Solid Elements with Rotational Degree of Freedom for Grand Rotation Problems in LS-DYNA®

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Abstract

The goal of this paper is to further enhance the solid elements with rotational degree of freedom (DOF). Three-dimensional finite elements with rotational degree of freedom have been proposed elsewhere, however, these elements are restricted to linear analysis. In this paper, by improving the mid-side node velocity update algorithm, we enhance the elements performance. Numerical results are presented, showing that the enhanced elements are capable of dealing with grand rotation problem. The enhanced formulation has been implemented into LS-DYNA® for solid element 3 and solid element 4.

Introduction

Using corner rotation to improve the computational efficiency of 8-node hexahedron element and 4-node tetrahedron element has been presented in References 1-2. The proposed elements are developed by transforming the mid-side nodal DOF of a high order 20-node and 10-node isoparametric element in terms of corner nodal translations and rotations of low order 8-node hexahedron and 4-node tetrahedron element. The inclusion of corner rotational DOF allows the normal motion along elements edges to be quadratic. Although the added rotational DOF are more expensive as compared with the conventional 8-node hexahedron and 4-node tetrahedron translation only element, the improved accuracy of the element outweighs this shortcoming by reducing the number of elements needed.

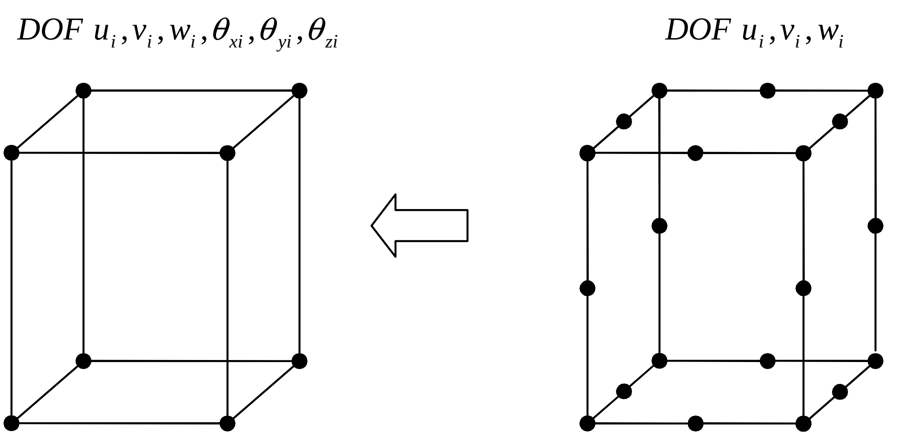
However, when using corner nodal translations and rotations to represent mid-side nodal DOF, References 1-2 based on the assumption that the deformation and rotation are infinitesimal, thus the developed elements can only be applied to linear analysis, which severely limit there application. This paper improves these elements by modifying the mid-side nodal velocity update algorithm such that they are capable of dealing with non-linear analysis.

Basic Development and Limitations

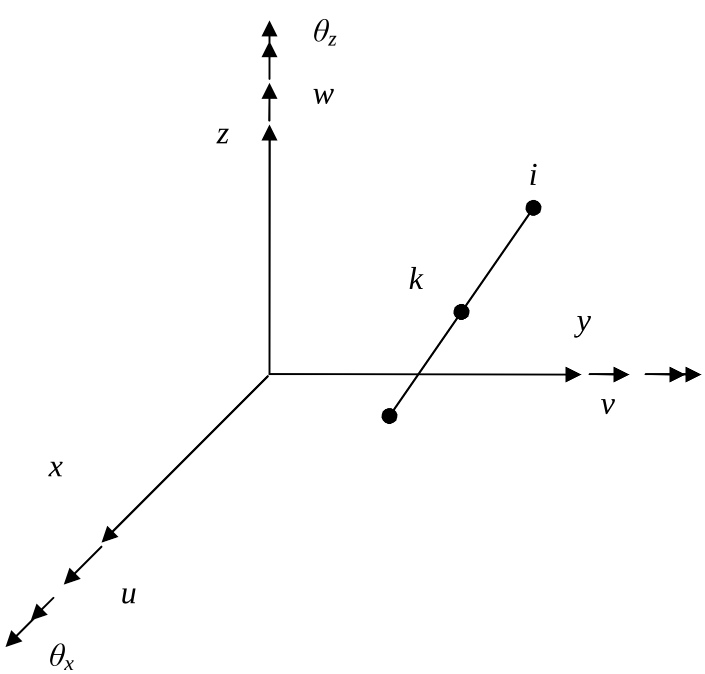
The formulation of the shape function follows a procedure given in Ref 1-2, for completeness, it is also reported here. For demonstration, only 8-node hexahedron element is considered in this paper. The 8-node forty-eight DOF brick element is derived from 20-node sixty degree DOF solid element through a transformation of the nodal velocity and rotations of the element. For a typical edge *i-j* shown in Figure1, the velocity of the midpoint *k* is given as the function of the corner node velocities as

