Mathematical Modeling of Asteroid Falling into the Ocean

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Abstract

Today, experimental information about large-scale collision tsunami is not available. That is why one of the main tools of studies is mathematical modeling. This paper considers falling of stone asteroid with diameter 1 km into the ocean 4 km deep. This asteroid collides with the Earth at a speed of 22 km/s at angles 30, 60 and 90 degrees.

Calculation of space body collision with a barrier is split into two stages. At the first stage, using finite-element code LS-DYNA® [1] and super computer SKIF-URAL of South-Ural State University under the support of OCC "STRELA", the process of interaction of body with the barrier was calculated.

Analysis of calculation data shows that for the angle of incidence 60 and 90 degrees, the results differ slightly. Even for the angle of incidence 30 degrees, we do not have big difference. That is why, one is to expect that the impact of tsunami on the sea shore for these angles of collision will be practically the same. Due to this reason, at the second stage of calculations, we considered the case of axisymmetric penetration of asteroid into the ocean. For describing cylindrically diverging surface wave and its impact on the shore with regard for the shelf profile, a special code was developed, in which approximation of shallow water was realized [3, 4].

It was given empirical formulae for calculation of the height of remote wave that is formed with underwater nuclear explosions [5]. Compared values are in good agreement. This means that using the approach to assessing the parameters of tsunami, which is proposed in this paper, is acceptable both for qualitative and quantitative description of this physical phenomenon.

As tentative assessments showed, the aftereffects of the falling of a stone asteroid with diameter ~1 km may be destructive for the ocean shore. Calculations showed that the wave height on the shelf increases from 60 to 100 m. Then the wave height on the shallow water decreases.

Impact-Drop Testing

Currently recognized is the danger for civilization resulting from asteroid collision with the Earth. Falling of hazardous space objects (HSO) into the ocean is of special attention. First of all, because the probability of falling into the ocean is twice as higher. Presence of thick layer of ocean water creates striking factor, i.e. gigantic tsunami. Additional stimulus for such studies was the discovery on the ocean bottom of several "collision" young craters (Fig. 1).

