. Although cavitation occurs after the shock, these spurious oscillations can induce a negative strain rate (i.e. the first case in Equation 1) and thus can be reduced by using the bulk viscosity. The bulk viscosity term with $Q_1 = 0$ in Equation (1) is very similar to the term used by Felippa & DeRuntz (1984). The difference is that there is no restriction on strain rate in their work. The other approach is through using the damping coefficient in a material law. The specific technique for this purpose depends on which material law is used. In this section, it is assumed that the option *MAT_ELASTIC_FLUID (Keyword Manual, 1999) is employed, in which the pressure and deviatoric stress are expressed as:

$$\dot{p} = -K\dot{\varepsilon}_{ii} \tag{2a}$$

$$S_{ij} = V_c lc\rho\dot{\varepsilon}'_{ij} \tag{2b}$$

where $K = \rho c^2$ is the bulk modulus of fluid, $\dot{\varepsilon}_{ii}$ is the strain rate as before, $\dot{\varepsilon}'_{ij}$ is the deviatoric strain rate, $S_{ij}$ is the deviatoric stress and $V_c$ is an selected damping coefficient.

Figures 5 to 7 illustrate the velocities of the plate obtained by using different methods for suppressing the spurious oscillations. It can be seen from these figures that using the two methods with proper values of parameters may effectively reduce the spurious oscillations, and the oscillations become smaller as the damping coefficients are increased. However, the peak of the structural response is decreased at the same time when the damping coefficients in all cases are increased. This makes the computational results deviate from their analytically derived values. For example, when the value of $V_c$ is increased from 0.015 to 0.225, the peak values of the velocity is reduced from 29.7 in/sec to 28.3 in/sec, this differing more noticeably from the analytical value of 29.76 in/sec than with the lower value of $V_c$. One reason for this may be that the application of damping to the fluid domain from beginning leads to dispersion or dissipation of the shock wave.

Therefore, it is necessary to seek a better way to remove the spurious oscillations without significantly affecting accuracy. To this end, it is worth noting that spurious oscillations are associated only with cavitation, and therefore a large value of damping coefficient is required to suppress these oscillations just for this stage of process. Given this, it is suggested that the problem is calculated in two steps, in a similar manner to that proposed for dealing with the boundary condition in the previous section. In the first step, a very small damping coefficient (for example the real viscosity of the fluid) is used, while in the second step a larger value is employed. Clearly, the first step must stop before the spurious oscillations occur. From several numerical tests, it is proposed that the computation time for the first step can be set as

$T_1 s = L/c + 0.5\tau$ for this purpose. Bearing in mind that the period of the first step in the previous case, with the truncated boundary condition, must satisfy the relationship: $-\tau \ln \alpha \le T_{1b} \le 2L / c$, then the computation time for the first step in this instance should be chosen according to the following formula:

$$\begin{cases} T_1 = L/c + 0.5\tau \\ -\tau \ln \alpha \le T_1 \le 2L/c \end{cases} \tag{3}$$

This can be satisfied by adjusting the value of $L$ once the values $\tau$ and $\alpha$ are specified.

Utilizing these techniques for the one-dimensional problem, one can obtain results that correlate well with the analytical solution of Bleich and Sandler (1970), as shown in Figure 3. In this figure, the parameters for the first step are:$T_1$=3.20E-03sec, $V_c$ =1.307E-12, $Q_1$ =0.0 and $Q_2$ =0.0; while in the second step they are: $V_c$ =0.5, $Q_1$ =0.0 and $Q_2$ =0.0. Other values of these parameters may also be used in the second step to obtain similar results. When plotting the analytical solution, the difference in starting time has been considered. For the analytical solution of Bleich and Sandler (1970), the shock wave hits the structure at $t$ = 0; while in the computation reported here, the shock wave needs to propagate though the whole computational fluid domain before it reaches the structure. In order to make the two starting times correspond to each other, the time for the analytical solution is adjusted when plotting Figure 3 so that the time corresponding to the peak value of the analytical solution is the same as that of the LS-DYNA numerical result.

## Discussions on the implementation of material laws

In addition to the material law given in Equation (2), there are other laws available in LS-DYNA, which can be utilised to simulate the fluid behaviour associated with the interaction between structures and shock waves created by underwater explosions. The discussions in this section will be devoted to two other material laws.

