Variable Screening Using Global Sensitivity Analysis

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Abstract

The cost of optimization increases with the dimensionality of the problem irrespective of using metamodels or direct methods. It is recommended to explore the opportunities to reduce the number of variables. One method to reduce the dimensionality is to fix the variables that do not influence the response significantly. ANOVA based on polynomial response surfaces is often used to identify the least important design variables. The global sensitivity analysis, proposed by Sobol, is another very useful technique to reduce the dimensionality. This method can be used with any surrogate model and is often used as a variable screening tool. While the dimensionality reduction based on a single response is widely used, this study presents an easy approach, facilitated by LS-OPT®, to reduce the number of design variables when a system comprising of multiple responses is considered. The benefits of reducing the problem dimensionality are demonstrated using a crashworthiness example.

Introduction

The cost of optimization increases significantly with the number of variables due to a polynomial increase in the size of the design space i.e., the number of designs to be evaluated significantly increases with the dimensionality of the problem [1]. Generally, it is very expensive to evaluate design alternatives for industrial problems such that a direct coupling of optimization method with the simulation model may be uneconomical. To alleviate the design evaluation cost, approximation models (also known as surrogate models or metamodels) of different responses are developed using the simulation data. The quality of such approximations depends on the number of data points available for approximations. While increasing the data mostly improves the quality of approximation, the minimum amount of data required to construct reasonably accurate approximation models increases polynomially with the number of design variables. It can be summarized that there is a tradeoff between the optimization cost governed by the number of design variables and the quality of final solutions.

The ever-increasing complexity of engineering systems leads to numerous coupled parts and a large number of controllable parameters. However, mostly a few key variables drive the performance of these systems such that the identification of these variables would result in a good solution on the *cost-vs.-optimum* tradeoff curve. Since the identification of these effective variables usually requires a lot of experience which might not be readily available for novel systems, mathematical models are very helpful for the variable screening.

There are many numerical tools to identify important variables for a response such as, the analysis of variance (ANOVA) [2] or the global sensitivity analysis [3]. These tools are most effectively used with approximation models to objectively quantify the importance of each design variable for a response. While most studies have used the global sensitivity analysis to screen variables using one response at a time, a system-level variable screening is presented in this paper for a crashworthiness optimization problem with large number of variables.

Global Sensitivity Analysis

The global sensitivity analysis was first presented by Sobol in 1993 [3]. This method is used to estimate the effect of different variables on the total variability of the function. Some of the advantages of conducting a global sensitivity analysis include,

1. Assessing importance of the variables,
2. Fixing non-essential variables (which do not affect the variability of the function) thus, reducing the problem dimensionality.

Homma and Saltelli (1996) (analytical functions and study of a chemical kinetics model) [4], Saltelli et al. (1999) (analytical functions) [5], Vaidyanathan et al. (2004) (liquid rocket injector shape design) [6], Jin et al. (2004) (piston shape design) [7], Jacques et al. (2004) (flow parameters in a nuclear reactor) [8], and Mack et al. (2005) (bluff body shape optimization) [9] presented some applications of the global sensitivity analysis. The theoretical formulation of the global sensitivity analysis is given as follows:

A function *f*(**x**) of a square integrable response as a function of a vector of independent uniformly distributed random input variables **x** in domain [0, 1] is assumed. The function can be decomposed as the sum of functions of increasing dimensionality as

$$f(x) = f_0 + \sum_i f_i(x_i) + \sum_{ij} f_{ij}(x_i, x_j) + \cdots + f_{12\ldots N_v}(x_1, x_2, \cdots x_{N_v}), \tag{1}$$

where $f_0 = \int_0^1 f d\mathbf{x}$. If the following condition

$$\int_0^1 f_{i_1 \ldots i_s} dx_k = 0, \tag{2}$$

is imposed for $k = i_1 \ldots i_s$, the decomposition described in Equation (1) is unique. In context of the global sensitivity analysis, the total variance of function *f*, denoted by *V*(*f*), can be shown equal to

$$V(f) = \sum_{i=1}^{N_v} V_i + \sum_{1 \le 1, j < N_v}^{N_v} V_{ij} + \cdots + V_{1\ldots N_v}, \tag{3}$$

where $V(f) = E\left((f - f_0)^2\right)$, and each term in Equation (3) represents the partial contribution or the partial variance of each independent variable (V_i) or a set of variables (e.g., V_{ij}) to the total variance, and provides an indication of their relative importance. The partial variances can be calculated using the following expressions:

$$\begin{aligned} V_i &= V(E[f \mid x_i]), \\ V_{ij} &= V(E[f \mid x_i, x_j]) - V_i - V_j, \\ V_{ijk} &= V(E[f \mid x_i, x_j, x_k]) - V_i - V_j - V_k, \end{aligned} \tag{4}$$

and so on, where *V* and *E* denote variance and expected value, respectively. Note that,

$$\begin{aligned} E[f \mid x_i] &= \int_0^1 f_i dx_i, \\ V(E[f \mid x_i]) &= \int_0^1 f_i^2 dx_i. \end{aligned} \tag{5}$$

This formulation facilitates the computation of the sensitivity indices corresponding to the independent variables and set of variables. For example, the first and second order sensitivity indices can be computed as,

$$S_i = \frac{V_i}{V(f)},\ S_{ij} = \frac{V_{ij}}{V(f)}. \tag{6}$$

Under the independent model input assumptions, the sum of all the sensitivity indices is equal to one. The first order sensitivity index (S_i) for a given variable represents the main effect of the variable, but it does not take into account the effect of the interaction of the variables. The total contribution of a variable to the total variance is given as the sum of all the interactions and the main effect of the variable and correspondingly, the total sensitivity index (S_i^{total}) is defined as,

$$S_i^{total} = \frac{V_i + \sum_{j,j\neq i} V_{ij} + \sum_{j,k,j\neq i,k\neq i}\sum V_{ijk} + \cdots}{V(f)}. \tag{7}$$

The above referenced expressions can be easily evaluated using the metamodels of any response function.