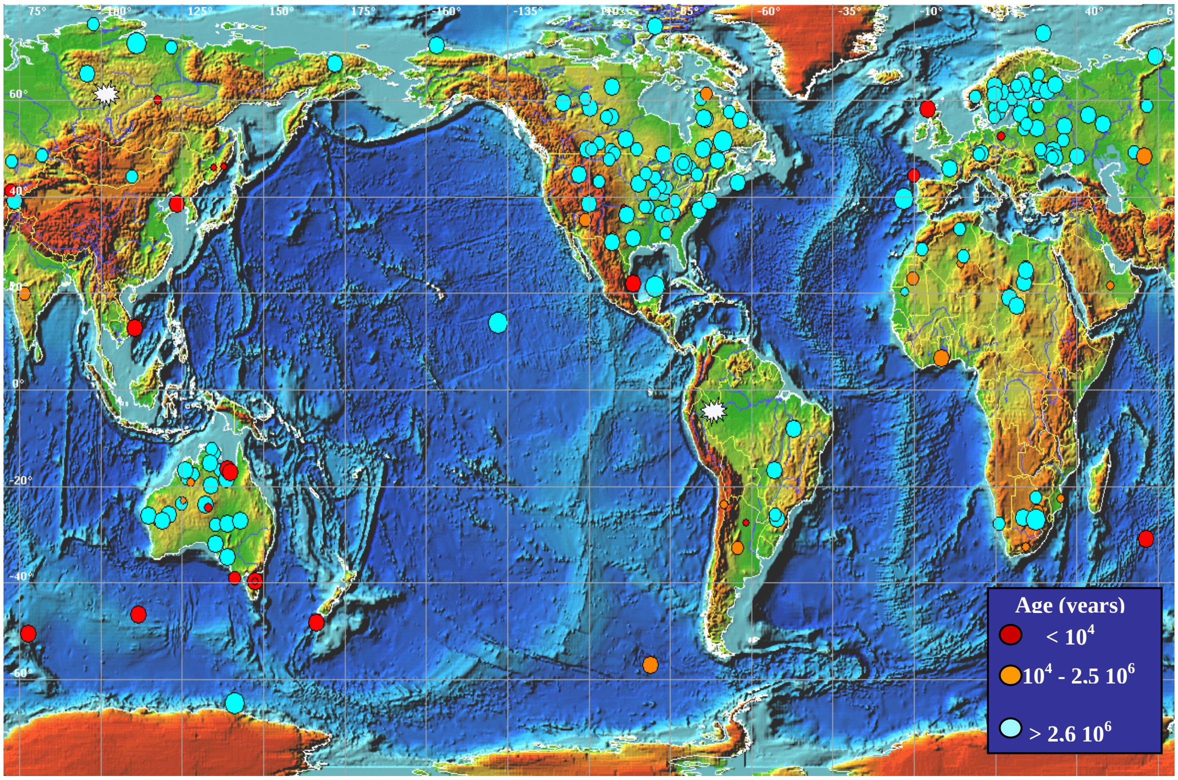


Fig. 1. Map of known impact structures

The map of known impact structures on the land surface (180 structures) and on the bottom of seas and oceans (24 structures). The size is proportional to the crater diameter. The color indicates the period of structure formation (age in billions of years). White stars show epicenters of Tunguska (1908) and Brazil (1930) impacts. Sixteen of currently known sea craters were discovered in 2005-2006 as a result of systematic search for underwater impact structures.

Friday, April 13, 2029.

This day may appear fatal for the planet Earth. At 4:36 according to Greenwich, asteroid Apophis (Fig.2) with mass 50 million tons and diameter 320 m will cross the Moon orbit and rush to the Earth with the speed 45,000 km/h. According to preliminary assessments, the place of Apophis falling will be a 50 km wide band, which runs across Russia, Pacific Ocean, and Central America and goes further into Atlantics. The most probable place for falling is a point in the ocean at several thousands of kilometers from the West coast of America.

As the result, Florida coast may be subjected to wave impact. According to NASA version, asteroid will approach the Earth at a distance 32.5 thousand kilometers (closer, than the Moon) and will be seen by unaided eye. The probability of collision with the Earth is 0.002.

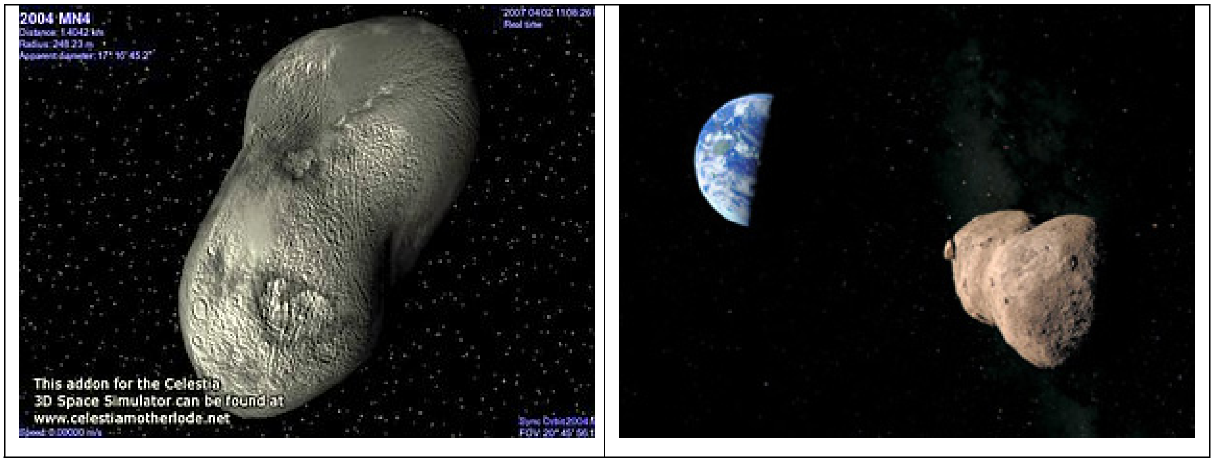


Fig. 2. Asteroid Apophis

Currently, there is no experimental information about large-scale collision tsunamis. That is why one of the main instruments of studies is mathematical modeling. In our work, we considered falling into the ocean with the depth 4 km of 1 km diameter stone asteroid, which collides with the Earth at a speed 22 km/s.

