Isogeometric Analysis in LS-DYNA

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Introduction

• Isogeometric analysis: finite element analysis performed using the same basis functions as in computer aided design (CAD).

• CAD basis functions:
  – NURBS: accepted standard for many years.
  – T-Splines: newcomer with advantages.
  – Subdivision surfaces: from animation industry. Future in CAD and analysis unclear.
  – It is clear that basis functions are a very active area of research for both the CAD and computer animation industries.

• Implementing elements for specific basis functions is
  – Extremely time consuming.
  – Software may quickly become obsolete as new basis functions are introduced.

• Desire an ability to rapidly prototype new elements.
B-Spline Basis Functions

• Piecewise polynomials in space.
• Degree determined by the knot vector:
  \[ \Xi = \{\xi_1, \ldots, \xi_{n+p+1}\} \]
  
• Coefficients of polynomials are points in space, referred to as control points, \( B_i \).
• Basis functions are generated recursively using the knot vector starting at \( p=0 \) (piecewise constants).

\[
N_{i,0}(\xi) = \begin{cases} 
1 & \text{if } \xi_i \leq \xi < \xi_{i+1} \\
0 & \text{otherwise.}
\end{cases}
\]

\[
N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi).
\]
B-Spline Basis Functions

• Each increase in degree typically increases the continuity too:
  – Linear B-spline: $C_0$
  – Quadratic B-spline: $C_1$
  – Cubic B-Spline: $C_2$

• Example: Euler-Bernoulli beams require $C_1$ continuity.
  – Conventional FEM: Cubic Hermitian polynomials.
  – B-spline: Quadratic w/o rotational DOF
B-Spline Basis Functions

• 1-D: For $n$ elements with degree $p$ polynomials and a continuity of $c$, then number of basis functions $N$ is

$$N = n \cdot (p + 1) - (c + 1) \cdot (n - 1)$$

• Example: 10 quadratic ($p=2$) elements
  – Lagrange polynomial: $10(2+1)-(0+1)(10-1)=21$
  – B-spline: $10(2+1)-(1+1)(10-1)=12$
B-Spline Basis Functions

• Fewer basis functions means fewer integration points ➔ cheaper higher order elements.

• Continuation of 1-D Example:
  – Lagrangian: 21/10 ~ 2 points/element.
  – B-Spline: 12/10 ~ 1 point/element.

• Multi-D:
  – 2x2x2 for quadratic B-Spline solids.
  – 2x2 for quadratic shells.
Properties of B-Splines

- B-splines sum to 1 like Lagrange interpolation functions.
  \[ \sum_{i=1}^{n} N_{i,p}(\xi) = 1 \quad \forall \xi \]

- The support of each \( N_{i,p}(\xi) \) compact and contained in the interval \([\xi_i, \xi_{i+p+1}]\) similar to Lagrange interpolation polynomials.

- B-spline basis functions are non-negative:
  \[ N_{i,p}(\xi) \geq 0 \quad \forall \xi \]
  (in contrast to higher order Lagrange polynomials).
Cubic B-Spline Basis Functions

\[ \Xi = \{0, 0, 0, 0, 1/6, 1/3, 1/2, 2/3, 5/6, 1, 1, 1, 1\} \]
B-Spline Surfaces and Solids

• Surfaces and solids are described in terms of tensor products of one-dimensional basis functions as is standard with Lagrange interpolation functions in standard FEA.

\[
S(\xi, \eta) = \sum_{i=1}^{n} \sum_{j=1}^{m} N_{i,p}(\xi) M_{j,q}(\eta) B_{i,j} \quad \text{Surface}
\]

\[
S(\xi, \eta, \zeta) = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} N_{i,p}(\xi) M_{j,q}(\eta) L_{k,l}(\zeta) B_{i,j,k} \quad \text{Solid}
\]
NURBS Basis Functions

- Non-Uniform Rational B-Splines (NURBS)
- Control points are homogenous coordinates.