Sobol (1993) proposed a variance based non-parametric approach to estimate the global sensitivity for any combination of design variables using Monte-Carlo methods. To calculate the total sensitivity of any design variable x_i, the design variable set is divided into two complimentary subsets of x_i and $Z\left(Z = x_j; \forall j = 1, N_v;\ j \neq i\right)$. The purpose of using these subsets is to isolate the influence of x_i from the influence of the remaining design variables included in the subset Z. The total sensitivity index for x_i is then defined as,

$$S_i^{total} = \frac{V_i^{total}}{V}; V_i^{total} = V_i + V_{i,Z}, \tag{8}$$

where V_i is the partial variance of the response with respect to x_i and $V_{i,Z}$ is the measure of the response variance that is dependent on interactions between x_i and Z. Similarly, the partial variance for Z can be defined as V_z. Therefore, the total response variability can be written as,

$$V = V_i + V_Z + V_{i,Z}. \tag{9}$$

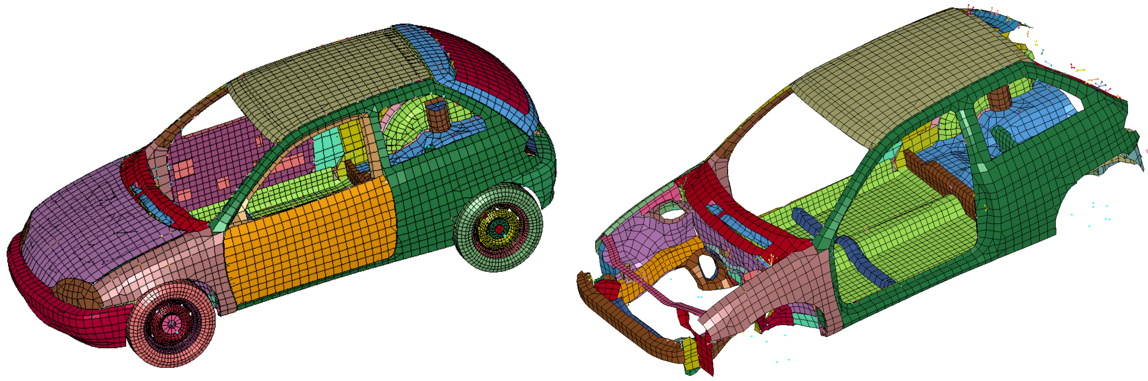
These expressions can be computed analytically for polynomial response surface approximations and radial basis functions with Gaussian basis functions. For all other metamodels, the Monte Carlo analysis is used to estimate the sensitivity indices. Since the sensitivity indices are non-dimensional entities, the system level importance of any variable can be easily estimated by simply adding all the corresponding indices as follows,

$$\begin{aligned} S_{i,system}^{total} &= \sum_j S_{i,j}^{total}; \\ S_{i,system} &= \sum_j S_{i,j}. \end{aligned} \tag{10}$$

where $S^{total}_{i,j}$ and $S_{i,j}$ represent the total and main sensitivity indices of the i^{th} variable for the j^{th} response, respectively. This feature is used to identify the most important design variables for the crashworthiness optimization problem described in the next section.

Problem Description

The performance of a vehicle undergoing the frontal impact crash is optimized for crashworthiness. The full frontal impact crash is simulated using the finite element model (Figure 1(A)) acquired from the NCAC at The George Washington University [10]. The finite element model consists of approximately 24,500 elements. Figure 1(B) shows the 100 thickness design variables representing the structural components of the vehicle that were selected to optimize the crash performance.



A) Complete vehicle model B) Structural members affected by design variables

Figure 1: Finite element model of the vehicle used to analyze full frontal impact crash.

The performance of the vehicle was characterized by the peak acceleration of the center of gravity (CG), maximum intrusion measured at the firewall, mass of the structural components, three stage pulses, and the time to reach zero velocity. Consequently, a multi-objective optimization problem is formulated as follows:

Table 1: Design objectives and constraints with corresponding scale factors.

Objectives	**Type**	**Scale Factor**	**Constraints**	**Upper Bound**
Mass	Minimize	0.35	*Subject to:*	
Maximum Intrusion	Minimize	300	Stage 1 pulse (SP1)	12.0g
Peak Acceleration	Minimize	6.5e5	Stage 2 pulse (SP2)	14.5g
Time_{0V}	Maximize	0.07	Stage 3 pulse (SP3)	35.0g

The bounds on the design variables are given in the Appendix along with the starting baseline design. The constraints were scaled using the target values to balance the violations of the all constraints and the objectives were scaled using the baseline design data.

The three stage pulses are calculated from the SAE filtered (60Hz) acceleration $\ddot{x}$ and displacement x of a left-rear-sill node in the following fashion:

$$\text{Stage } j \text{ pulse} = -\frac{k}{(d_2 - d_1)}\int_{d_1}^{d_2} \ddot{x}dx; k = 0.5 \text{ for } j = 1, k = 1.0 \text{ otherwise;} \quad (11)$$

with the limits (d_1;d_2) = (0;200); (200;400); (400;Max(x)) for i = 1, 2, 3 respectively. All displacement units are given in *mm* and the *minus* sign converts acceleration to deceleration.