The results of space body collision with the Earth sufficiently depend on many factors: body size, its composition, velocity of the Earth approaching, angle of entry into atmosphere, and matter properties in the area of falling.

Space body motion in atmosphere is accompanied by formation of air shock wave of high intensity. Due to high velocity of body motion (15-25 km/s), shock front in the air will be at some distance from the object. That is why, while considering body collision with water, one can neglect air presence for simplifying calculation model.

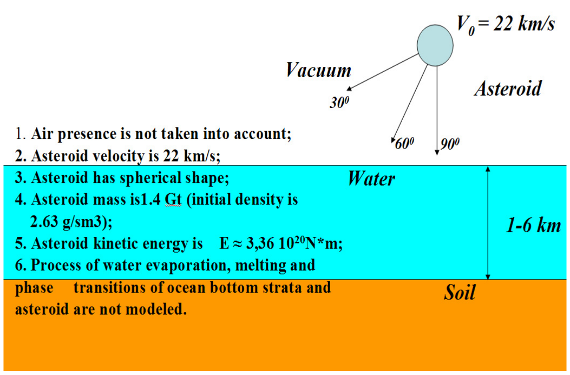
When asteroid collides with barrier, such physical processes as water evaporation, shock loading, melting and phase transitions of ocean bottom strata, mechanical destruction of bottom and asteroid will run in the barrier. Typical time of dynamic processes in the vicinity of strike makes about tens of seconds, and typical time of tsunami wave motion in the ocean makes several thousands of seconds. It is impossible to calculate the whole process for such times with the help of multi-dimensional finite-difference or finite–element techniques with acceptable accuracy even using the most powerful PC. That is why calculation of space body collision with barrier is split into two stages.

One of the first work in RF on modeling asteroid or comet falling into the ocean in 2D set-up was performed at RFNC-VNIITF in 2007 [4] in gas-dynamic approximation with the help of code MECH – finite-difference technique for angle of falling into the ocean 90 degrees. The code that was used in Snezhinsk did not permit to calculate space body falling at various angles and to consider different versions of asteroid collision with the Earth. Let us fill this gap.

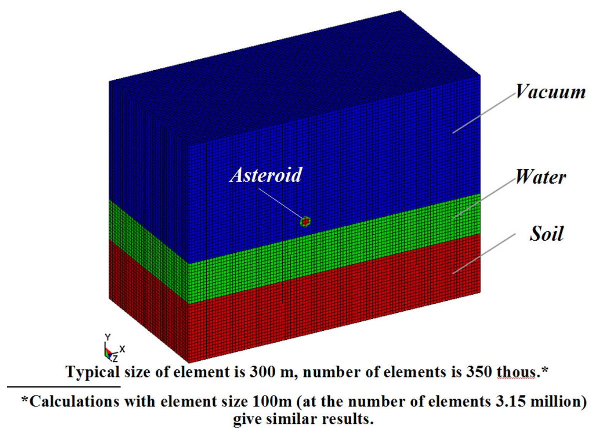
At the first stage, using finite–element code LS-DYNA [1] and super computer SKIF-URAL of South-Ural State University and working station under the support of LLC "STRELA", processes of space hazardous object interaction with barrier were calculated.

Interaction water –asteroid was modeled in method of finite elements using algorithm of coupling; water on immovable mesh – in Eulerian formulation, and asteroid and ground – on deformable mesh using Lagrangian formulation. Algorithm of coupling permits to calculate forces of coupling at the interface of fluid medium with asteroid and ground. These forces are added to fluid medium (Eulerian space) and to nodes of asteroid and ground (Lagrangian space). Both problems are solved using explicit method of integration by time, which is good for pulse loading problems.

While describing granite and water, EOS of Mie Gruneisen type were used with corresponding parameters [2, p. 314-317].