$$
\begin{aligned}
\dot{u}_k &= \frac{1}{2}\left(\dot{u}_i + \dot{u}_j\right) + \frac{y_j - y_i}{8}\left(\dot{\theta}_{zj} - \dot{\theta}_{zi}\right) + \frac{z_j - z_i}{8}\left(\dot{\theta}_{yi} - \dot{\theta}_{yj}\right) \\
\dot{v}_k &= \frac{1}{2}\left(\dot{v}_i + \dot{v}_j\right) + \frac{z_j - z_i}{8}\left(\dot{\theta}_{xj} - \dot{\theta}_{xi}\right) + \frac{x_j - x_i}{8}\left(\dot{\theta}_{zi} - \dot{\theta}_{zj}\right) \\
\dot{w}_k &= \frac{1}{2}\left(\dot{w}_i + \dot{w}_j\right) + \frac{x_j - x_i}{8}\left(\dot{\theta}_{yj} - \dot{\theta}_{yi}\right) + \frac{y_j - y_i}{8}\left(\dot{\theta}_{xi} - \dot{\theta}_{xj}\right)
\end{aligned}
\tag{1}
$$

Where $u, v, w, \theta_x, \theta_y$ and θ_z are the translational and rotational velocity of corner node in the global coordinate.



(a) 8-Node Hexahedron (b) 20-Node Hexahedron



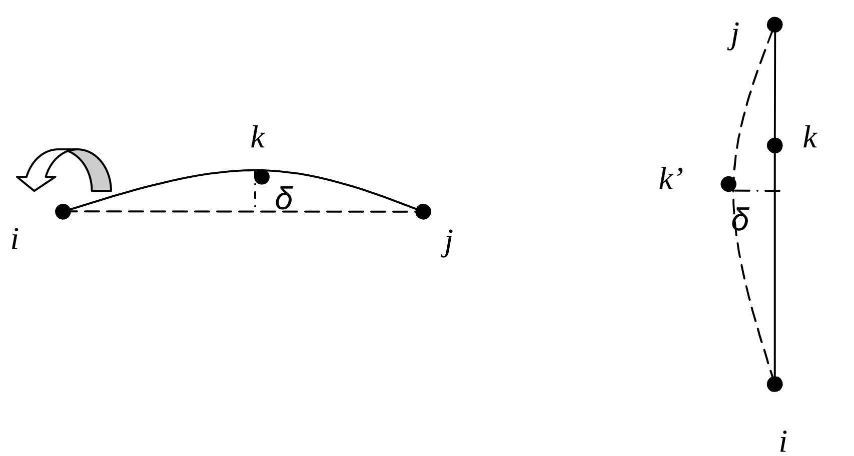
(c) Typical Element Edge and DOF Orientation

Figure 1. Solid Hexahedron Element

The velocity field for the 20-node hexahedron element in terms of nodal velocities is

$$\begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{Bmatrix} = \begin{bmatrix} \mathrm{N}_1 & \mathrm{N}_2 & \cdots & \mathrm{N}_{20} & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \mathrm{N}_1 & \mathrm{N}_2 & \cdots & \mathrm{N}_{20} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \mathrm{N}_1 & \mathrm{N}_2 & \cdots & \mathrm{N}_{20} \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \vdots \\ \dot{u}_{20} \\ \dot{v}_1 \\ \vdots \\ \dot{v}_{20} \\ \dot{w}_1 \\ \vdots \\ \dot{w}_{20} \end{Bmatrix} \quad (2)$$

Where N_i are the shape functions for the 20-node hexahedron element. Careful examination of the element shape function reveals that Eq. 1 is not suitable for finite deformation and rotation problem. For a typical edge *i-j* shown in Figure 2, assuming initially there is a deflection at midpoint. Let the element undergo a rigid body rotation around node *i*, based on Eq. 1, the mid-side node move to location k instead of k', thus, the above formulation can not preserve rigid body rotation and generate spurious strain for large rotation problem.