Simulation Procedure: The explicit MPP version of LS-DYNA® [11] was used to analyze the performance of the vehicle. Each frontal crash simulation took approximately 1800s on four 3.6 GHz P4 Xeon processors with 16GB memory. The Sun Grid Engine® queuing system was used to schedule jobs on an in-house 32 processor cluster.

Optimization Procedure: This computationally expensive problem is optimized using metamodeling approach in LS-OPT® [12]. The metamodels were constructed in two stages.

i) **Screening Stage –** In the screening stage, all 100 design variables were considered and 300 simulation points were sampled using space filling design of experiments (DOE) method. The radial basis function approximation models were fit to the resulting response data and a global sensitivity analysis was carried out to estimate the influence of each design variable on various responses. All variables which contributed to more than 1% variability of the system of all responses representing objectives and constraints were considered important.

ii) **Optimization Stage –** After screening, the non-important design variables were fixed at the optimum design variable values obtained from the Pareto optimal set in the screening stage. An iterative metamodel based optimization was carried out using the screened variables. At each iteration, 32 design points were added using the space filling DOE such that the minimum distance between all points, including those selected in previous iterations, was maximized. As before, the radial basis function approximations for all responses were constructed using all simulated data points. A maximum of 10 iterations were allowed. The elitist non-dominated sorting genetic algorithm (NSGA-II) was used for optimization. A population of 100 individuals was evolved for 250 iterations using a binary tournament selection, SBX crossover, and polynomial mutation operators.

To demonstrate the effectiveness of screening procedure, an iterative optimization procedure was also carried out with 100 design variables. 160 points were sampled at each iteration and radial basis function networks approximations were developed using all available data points. The optimization parameters were kept the same as used with the screened variables.

Results

The results of this study are presented in two subsections. Firstly, the effectiveness of variable screening procedure is demonstrated and the optimization results are presented later.

Variable Screening

In the screening stage, 300 simulations points were obtained using the space filling experimental design method in 100 design variables space. Five simulations failed to converge. All responses were approximated by the radial basis function networks with Gaussian basis functions using the simulation data for 295 points. The accuracy of metamodels, listed in Table 2, was reasonable for most responses.

Table 2: Accuracy of approximations in the screening stage (based on 295 simulations).

Response	Mean	COV	RMSE	% RMSE	Max. Resid.	sPRESS	% sPRESS	R^2
Mass	0.33	0.01	0.00	0.00	0.00	0.00	0.00	1.00
Peak Accel.	6.23e5	0.18	7.16e4	1.15e1	3.09e5	1.05e5	16.90	0.53
Intrusion	309.30	0.04	0.16	0.05	0.74	7.92	2.56	0.99
$Time_{0v}$	0.063	0.05	0.002	2.59	0.006	0.003	3.97	0.64
SP-1	10.40	0.07	0.17	1.62	0.52	0.26	2.47	0.95
SP-2	12.09	0.10	0.59	4.83	1.79	0.86	7.12	0.70
SP-3	33.27	0.06	0.98	2.96	3.02	1.47	4.42	0.65

The top variables identified using the global sensitivity analysis for responses relevant to the optimization problem are shown in Figure 2 and Figure 3. The important deductions are as follows:

1. The top 10 design variables accounted for >70% of the variability in the responses, and the remaining 90 variables contributed to <30% of the total variability.
2. The most important variables for various responses were generally different which made it difficult to identify the most important variables for the optimization problem.

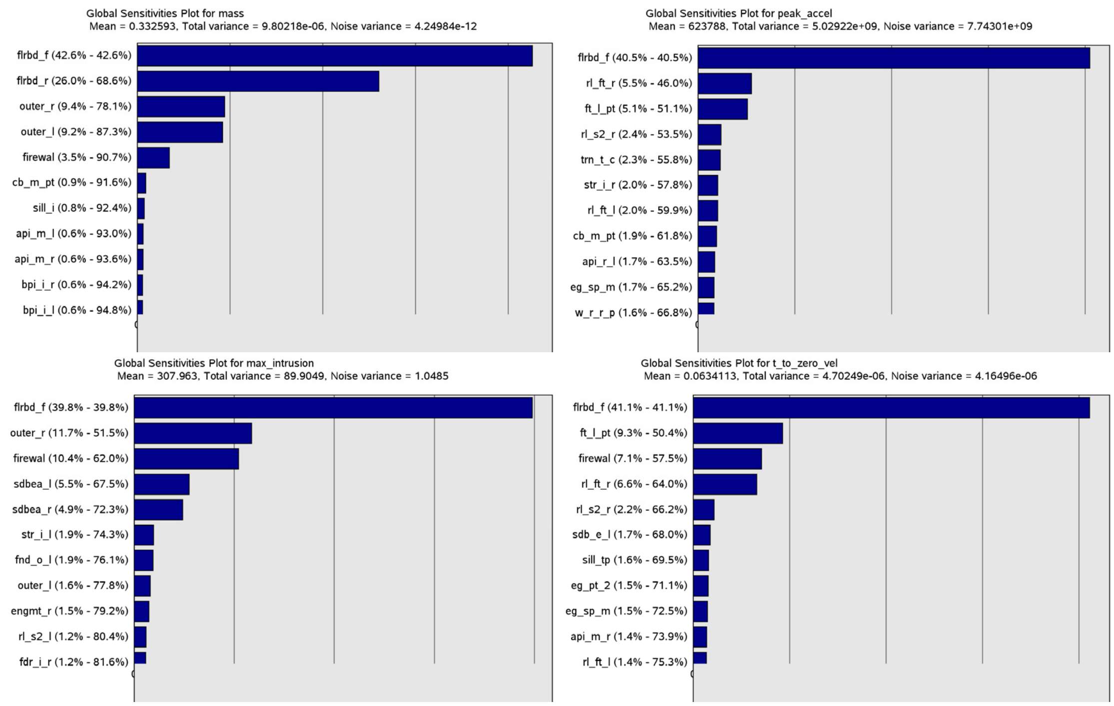


Figure 2: Total sensitivity indices of top variables for objectives in the screening stage.