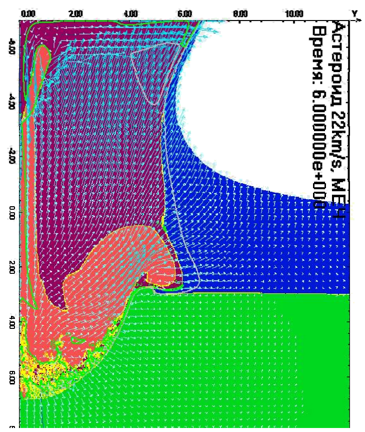
Calculation scheme, accepted assumptions



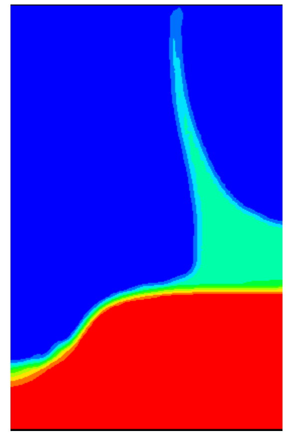
Finite-element model

Fig. 3. Calculation scheme and finite-element model

Comparison of solutions obtained according to MECH [4] and LS-DYNA codes (using super computer SKIF-Ural and working station of LLC “STRELA”) shows satisfactory coincidence of the results of calculations by the wave height, horizontal component of wave motion velocity, radius of water cavity (Fig. 4). This permits to perform further calculations according to LS-DYNA code and to determine primary parameters of collision tsunami wave using verified finite-element model. Water evaporation, melting and phase transitions of ocean bottom strata and asteroid are not modeled, and this does not lead to sufficient error of calculations of lifted wave parameters and parameters of formed crater.



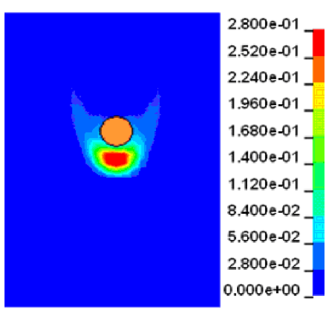
Results of solution according to MECH code



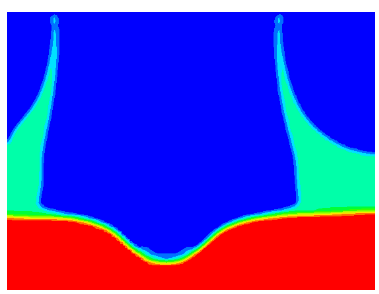
Results of solution according to LS-DYNA code

Fig. 4. Results of verification of created calculation model

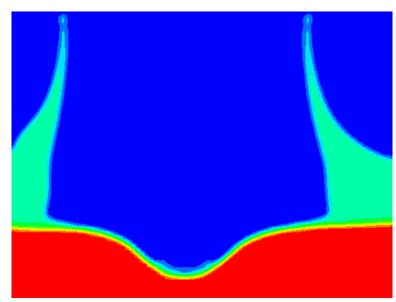
Fig. 5 shows the results of calculations according to LS-DYNA code with regard and without regard for atmospheric air. Comparison of the results shows that when atmosphere is not taken into account, there is no sufficient error of wave parameters and crater size, while modeling asteroid falling into the ocean.



Location of shock waves around asteroid (GPa)



Vacuum



Air

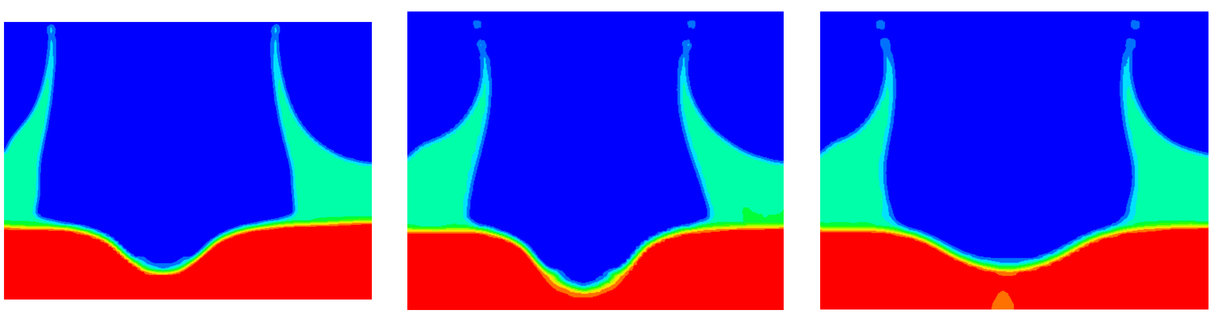
Fig. 5. Analysis of accepted suppositions