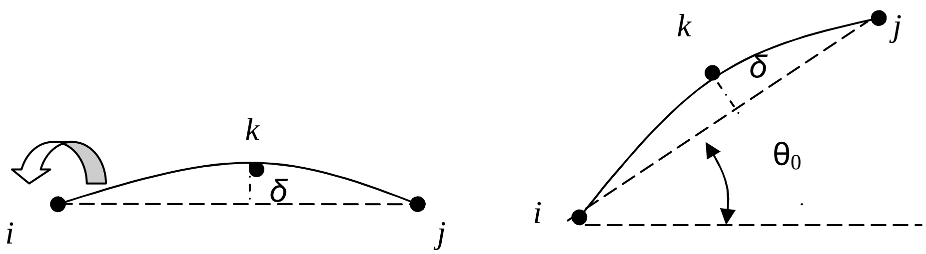


(a) Initial Configuration of Element Edge (b) Rotated Configuration of Element Edge

Figure 2. Element Edge Rotation

Modifications of the Basic Formulation

To account for the large rotation, Eq. 1 needs to be modified to preserve rigid rotation. Consider a typical edge of the element as shown in Figure3 (a), the length of the edge is L , initially there is a deflection δ at midpoint, the rotation angles are θ_i and θ_j respectively. For simplicity, let node *i* locate at origin, then the midpoint coordinate is $\left(\frac{1}{2}L, \delta\right)$,



(a) Initial Configuration of Element Edge (b) Rotated Configuration of Element Edge

Figure 3. Element Edge Rotation with initial deflection

the deflection δ is given as the function of the corner node by $\delta = H_1 y_i + H_2 y_j + H_3 \frac{dy}{dx}|_i + H_4 \frac{dy}{dx}|_j$, where H_{1-4} are Hermite shape function. For mid-side node, $\delta = \frac{1}{8}\tan\theta_i - \frac{1}{8}\tan\theta_j$. Suppose the element undergo rigid body rotation as shown in Figure 3(b), then the current coordinate of midpoint is given by

$$\begin{bmatrix} x_k \\ y_k \end{bmatrix} = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 \\ \sin\theta_0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} \frac{L}{2} \\ \delta \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(x_i + x_j) \\ \frac{1}{2}(y_i + y_j) \end{bmatrix} + \begin{bmatrix} -\frac{L}{8}\sin\theta_0(\tan\theta_i - \tan\theta_j) \\ \frac{L}{8}\cos\theta_0(\tan\theta_i - \tan\theta_j) \end{bmatrix} \tag{3}$$

Assume the deflection angle is small, i.e., $\tan\theta_i \approx \theta_i$, and $\tan\theta_j \approx \theta_j$. The velocity of midpoint is given by the time derivative of Eq. 3

$$\begin{bmatrix} \dot{u}_k \\ \dot{v}_k \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(\dot{u}_i + \dot{u}_j) \\ \frac{1}{2}(\dot{v}_i + \dot{v}_j) \end{bmatrix} + \begin{bmatrix} -\frac{L}{8}\sin\theta_0(\dot{\theta}_i - \dot{\theta}_j) \\ \frac{L}{8}\cos\theta_0(\dot{\theta}_i - \dot{\theta}_j) \end{bmatrix} + \begin{bmatrix} -\frac{L}{8}\cos\theta_0(\theta_i - \theta_j)\dot{\theta}_0 \\ -\frac{L}{8}\sin\theta_0(\theta_i - \theta_j)\dot{\theta}_0 \end{bmatrix}$$

Which can be rewritten as

$$\begin{bmatrix} \dot{u}_k \\ \dot{v}_k \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(\dot{u}_i + \dot{u}_j) \\ \frac{1}{2}(\dot{v}_i + \dot{v}_j) \end{bmatrix} + \begin{bmatrix} -\frac{y_j - y_i}{8}(\dot{\theta}_i - \dot{\theta}_j) \\ \frac{x_j - x_i}{8}(\dot{\theta}_i - \dot{\theta}_j) \end{bmatrix} + \begin{bmatrix} -\frac{L}{8}\cos\theta_0(\theta_i - \theta_j)\dot{\theta}_0 \\ -\frac{L}{8}\sin\theta_0(\theta_i - \theta_j)\dot{\theta}_0 \end{bmatrix} \tag{4}$$

From the above equation it can be seen that the first two terms of RHS of Eq. 4b is exactly the same as the two-dimensional version of original formulation Eq.1, while there are one additional term that account for the rigid rotation of element edge.

Numerical Results

The three-dimensional simulation model in LS-DYNA was built up and shown in Figure 4. The model consist of a rotating cylinder with prescribed angular velocity 5000 *rad/s* using *LOAD_BODY_RZ. The pressure of the cylinder is 1000N/mm^2 applied using the *LOAD_SEGMENT keyword.