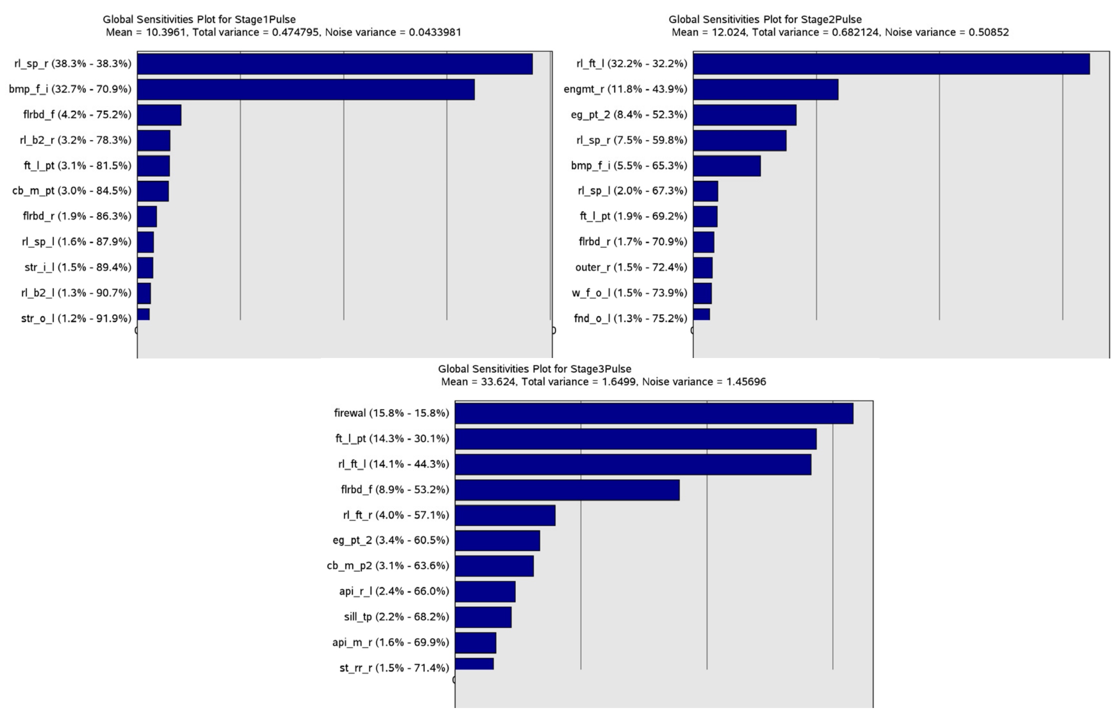


Figure 3: Total sensitivity indices of top variables for constraints in the screening stage.

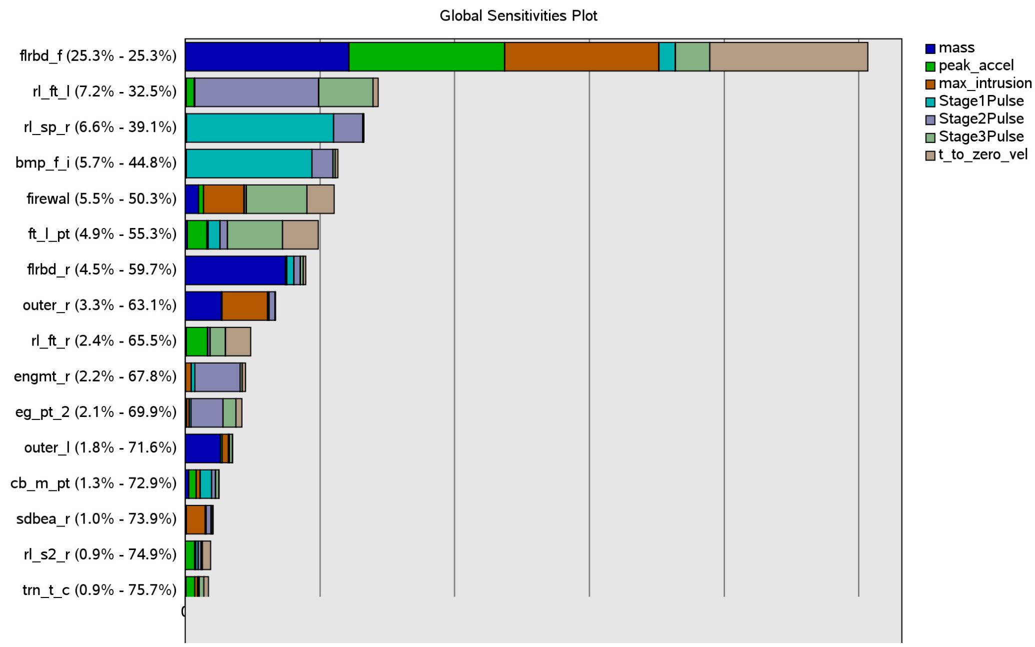


Figure 4: Top variables for the optimization problem in screening stage.