Calculations model asteroid by a body that has spherical shape with diameter 1 km and mass M ≈ 1.4 Gt, with initial density ρ = 2.63 g/cm^3. Let us consider how HSO shape influences wave parameters and crater size. For this purpose, let us perform three calculations, when HSO, falling vertically into the ocean with the speed 22 km/s, with mass M ≈ 1.4 Gt, is a sphere (Ø = 1 km), cylinder (Ø = 0.6 km and height 1.85 km) or "washer" (Ø = 1.6 km and height 0.26 km).

The results of obtained solutions for wave parameters and crater are given in Table 1 and Fig. 6. Comparison shows that HSO shape has week influence on wave parameters, crater size and water cavity. This can be explained by the fact that these parameters mainly depend on kinetic energy of HSO, and in these examples, it was constant and equal to $3.36 \cdot 10^{20}$ N·m.

Table 1. The results of obtained solutions on determination of wave parameters and cater for HSO in the shape of sphere, cylinder and "washer"

Parameter	Sphere	Cylinder	"Washer"
Ocean depth in the point of asteroid falling , km	4	4	4
Wave height, m	9400	9000	9100
Horizontal component of velocity vector, m/s	150	145	180
Crater diameter, m	14400	12000	19000
Primary crater depth, m	3300	3900	2100
Diameter of water cavity, m	18900	17000	19500



Sphere Cylinder "Washer"

Fig. 6. Changes in parameters of wave and crater size for the case of falling of HSO in the shape of sphere, cylinder and "washer"

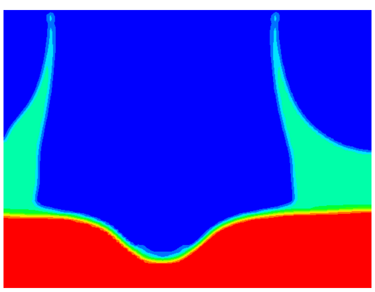
The results of calculations of wave height, horizontal component of wave motion velocity, radius of water cavity and parameters of formed crater for various angles of asteroid entry into the Earth atmosphere are given in Table 2 and Fig. 7.

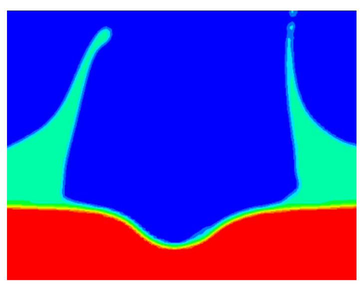
Analysis of calculation data shows that for falling angles 60° and 90°, the results slightly differ from each other. Even for falling angle 30°, there is no drastic difference. It is necessary to expect that collision tsunami impact on the coast for considered angles will be practically the same. As far as primary parameters of formed crater do not decrease with decrease of asteroid entry angle, it was proposed to consider only the case of axisymmetric falling of asteroid into the ocean.

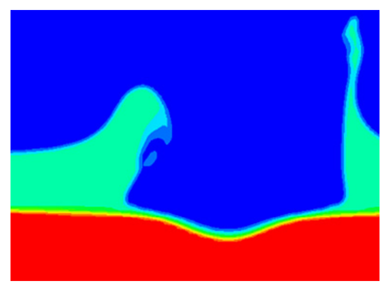
Table 2. The results of obtained solutions for parameters of wave and crater depending on angle of asteroid entry into atmosphere

Angle of asteroid falling	30°	45°	60°	90°
Wave height, m	9200 and 4700	10500 and 6600	9500 and 9200	9400
Horizontal component of velocity vector, m/s	180 and 170	170 and 175	160 and 150	150
Crater diameter, m	10500	12300	13000	14400
Primary crater depth, m	1400	2100	2700	3300
Diameter of water cavity, m	15000	17400	18600	18900

The results of calculations of wave height, horizontal component of wave motion velocity, radius of water cavity and parameters of formed crater depending on asteroid diameter are given in Table 3 and Fig. 8.