The first is the elastic-plastic hydrodynamic law represented in LS-DYNA by keyword *MAT_ELASTIC_PLASTIC_HYDRO. In this law, deviatoric stresses are given by:

$$S_{ij} = 2\mu_w \varepsilon'_{ij} \tag{4}$$

where $\mu_w$ is the dynamic coefficient of viscosity of fluid. When this law is applied to a material with the possibility of plastic deformation, this modification is automatically made if the calculated stress exceeds specified yield stress. When applying it to a fluid, the plastic deformation does not need to be considered. To avoid modification in this case, the yield stress required in this law should be set to a very large value, far beyond the range of possible stresses. This is chosen to be 1.E+30 in this simulation. Equation (4) alone does not provide information for pressure and so is not enough to determine the stress field, unlike the keyword *MAT_ELASTIC_FLUID. An equation of state (EOS) is required together with this law to calculate the pressure. Several options for pressure computation are available in LS-DYNA. The following equation is used in this work:

$$p = C_0 + C_1\lambda + C_2\lambda^2 + C_3\lambda^3 + (C_4 + C_5\lambda)E_n \tag{5}$$

where $\lambda = \rho / \rho_0 - 1$; $E_n$ is energy; $C_i$ $(i = 0,1,\Lambda\ 5)$ are the hydrodynamic constants. $C_i$ is set to $C_i = \rho_0 c^2$ with the other four being zero in the following investigation.

The difference between this material law and that represented by keyword *MAT_ELASTIC_FLUID comes from two sources. One is the expression of deviatoric stress. Although Equation (5) is very similar to Equation (2b), the coefficients before deviatoric strain rate could not be equivalent. That is because 2 $\mu_w$ is a constant throughout the whole computation process while the value of $V_c l c \rho$ may change with time although $V_c$ is kept as a constant. The other reason for the difference between the two laws is due to the fact that $\rho_0 c^2(\rho / \rho_0 - 1)$ in Equation (5) is not generally equal to($-\varepsilon_{ii}$ in Equation (2a) unless the stain rate is very small.

An investigation has been carried out by numerical tests on the prediction using the material law represented by *MAT_ELASTIC_PLASTIC_HYDRO with EOS in Equation (5). Figure 8 illustrates the results for different values of $\mu_w$ together with the analytical solution. It shows that when $\mu_w$ is small, spurious oscillations are not suppressed; on the other hand when it is big enough to suppress the oscillations, the computational velocity tends to diverge from the analytical result. That means that the use of $\mu_w$ alone may not effectively suppress the oscillations without reducing the accuracy of results, unlike the use of $V_c$ in previous formulation of material law (i.e. *MAT_ELASTIC_FLUID). The reason for this may be due to the difference between the two laws, particular the difference in the coefficients before deviatoric strain rate. In order to improve the performance using this law, further investigations were carried out by combining effects of the bulk viscosity term $Q_2$ in Equation 1 and $\mu_w$ above. The combination of these two terms gives satisfactory results, as is shown in Figure 9.

The null material law, represented by keyword *MAT_NULL, is considered next. In this model, the strength of the material is neglected and deviatoric stresses are not calculated. Nevertheless, it allows the inclusion of an equation of state for the estimation of pressure (Equation 5 is used in the following). In addition, an optional viscous stress may be computed by the formula:

$$\sigma_{ij} = \mu \varepsilon'_{ij} \tag{6}$$

as is given in the theory manual of LS-DYNA, where $\sigma_{ij}$ is the fluid Cauchy stresses and μ is the viscosity coefficient. The main difference between Equation 6 and Equations 2b and 4 is due to the stress ($\sigma_{ij}$) being calculated, instead of deviatoric stress ($S_{ij}$).

A number of numerical tests with various parameters have been carried out in order to investigate the performance of this law in simulating the shock wave propagation in the fluid and the response of the one-dimensional structure. The results are not presented here for brevity and since they are broadly similar to the results presented here. From these results it can be concluded that a similar performance can be obtained for this *MAT_NULL law to that using *MAT_ELASTIC_FLUID, i.e. Equation 2, when the values of $V_c$ and μ are the same. This is surprising since the two parameters are not even of the same dimensions - one of them is non-dimensional and the other has the dimension of the coefficient of dynamic viscosity. When the two parameters have the same value as in this paper, the calculated values of stresses in Equation 2b should be several times larger than those from Equation 6 if the two formulas are correctly given in the manual of LS-DYNA. This inconsistency may not obstruct the application of the material law but one should be aware of it when choosing a value for μ.