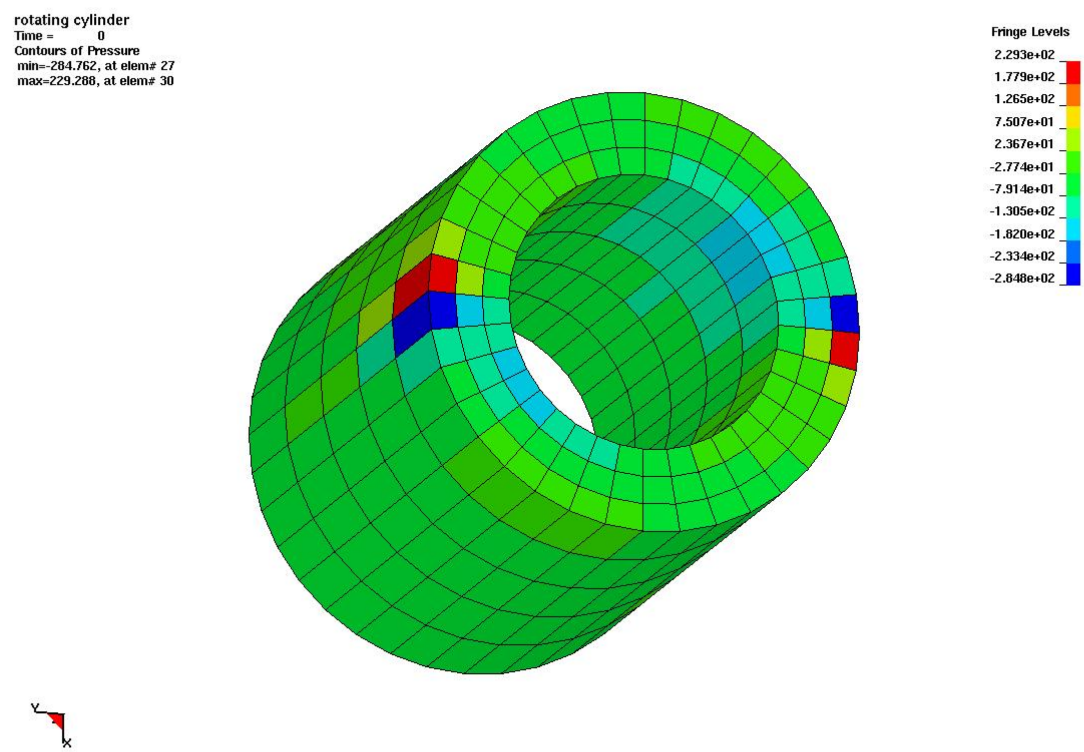
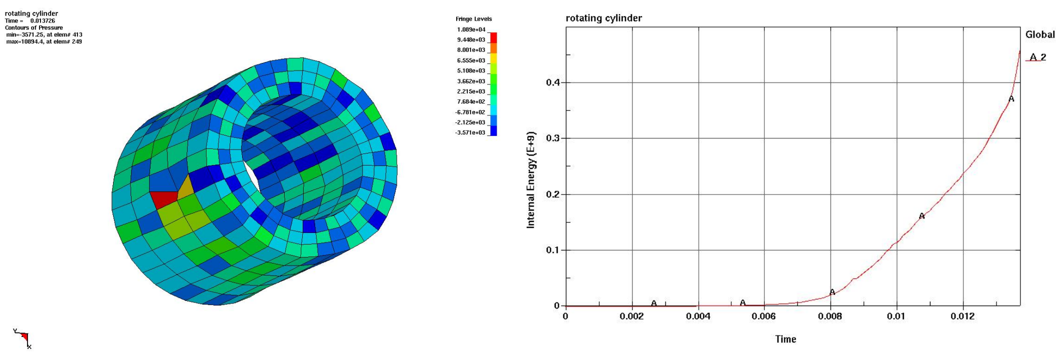