Angle of asteroid falling 90°


Angle of asteroid falling 60°


Angle of asteroid falling 30°

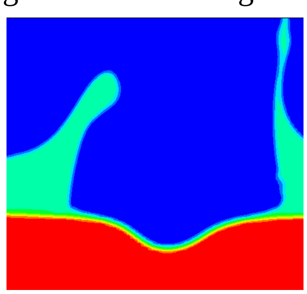
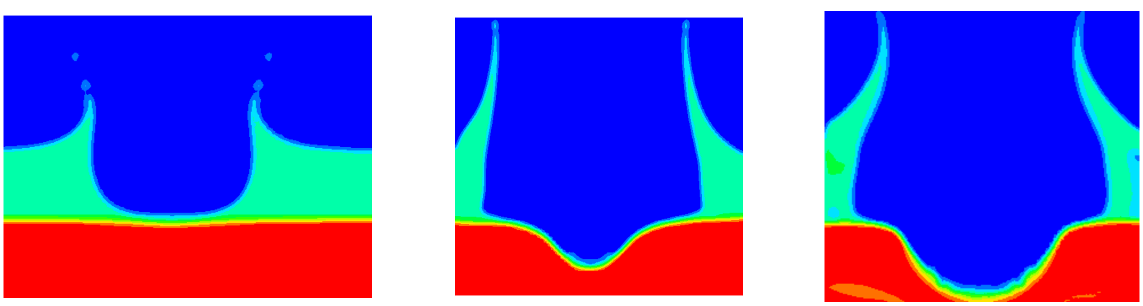

Angle of asteroid falling 45°

Fig. 7. Changes of wave parameters and crater size depending on angle of asteroid entry into atmosphere

Table 3. The results of obtained solutions for parameters of wave and crater for various diameters of asteroid

Parameter	Diameter of asteroid km		
	0.5	1	1.5
Wave height, m	7800	9400	10000
Horizontal component of velocity vector, m/s	100	150	175
Crater diameter, m	-	14400	13500
Primary crater depth, m	-	3300	5000
Diameter of water cavity, m	9600	18900	18000

Analysis of calculation data shows that initial height of maximum wave, horizontal component of its motion velocity, radius of water cavity, primary parameters of formed crater decrease with decrease of asteroid diameter. Asteroid with diameter 0.5 km does not form big crater on the bottom.



Diameter of asteroid 0,5 km — Diameter of asteroid 1 km — Diameter of asteroid 1,5 km

Fig. 8. Change of wave parameters and crater sizes for various diameters of asteroid

The results of calculation of wave height, horizontal component of wave motion velocity, radius of water cavity and parameters of formed crater depending on ocean depth in the point of asteroid falling are given in Table 4 and Fig. 9.

Analysis of calculations data shows that initial height of maximum wave, horizontal component of wave motion velocity, radius of water cavity first increase with increase of ocean depth in the point of asteroid falling, and then acquires practically the same value. Primary parameters of formed crater decrease with increase of ocean depth in the point of asteroid falling.

Modeling of this problem in LS-DYNA code revealed the change of kinetic and internal energy of falling of asteroid with diameter 1 km into the ocean with 4km depth. Kinetic energy of asteroid is $3.36 \cdot 10^{20}$ N·m, and energy of lifted wave makes about 25% of asteroid energy, i.e. $0.767 \cdot 10^{20}$ N·m.

Analysis of calculation data shows that the highest parameters of wave were obtained for the case of falling of asteroid with diameter 1 km into the ocean with 4 km depth. Let us consider parameters of developed tsunami for this case.

Table 4. Parameters of wave and crater for various ocean depths in the point of asteroid falling

Parameter	Ocean depth in the point of asteroid falling, km					
	1	2	3	4	5	6
Wave height, m	3300	6600	7200	9400	9000	8700
Horizontal component of velocity vector, m/s	220	195	170	150	127	100
Crater diameter, m	8400	10200	12300	14400	12900	12700
Primary crater depth, m	6900	4500	3700	3300	3000	2800
Diameter of water cavity, m	9000	17700	18400	18900	16800	16700