## Cavitation effect

For the problem considered in this work, the effect of cavitation is well know since it has been discussed by Bleich and Sandler (1970) theoretically. What is considered here is how to use LS-DYNA to investigate the cavitation effect. In all three material laws discussed above, there is an option to set the value of cutoff pressure (see Keyword Manual, 1999). During the calculation, the pressure is forced to be equal to the cutoff pressure when the calculated pressure is smaller than the latter. Therefore, when it is intended to consider the effect of cavitation, the cutoff pressure must be set to a realistic value, such as atmospheric pressure, as discussed above. When it is intended to exclude the effect of cavitation, this value should be set at a large negative value, such as -1.0E+20, which greatly exceeds the possible range of pressure in a typical problem.

Figure 10 presents the vertical velocity and displacement of the plate with or without the pressure cut-off. The results with the pressure cut off are obtained using exactly the same parameters as for Figure 3, while the results without the pressure cut-off are obtained by setting the cutoff pressure equal to -1.0E+20 and keeping all the other parameters unchanged. For comparison, the analytical solution for velocity is also included in Figure 10(a). It can be seen that the numerical results correlate well with the analytical solution for both the cases, namely with and without considering the effect of cavitation.

## Summary

Investigations have been carried out on techniques used for simulating by LS-DYNA the effects of cavitation associated with the interaction between structures and underwater water explosions. A two-step technique has been suggested in order to resolve the difficulty associated with the application of a boundary condition on truncated boundary and with the utilisation of artificial damping to suppress spurious oscillations. The computational time of the first step for this purpose is determined by a formula based on the speed of sound, the length of the fluid domain and the decay time of an incident shock wave. In addition, a discussion has been presented on the implementation of three material laws and on the method of studying the effect of cavitation. The results show that with the use of the technique, LS-DYNA can effectively simulate the response of a structure excited by a shock wave with cavitation conditions arising.

## References

Bleich, H.H. & Sandler, I.S., 1970, “Interaction between structures and bilinear fluids”, *Int. J. Solids Structures*, Vol. 6, pp. 617-639.

Felippa, C.A. & Deruntz, J.A., 1984, “Finite element analysis of shock-induced hull cavitation”, *Computer Methods in Applied Mechanics and Engineering*, Vol. 44, pp. 297-337.

Hallquist, J.O., 1998, “LS-DYNA theory manual”, Livermore Software Technology Corporation.

Keyword Manual, 1999, “LS-DYNA keyword user's manual”, Livermore Software Technology Corporation.