To alleviate the difficulty in screening design variables for a system (many responses), the cumulative total sensitivity indices for objectives and constraints were plotted in Figure 4. As before, the top 10 variables accounted for nearly 70% of the variability. 13 variables that influenced the optimization problem by more than one percent each were selected for the optimization (Table 3) and the remaining variables were fixed.

Table 3: Contribution of each selected variable to the total variability of different responses.

	Mass	Peak Accel.	Max Intrusion	T_{0v}	SP-1	SP-2	SP-3
Cabin mid plate	0.9	1.9	1.0	0.0	3.0	1.0	0.9
Engine mount plate-2	0.0	0.3	0.7	1.5	0.5	8.4	3.4
Engine mount plate-right	0.0	0.1	1.5	0.9	0.9	**11.8**	0.5
Firewall	3.5	1.3	**10.4**	7.1	0.1	0.5	**15.8**
Floorboard front	**42.6**	**40.5**	**39.8**	**41.1**	4.2	0.1	8.9
Floorboard rear	**26.0**	0.3	0.1	0.7	1.9	1.7	0.7
Front-lower plate	0.5	5.1	0.3	9.3	3.1	1.9	**14.3**
Outer-left frame	9.2	0.4	1.6	0.1	0.0	0.2	0.8
Outer-right frame	9.4	0.1	**11.7**	0.0	0.4	1.5	0.1
Rail – front-left	0.2	2.0	0.2	1.4	0.0	**32.2**	**14.1**
Rail – front-right	0.3	5.5	0.0	6.6	0.1	0.6	4.0
Support rail – front-left	0.0	0.2	0.1	0.0	**38.3**	7.5	0.3
Bumper – front-inner	0.0	0.0	0.3	0.7	**32.7**	5.5	0.6
Cumulative	**92.6**	**57.7**	**67.7**	**69.4**	**85.2**	**72.9**	**64.4**

The selected variables cumulatively accounted for nearly 73% variability of the optimization problem (Figure 4). The contributions of the selected variables on the total variability of all responses were tabulated in Table 3. These variables accounted for more than two-third variability for most responses under consideration. Relatively insufficient accuracy of the peak acceleration was explained by the fact that this was the least accurately modeled response and this response was influenced by a large number of variables.

The multi-objective optimization resulted in 1950 potential Pareto optimal solutions and a representative solution was selected by assigning unit weight to each objective function. The predicted baseline design and the final predicted optimum, shown in Table 4, clearly showed substantial improvements. The optimal design vector (Appendix, Table 7) was used as the baseline design for the iterative optimization with screened variables in the next stage.

Table 4: Predicted baseline and optimal design with unit weights for all objectives.

	Baseline	Optimal	Upper Bound
Mass	0.333	0.335	
Peak Accel.	6.31e5	1.17e5	
Max Intrusion	305.1	283.0	
$Time_{0V}$ *(maximize)*	0.063	0.078	
Stage 1 Pulse	10.4	9.4	12.0
Stage 2 Pulse	11.8	12.9	14.5
Stage 3 Pulse	34.8	26.3	35.0

In the optimization stage, the quality of metamodel approximations as measured by the PRESS errors is shown in Table 5. All error values were presented as a percentage of the mean response to facilitate easy comparison. Results indicated that all responses were accurately modeled. Relatively high error in the approximation of peak acceleration was acceptable as this response tends to be very noisy. The simulations after five iterations resulted in sufficient accuracy of metamodels.

Table 5: Square root of PRESS is presented as a %age of the mean value of various responses for iterative metamodeling with screened variables. The number of simulations includes the cost of screening stage.

Iteration #	1	2	3	4	5	6	7	8	9	10
Mass	1.0e-4	4.8e-4	4.6e-4	4.7e-4	4.9e-4	4.9e-4	4.8e-4	4.9e-4	4.9e-4	4.8e-4
Peak Accel.	16.90	14.00	12.10	11.50	11.50	12.00	11.60	12.00	11.80	12.10
Max Intrusion	3.13	2.23	1.94	2.07	2.08	2.06	2.09	2.03	1.99	1.99
$Time_{0V}$	3.50	3.30	3.35	3.11	3.05	3.00	3.09	3.05	3.01	3.02
SP-1	1.87	1.99	1.83	1.68	1.62	1.65	1.69	1.68	1.65	1.67
SP-2	5.31	4.60	6.04	5.64	5.61	5.33	5.44	5.28	5.56	5.43
SP-3	5.47	4.24	4.52	4.15	3.98	3.87	3.92	3.83	3.83	3.90
Total simulations	**332**	**364**	**396**	**428**	**460**	**492**	**524**	**556**	**588**	**620**

To assess the benefits of variable screening, the same set of responses was also approximated by considering all 100 design variables and the resulting quality of approximation is tabulated in Table 6. As expected, increasing the number of simulation data reduced approximation errors. A comparison of results in Table 5 and Table 6 clearly showed that the quality of approximations with the reduced number of variables was better than that obtained with all design variables.

Table 6: Square root of PRESS as a % of the mean value of various responses with 100 variables.