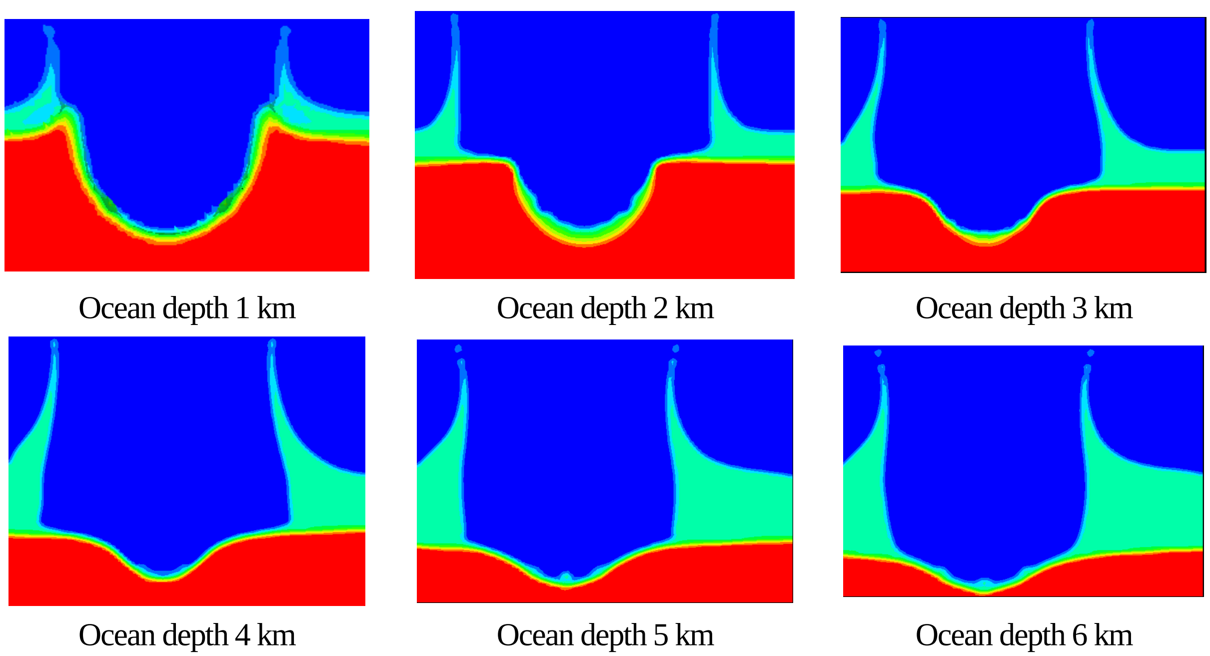


Fig. 9. Change of wave parameters and crater sizes depending on ocean depth in the point of asteroid falling

For our case, by the end of the first stage with duration about 30-50 seconds, in the system "ocean-rock" all strong nonlinear processes finished: shock waves attenuated, crater zone was formed, and asteroid was destructed. On water surface, one observes a wave with height 8…10 km and diameter of funnel 15…20 km. Density of water became 1,0...1,1 g/cm^3, vector of velocity of medium particle motion in water layer became almost horizontal and approximately constant by water depth. The scatter of velocity modulus by depth does not exceed 15%.

It is known that in case, when the length of wave propagating on water surface is much more than ocean depth, its propagation can be calculated by shallow water theory [3, 4].

The longer is the time of transition from calculation of the first stage according to code LS-DYNA to calculation according to shallow water theory (code SWAT), the less is an error of such problem approximation.

For initial data for code SWAT, profile of wave and funnel in the soil are specified, i.e. distribution of wave height $\eta(R,t)$ and ocean depth $h(R)$ depending on distance R, which is calculated from a point of body entry into the water.

At the moment of about 90 s, Fig. 10 shows water plume that is formed as the result of water flowing into bottom crater. Fig. 13 shows position of tsunami for three subsequent moments of time.