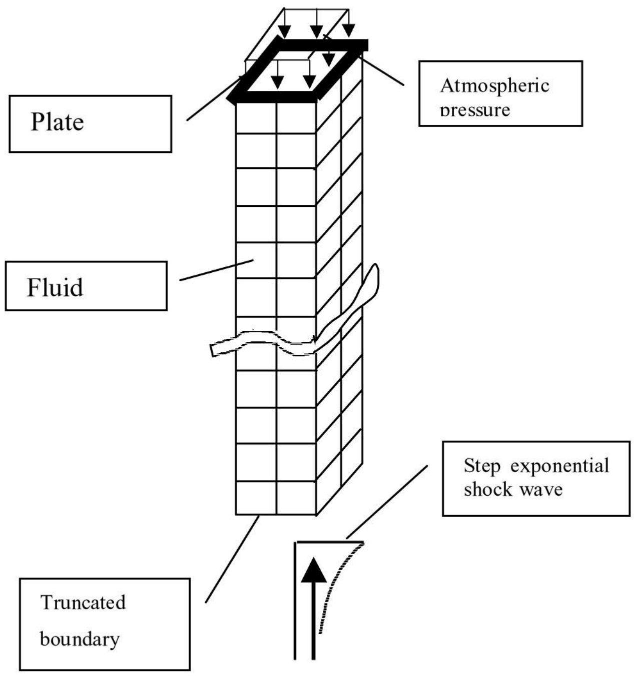


**Figure 1 Sketch of one-dimensional problem**

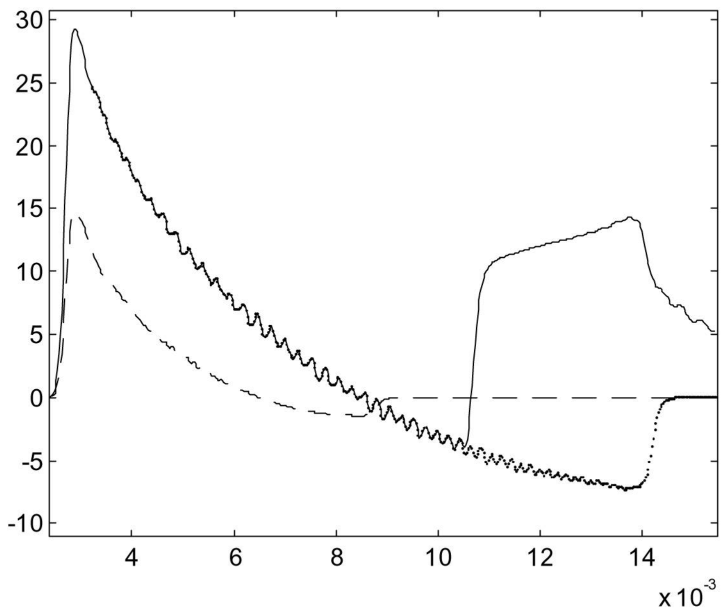


**Figure 2 Vertical velocity of the plate due to different implementation of the boundary condition (Solid line: without condition; Dashed line: with non-reflecting condition from beginning; Dotted line: with non-reflection condition at the second step)**

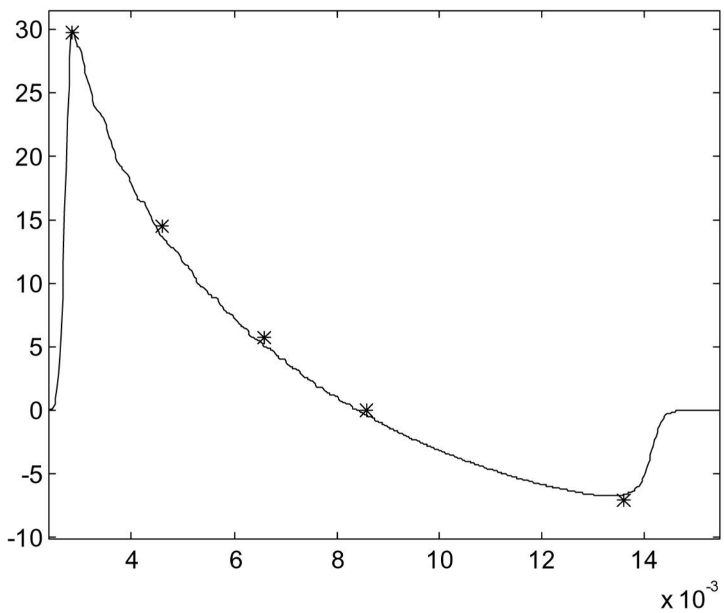


**Figure 3 Vertical velocity of the plate obtained by two step technique (solid line) and comparison with analytical solution (*) given by Bleich and Sandler (1970)**