Iteration #	1	2	3	4	5	6	7	8	9	10
Mass	9.4e-4	8.0e-4	7.2e-4	6.9e-4	6.8e-4	6.7e-4	6.7e-4	6.6e-4	6.6e-4	6.6e-4
Peak Accel.	26.40	18.90	17.70	17.60	17.10	16.90	16.90	17.00	16.70	16.70
Max Intrusion	2.58	2.27	2.20	2.16	2.12	2.11	2.09	2.08	2.12	2.11
$Time_{0V}$	5.85	4.28	3.98	3.88	3.86	3.83	3.80	3.85	3.83	3.88
SP-1	3.12	2.53	2.32	2.18	2.19	2.21	2.17	2.14	2.15	2.16
SP-2	9.01	6.72	6.44	6.14	6.03	6.01	6.01	5.99	5.94	5.89
SP-3	6.06	4.51	4.34	4.21	4.26	4.19	4.10	4.13	4.05	4.11
# of simulations	**160**	**320**	**480**	**640**	**800**	**960**	**1120**	**1280**	**1440**	**1600**

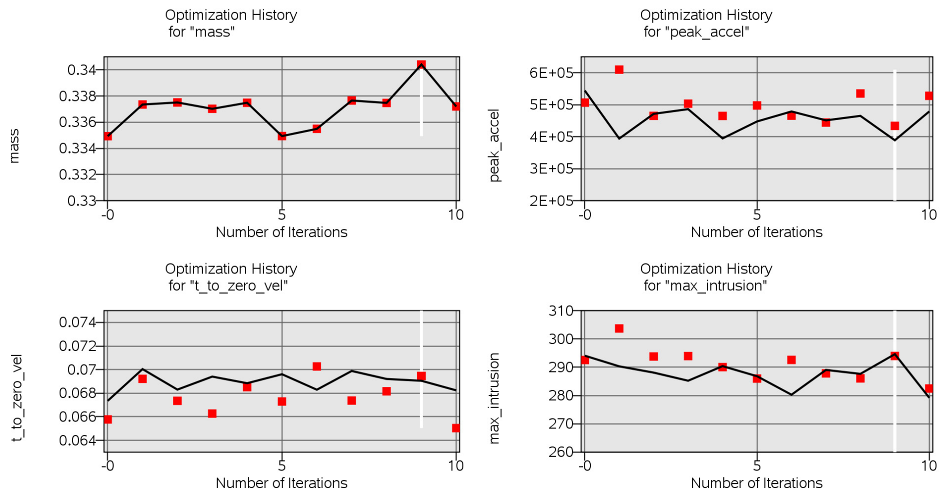


Figure 5: Optimization history for screened variables based optimization.

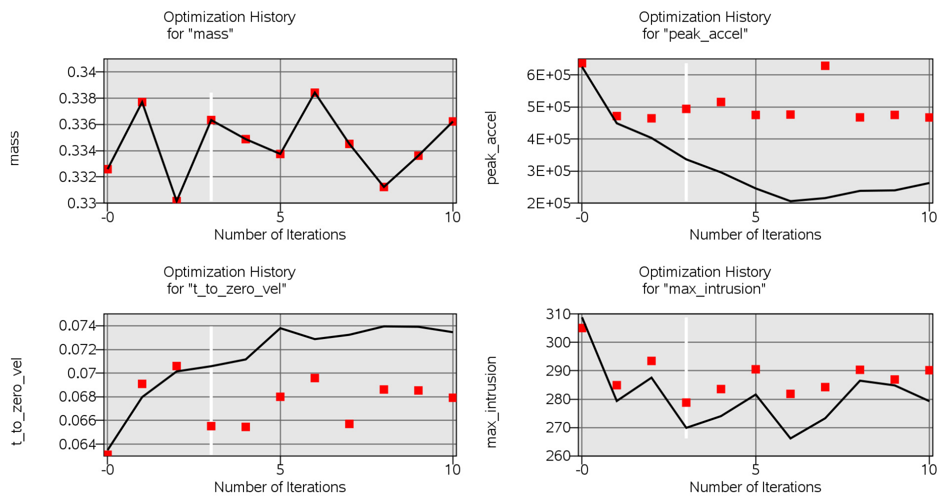


Figure 6: Optimization history for all variables based optimization.

A comparison of the history of responses corresponding to the minimum unit weighted sum of all objectives from the screened variable based optimization problem and the all design variables based optimization problem is shown in Figure 5 and Figure 6. The optimal results from both optimization processes were significantly improved compared to the baseline design (data at the 0^{th} iteration in the all-variables based optimization history). While the results were comparable from the two optimization processes, the prediction errors (differences in the predicted values represented by the solid line and the computed values represented by red dots) for the screened variables based approximation was significantly less than the prediction errors for the all design variables based metamodels. This improvement is attributed to the increase in effective sampling density for the screened variables based approximations.

The benefits of the variable screening procedure, quantified by computing the percentage reduction in the number of simulations, are shown in Figure 7. Due to the high cost of screening stage, the variable screening method was not beneficial in the first two iterations. Afterwards, a very significant reduction in the computational cost was observed.

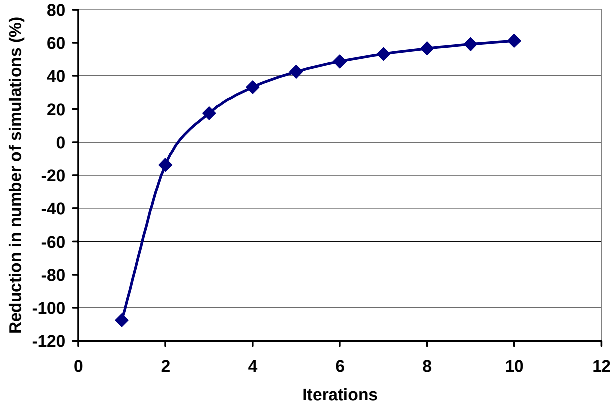


Figure 7: Percentage reduction in the number of simulations using variable screening.