Figure 4. Rotating Cylinder

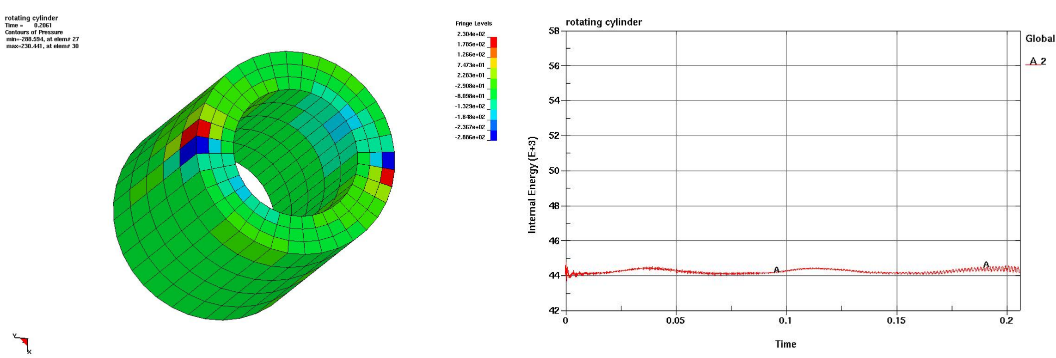
Figure 5a shows the rotated configuration at time=0.0137s based on formulation Eq. 1, which corresponding to about 11 cycles. It can be seen that cylinder severely distorted, the pressure distribution pattern is completely changed, implying a large stress build up. As a result, the internal energy increases sharply from 4.4E+03J to 4.6E+08J as shown in Figure 5b, thus Eq.1 failed to simulate this kind of problems with large rotation.



(a) Rotated Configuration (b) Internal Energy History Plot

Figure 5. Simulation Results based on Eq. 1

Figure 6 shows the rotated configuration at time=0.206s and internal energy history plot based on the enhanced formulation. The pressure distribution pattern of rotated configuration at 0.206s is identical to the pressure distribution pattern of initial configuration. Also the cylinder can rotate hundreds of cycles without an increase in the internal energy.



(a) Rotated Configuration (b) Internal Energy History Plot

Figure 6. Simulation Results based on Eq. 4

Figure 7 compares the internal energy history plot between the formulation Eq. 1 and formulation Eq. 4, showing the effectiveness of the enhanced formulation.

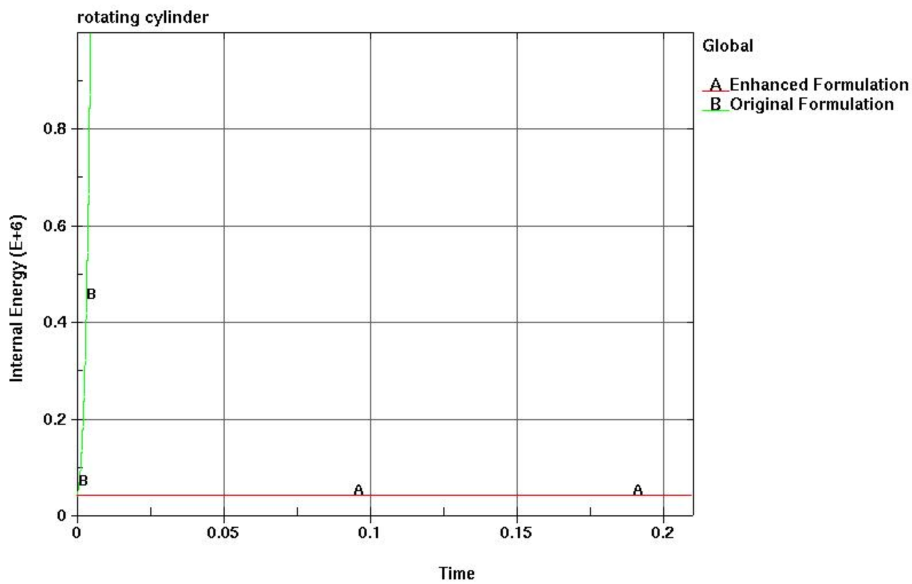


Figure 7. Comparison of Internal Energy History Plot between original formulation and enhanced formulation

Conclusion

A formulation that preserves rigid body rotation is developed for solid element with rotational degree of freedom. The formulation can be applied to both 8-node hexahedron and 4-node tetrahedron elements. When element distortion is not severe, the elements show good accuracy for large rotation problems. The formulation has been implemented into LS-DYNA[3].

References

[1] S.M. Yunus, T.P. Pawlak and R.D. Cook, 'Solid elements with rotational degrees of freedom, Part I-Hexahedron element', Int. J. Numer. Methods Eng., 31, 573-592 (1991).

[2] T.P. Pawlak, S.M. Yunus and R.D. Cook, 'Solid elements with rotational degrees of freedom, Part II-Tetrahedron element', Int. J. Numer. Methods Eng., 31, 593-610 (1991).

[3] H. Teng, and J. Hallquist, "Improved solid finite elements suitable for simulating large deformations and/or rotations of a structure", European Patent 09013299.4-2224, 21.10.2009