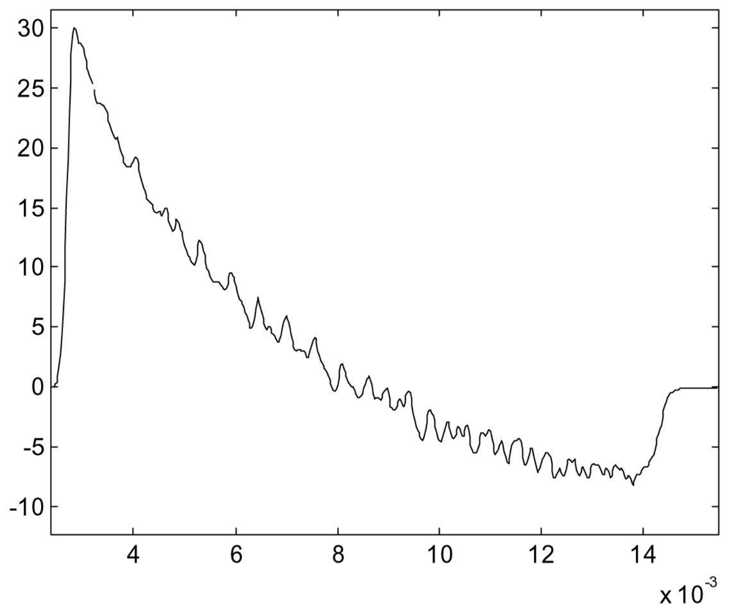


**Figure 4 Vertical velocity of the plate without suppression of spurious oscillations**

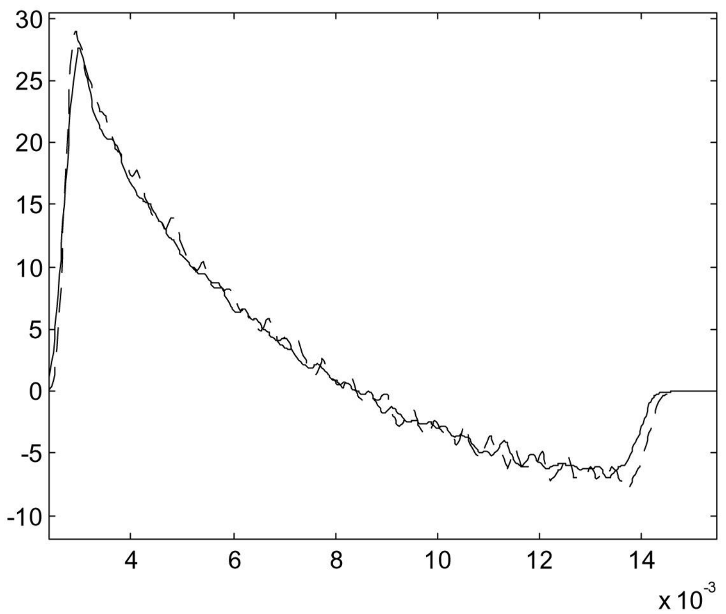


**Figure 5 Influence of bulk viscosity on vertical velocity of the plate by changing with $V_c$ =1.307E-12 and $Q_2$ = zero (Solid line with peak=27.6 in/sec: $Q_1$ =1.8E+04; Dashed line with peak = 29.0 in/sec: =0.3E+04,)**

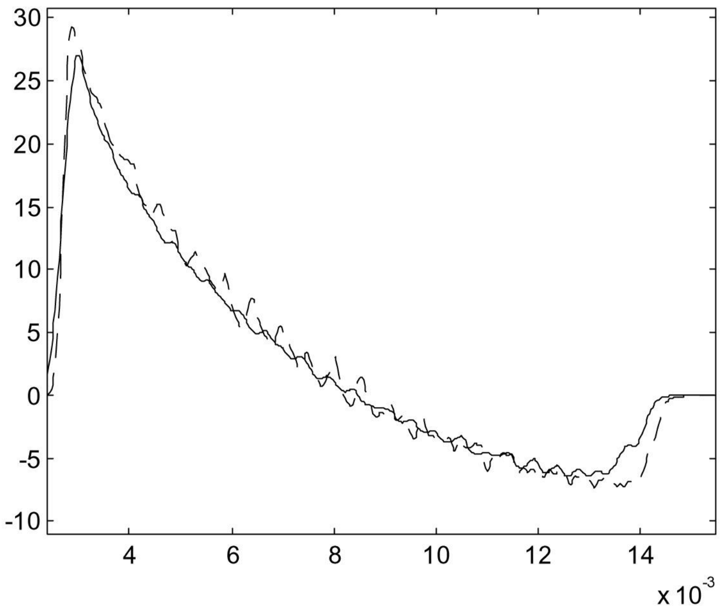


**Figure 6 Influence of bulk viscosity on vertical velocity of the plate by changing with $V_c$=1.307E-12 and $Q_1$= zero (Solid line with peak=27.0 in/sec: $Q_2$=0.5; Dashed line with peak = 29.3 in/sec: $Q_2$=0.1)**