\[(B_i)_j = \frac{(B_i^w)_j}{w_i}, \quad j = 1, ..., d \quad w_i = \text{weights}\]

- Basis functions:

\[R_i^p(\xi) = \frac{N_{i,p}(\xi)w_i}{\sum_{i=1}^{n} N_{i,p}(\xi)w_i}\]

- Curve:

\[C(\xi) = \sum_{i=1}^{n} R_i^p(\xi)B_i\]

- Previous comments about B-splines apply to NURBS
Intro to Generalized Elements

KEY IDEAS:

1. Elements are formulated in terms of *generalized coordinates*.

2. Software implementation is independent of the basis functions.

3. New basis functions can be used immediately, permitting rapid prototyping of elements.

4. Instantiations of the generalized elements are defined in the input data by specifying the values of the integration weights, the basis function values and their derivatives.
Kinematics & Semi-discrete Equations

• Similar to the standard FE formulation.

\[
x(s, t) = \sum_{A=1,N} N_A(s) q_A(t) \quad X(s) = x(s, 0) = \sum_{A=1,N} N_A(s) q_A(0)
\]

\[
\dot{x}(s, t) = \sum_{A=1,N} N_A(s) \dot{q}_A(t) \quad u(s, t) = x(s, t) - X(s)
\]

\[
\ddot{x}(s, t) = \sum_{A=1,N} N_A(s) \ddot{q}_A(t) \quad \delta x(s) = \sum_{A=1,N} N_A(s) \delta q_A
\]

• Everything else follows similarly.

\[
\sum_B M_{AB} \ddot{q}_B + \int_V B^T_A \sigma dV = \int_S hN_A dS + \int_\Omega bN_A dV
\]
Basis Functions for Shells & Solids

• Valid for:
  – NURBS
  – T-Splines
  – Lagrange polynomials (standard FEM)
  – Subdivision surfaces
  – X-FEM (in collaboration with Belytschko)
  – Normal modes
  – & More…

• All LS-DYNA isogeometric (NURBS & T-Spline) calculations currently performed with generalized elements.
Notes on Visualization

- NURBS control points don’t live on shell surfaces or solid volumes.
- LS-PREPOST only visualizes elements with linear basis functions.
- Interpolation elements: linear elements generated to visualize isogeometric results.
- Interpolation nodes: nodes defined for interpolation elements. Motion is linear function of control points.
- Usually represent isogeometric element of degree P with P x P patch of linear elements.
  - Quadratic: 2 x 2 patch of linear quadrilaterals.
  - Cubic: 3 x 3 patch of linear quadrilaterals.
LS-DYNA Input Structure

Generalized Shell Elements

Analysis

\[ \dot{\varepsilon}_i = B \cdot v \]

\[ \sigma_s = \sum_{i=1}^{ngp} w^\sigma_i \cdot \sigma_i \]

Output

Output & BC

Input for the generalized solid elements is similar.
Intro to Generalized Elements: Applications to Solids

- Geometry defined in terms of
  - Parametric coordinates on domain, \( s \)
  - Generalized coordinates in time, \( q_A(t) \)
  - Basis functions, \( N_A(s) \)

\[
x(s,t) = \sum_{A=1,N} N_A(s)q_A(t)
\]

- Generalized coordinates are not assumed to be interpolatory.
- Formulation is isoparametric and spatially isotropic.
## Square Taylor Bar Impact

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<th># Nodes/ CP</th>
<th>Peak Plastic Strain</th>
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- **1-Pnt Hex**: Standard LS-DYNA element
- **27 Node Quadratic**: Generalized Element
- **Quad. NURBS**: Generalized Element

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## Square Taylor Bar Impact Cost Comparisons

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Cost per node scales roughly linearly with the number of nodes in element.