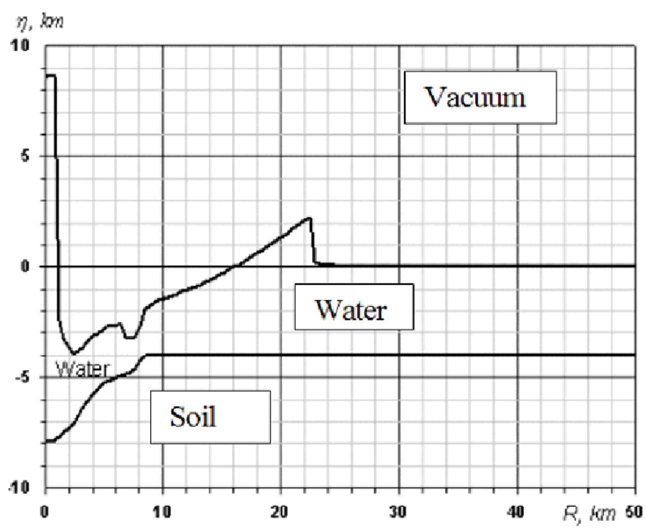


Fig. 10. Water plume η(R), R – distance from point of falling

It is seen that wave height increases from 60 up to 100 m on the shelf. Then, there is decrease of wave height on shallow water region. It is nothing, but wave modeling with the help equation of shallow water theory. Validity of this phenomenon modeling is shown in [3].

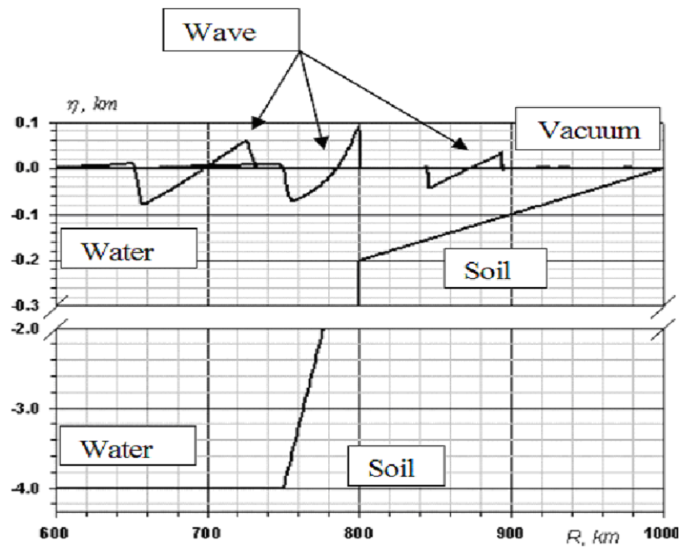


Fig. 11. Wave arrival to the shelf, R – distance from falling point

Modified theory of shallow [6, 7] water permitted to perform calculation of ocean wave uprush to the shore (Fig. 12). Calculations show that for obtained parameters of tsunami, which is formed as the result of falling of 1 km diameter asteroid into the ocean 4 km deep, can propagate on the shore having the slope of 32 degrees, at a distance of 1.5 km, rising to a height up to 80 m, that is hazardous for populated settlements along the coast.

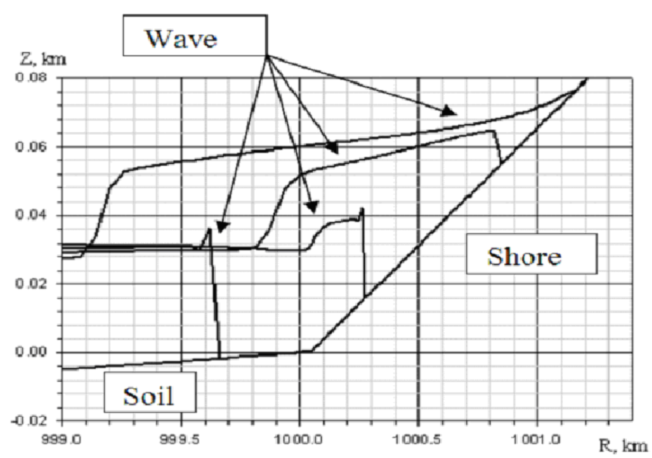


Fig. 12. Calculation of collision tsunami uprush to ocean shore having the slope of 32°

Using of approach to calculation of tsunami parameters, which is formed as the result of asteroid falling into the ocean, proposed in this work is quite acceptable both for qualitative, and for quantitative description of this physical phenomenon. As obtained results showed, aftereffects of stone asteroid falling may be destructive for the ocean coast.

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