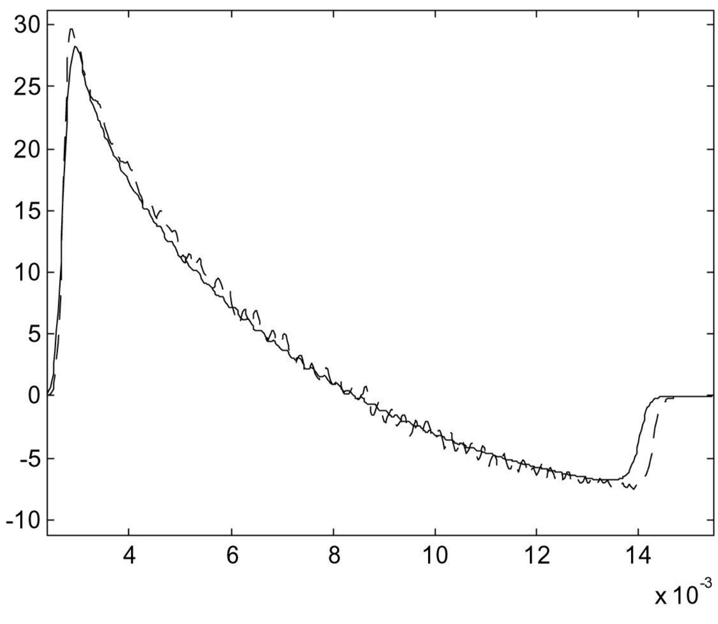


**Figure 7 Influence of viscosity in material law on vertical velocity of the plate with and $Q_1$ and $Q_2$ being zero (Solid line with peak=28.3 in/sec: $V_c$=0.225; Dashed line with peak = 29.7 in/sec: $V_c$=0.015)**

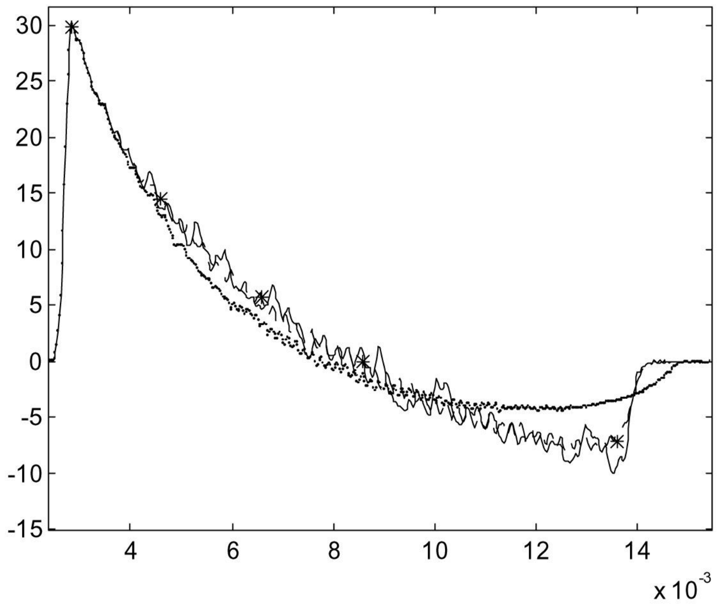


**Figure 8 Vertical velocity of the plate obtained using elastic-plastic hydrodynamic law with different damping in second step (Solid line: $\mu_w$=5, $Q_1$= 0 and $Q_2$=0.0; Dashed line: $\mu_w$=500, $Q_1$= 0 and $Q_2$=0.0; Dotted line: $\mu_w$=5000, $Q_1$= 0 and $Q_2$=0.0; *: analytical solution)**