Optimization with Screened Variables

The multi-objective optimization using the metamodel with 320 simulation points in the 13 variable space resulted in 4137 potential Pareto optimal solutions. A self-organizing map representation of the objectives, constraints and key variables is shown in Figure 8. The trade-offs among different objectives were immediately obvious (row 1 of left-Figure 8). The mass and intrusion; and the peak acceleration and time to reach zero velocity were in direct conflict. D-, U- and C- matrices suggested a uniform distribution of points on the entire Pareto optimal front without any gaps. While most design variables almost encompassed the entire ranges, the separation of design points in the variable space was noticeable e.g., the front floorboard thickness is close to the bounds. Similarly, the rear floorboard mostly assumes low values.

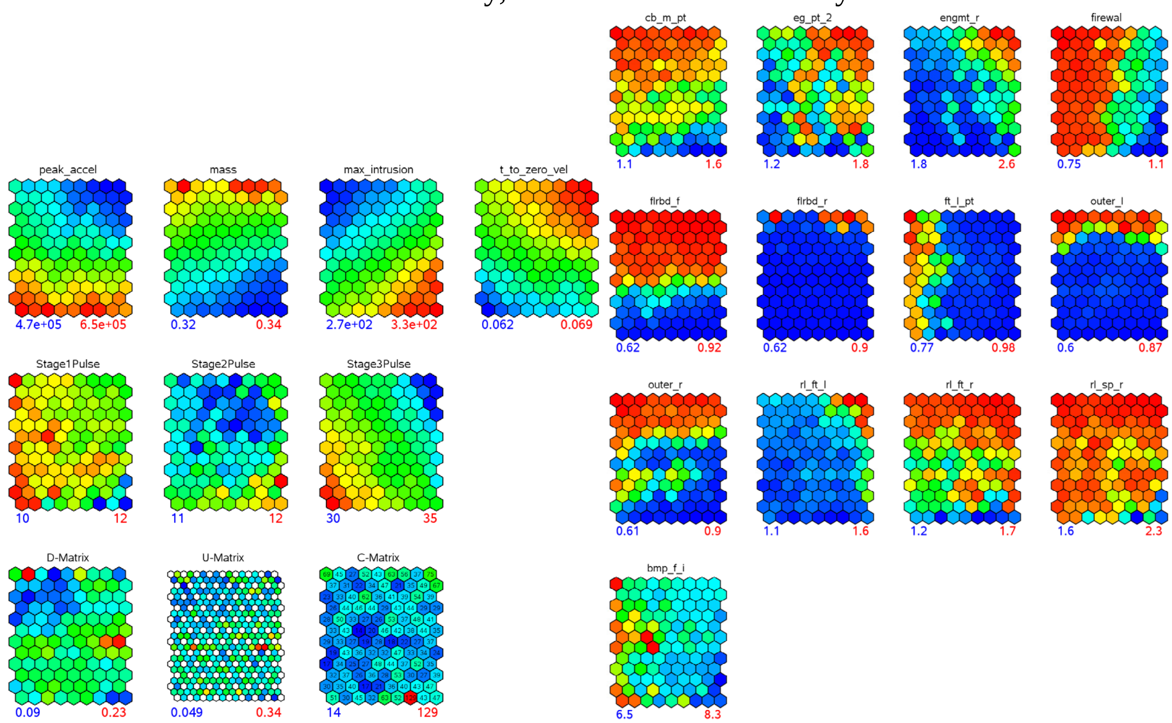


Figure 8: Self-organizing maps of the objectives and most important design variables on the Pareto front.

Summary

In this paper, a practical application of Sobol's global sensitivity analysis for dimensionality reduction is demonstrated with the help of a large crashworthiness optimization problem. For this 100 design variables problem, the optimization was carried out in two stages. In the first stage, all design variables were used to construct metamodels and these metamodels were used to identify the most important design variables for all relevant responses. The cumulative total sensitivity indices were applied to screen system-level design variables. In the second stage, a reduced set of 13 design variables was used to develop metamodels iteratively. The metamodels based on the reduced set of design variables were more accurate and had better prediction capabilities than the metamodels based on the full set of design variables. The cost of optimization quantified by the total number of simulations indicated significant benefits of using the variable screening with a moderate number of iterations.

The multi-objective optimization of this crashworthiness problem demonstrated noticeable trade-offs among all objectives. Compared to the baseline design, significant improvements in the peak acceleration, maximum intrusion, and time to reach zero velocity were attained for a vehicle with the same mass.

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Appendix

Table 7: List of variables, corresponding ranges and baseline values. Also shown are the optimum variables in the screening stage.