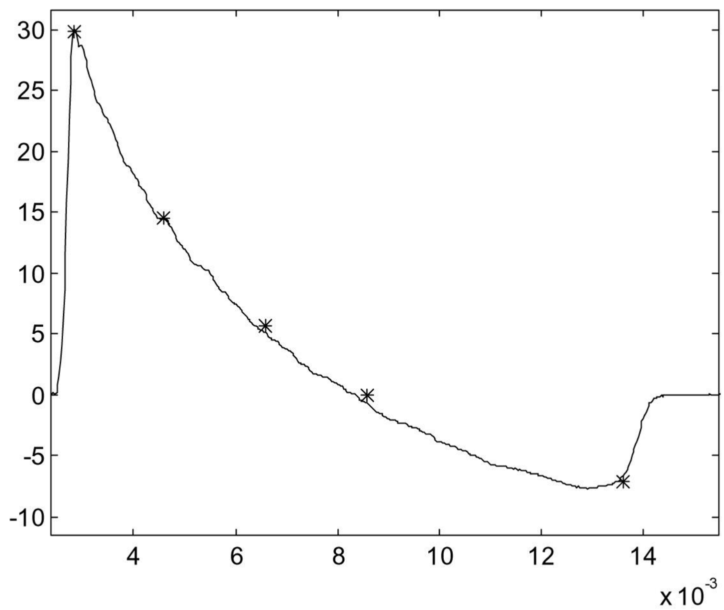


**Figure 9 Vertical velocity of the plate obtained using elastic-plastic hydrodynamic law with different parameters in second step (Solid line: $\mu_w$=500, $Q_1$= 0 and $Q_2$=0.5; *: analytical solution)**

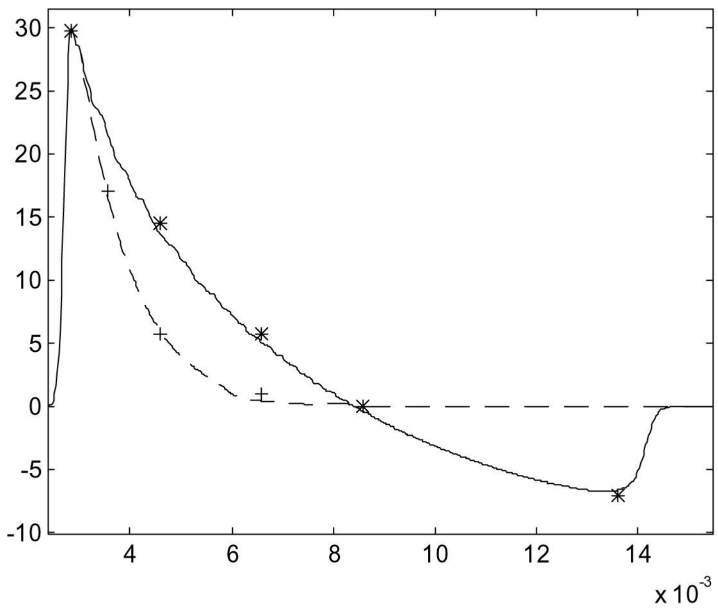


**Figure 10(a) Vertical velocity of plate**

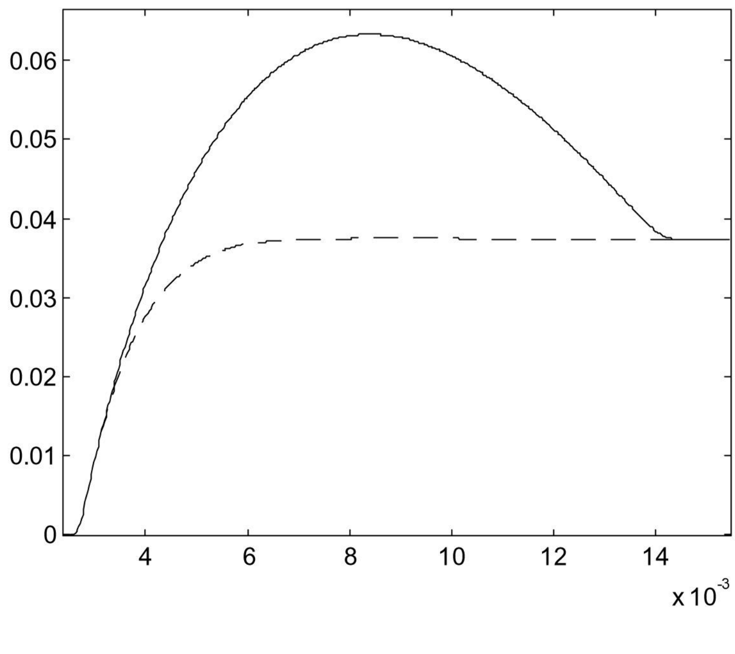


**Figure 10(b) Displacement of plate**

**Figure 10 Effects of cavitation (Solid line: with cavitation effect; Dashed line: without cavitation effect; * and +: analytical solutions by Bleich and Sandler)**