Name	Lower	Baseline	Upper	Optima-S1	Influential	Name	Lower	Baseline	Upper	Optima-S1	Influential
fdr_i_l	0.720	0.900	1.080	0.8399		rl_ft_r	1.118	1.397	1.676	1.67	YES
fdr_i_r	0.720	0.900	1.080	0.7669		rl_md_l	0.749	0.936	1.123	0.8109	
fd_sp_l	0.720	0.900	1.080	1.011		rl_md_r	0.749	0.936	1.123	1.107	
fd_sp_r	0.720	0.900	1.080	1.065		rl_pt_l	1.238	1.548	1.858	1.801	
rsus_tp	0.480	0.600	0.720	0.5784		rl_pt_r	1.238	1.548	1.858	1.723	
sdb_e_r	0.800	1.000	1.200	1.145		rl_rr_l	0.898	1.122	1.346	0.9068	
sdb_e_l	0.800	1.000	1.200	0.9368		rl_rr_r	0.910	1.137	1.364	1.046	
w_r_l_p	0.800	1.000	1.200	0.9333		rl_rr_m	0.957	1.196	1.435	1.303	
w_r_r_p	0.800	1.000	1.200	1.195		rl_radi	0.693	0.866	1.039	1.007	
api_m_l	1.123	1.404	1.685	1.219		rl_sp_l	1.510	1.887	2.264	2.252	
api_m_r	1.123	1.404	1.685	1.571		rl_sp_r	1.510	1.887	2.264	1.553	YES
api_i_l	0.678	0.847	1.016	1.011		rl_s2_l	1.280	1.600	1.920	1.91	
api_i_r	0.678	0.847	1.016	0.7278		rl_s2_r	1.280	1.600	1.920	1.912	
api_r_l	0.608	0.760	0.912	0.6127		rp_ft_l	0.765	0.956	1.147	1.05	
api_r_r	0.608	0.760	0.912	0.6319		rp_ft_r	0.765	0.956	1.147	1.126	
bpi_i_l	0.958	1.198	1.438	1.419		rf_rl_l	0.603	0.754	0.905	0.901	
bpi_i_r	0.958	1.198	1.438	0.9893		rf_rl_r	0.603	0.754	0.905	0.6082	
bpi_p_l	1.070	1.337	1.604	1.143		rfbow_f	0.612	0.765	0.918	0.9046	
bpi_p_r	1.070	1.337	1.604	1.123		st_rr_l	0.929	1.161	1.393	1.311	
bpi_w_l	0.790	0.988	1.186	0.8177		st_rr_r	0.929	1.161	1.393	1.312	
bpi_w_r	0.790	0.988	1.186	1.132		sdbea_l	0.680	0.850	1.020	1.011	
cb_m_pt	1.078	1.347	1.616	1.173	YES	sdbea_r	0.680	0.850	1.020	0.9556	
cb_m_p2	0.687	0.859	1.031	0.9817		sill_ft	0.594	0.742	0.890	0.8188	
cb_m_rl	1.070	1.337	1.604	1.422		sill_i	0.718	0.897	1.076	0.9343	
cb_st_r	0.693	0.866	1.039	0.7758		sill_tp	0.590	0.738	0.886	0.6471	
eg_bt_l	1.396	1.745	2.094	1.623		sill_sp	1.173	1.466	1.759	1.428	
eg_bt_m	0.830	1.038	1.246	1.007		str_i_l	1.194	1.492	1.790	1.775	
eg_bt_r	1.396	1.745	2.094	1.745		str_i_r	1.194	1.492	1.790	1.745	
eg_sp_m	0.978	1.222	1.466	1.151		str_o_l	1.138	1.422	1.706	1.593	
eg_pt_1	1.226	1.533	1.840	1.829		str_o_r	1.138	1.422	1.706	1.408	
eg_pt_2	1.226	1.533	1.840	1.594	YES	trn_t_c	0.946	1.182	1.418	0.9581	
engmt_r	1.769	2.211	2.653	2.505	YES	w_f_i_l	0.694	0.868	1.042	1.001	
fd_sh_l	1.548	1.935	2.322	1.616		w_f_i_r	0.694	0.868	1.042	0.965	
fd_sh_r	1.548	1.935	2.322	1.728		w_f_o_l	0.653	0.816	0.979	0.7784	
firewal	0.746	0.932	1.118	0.8036	YES	w_f_o_r	0.653	0.816	0.979	0.7075	
fw_i_up	1.246	1.558	1.870	1.869		ww_rr_l	0.582	0.727	0.872	0.6075	
Flrbd_f	0.614	0.768	0.922	0.9172	YES	w_r_o_l	0.598	0.748	0.898	0.8631	
Flrbd_r	0.617	0.771	0.925	0.6429	YES	w_r_o_r	0.598	0.748	0.898	0.7332	
ft_l_pt	0.766	0.958	1.150	0.7982	YES	ww_rr_r	0.570	0.712	0.854	0.5957	
hl_bt_l	0.595	0.744	0.893	0.6048		w_s_f_l	0.704	0.880	1.056	1.049	
hl_bt_r	0.595	0.744	0.893	0.8447		w_s_f_r	0.704	0.880	1.056	0.7438	
outer_l	0.603	0.754	0.905	0.8476	YES	wnd_r_l	0.590	0.737	0.884	0.7307	
outer_r	0.603	0.754	0.905	0.9007	YES	wnd_r_r	0.590	0.737	0.884	0.8597	
rsus_bo	0.856	1.070	1.284	0.8727		wdr_r_l	0.593	0.741	0.889	0.7444	
rsus_bt	1.178	1.472	1.766	1.65		wdr_r_r	0.593	0.741	0.889	0.6164	
rl_b1_l	2.246	2.808	3.370	3.338		bmp_f_i	6.424	8.030	9.636	6.941	YES
rl_b1_r	2.246	2.808	3.370	2.505		fnd_o_l	0.560	0.700	0.840	0.8019	
rl_b2_l	0.773	0.966	1.159	0.8179		fnd_o_r	0.560	0.700	0.840	0.5638	
rl_b2_r	0.773	0.966	1.159	1.114		roof	0.600	0.750	0.900	0.8177	
rl_ft_l	1.108	1.385	1.662	1.608	YES	rdssp_i	1.192	1.490	1.788	1.238	