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Isogeometric X-FEM

• Higher order linear static fracture analysis in collaboration with Ted Belytschko.
• Enriched degrees of freedom treated as additional nodes.
• Enrichment functions are not spatially isotropic, therefore constraints are added to eliminate unwanted enrichment contributions.
Enrichment Functions

\[ u(x) = u^{\text{IsoGeo}}(x) + \sum_{J} N_{J}(x) \left[ H(y) - H_{J} \right] a_{J} + K_{I} u^\infty_{K_{I}}(x) - \sum_{L} N_{L}(x) u^\infty_{K_{I},L} \]

\[ u^\infty_{K_{I}}(x) = \begin{bmatrix} u_{x}^\infty \\ u_{y}^\infty \end{bmatrix} = \frac{\sqrt{r}}{2 \sqrt{2} \pi \mu} \begin{bmatrix} (-1/2 + \kappa) \cos \left( \frac{\theta}{2} \right) - 1/2 \cos \left( \frac{3\theta}{2} \right) \\ (1/2 + \kappa) \sin \left( \frac{\theta}{2} \right) - 1/2 \sin \left( \frac{\theta}{2} \right) \end{bmatrix} \]

\( y = \) level set distance function from crack

\( \kappa = 3 - 4\nu \)

Generalized Element Format

\[ u(x) = \sum_{A} N_{A} q_{A} + \sum_{J} N'_{J} a_{J} + N''_{K_{Ix}} K_{Ix} + N''_{K_{Iy}} K_{Iy} \]

\[ N'_{J} = N_{J}(x) \left[ H(y) - H_{J} \right] \]

\[ N''_{K_{Ix}} = u^\infty_{K_{Ix}}(x) - \sum_{L} N_{L}(x) u^\infty_{K_{Ix},L}, \quad N''_{K_{Iy}} = u^\infty_{K_{Iy}}(x) - \sum_{L} N_{L}(x) u^\infty_{K_{Iy},L} \]

\[ q_{A}, a_{J}, K_{Ix}, K_{Iy} : \text{Nodal variables have dimension 3} \]

\[ K_{Ix2} = K_{Ix3} = 0, \quad K_{Iy1} = K_{Iy3} = 0 \]
X-FEM + Isogeometric for Linear Fracture
Exact $K_I=100$

3x3 Mesh
Quintic
Isogeometric
Elements $K_I=97$

11x11 Mesh
Quintic
Isogeometric
Elements $K_I=99.49$

$\varepsilon_{xx}$

$\varepsilon_{yy}$

Emmanuel De Luyckyer, UCSD
X-FEM + Isogeometric Convergence in $H_1$ Norm
Generalized Shells

• Shear deformable and thin shell theories have been implemented.

• Shear deformable implementation is a hybrid of two formulations:
  – Degenerated solid of Hughes-Liu for basic kinematics.
  – Use normal vector instead of fiber vector as in Belytschko-Tsay to avoid ambiguities at shell intersections and to enhance the robustness for explicit calculations.
Shell Formulation With Rotations

• Geometry:
  \[ x(s, t) = \sum_A N_A(s) \left( q_A + \frac{h}{2} s_3 n_A \right) \]

• Velocity field:
  \[ \dot{x}(s, t) = \sum_A N_A(s) \left( \dot{q}_A + \frac{h}{2} s_3 \omega_A \times n_A \right) \]

• Definition of normal:
  \[ n_A = \frac{p}{|p|}, \quad p = \frac{\partial x}{\partial s_1} \times \frac{\partial x}{\partial s_2} \]

• Current input restricted to constant thickness shells, but not a theoretical limitation.
Thin Shell Formulation Without Rotations – Formulation 1

• Geometry:  \[ x(s,t) = \sum A N_A(s) \left( q_A + \frac{h}{2} s_3 n_A \right) \]

• Velocity field:
  \[ \dot{x}(s,t) = \sum A N_A(s) \left( \dot{q}_A + \frac{h}{2} s_3 \dot{n}_A \right) \]

• Definition of normal:
  \[ n_A = \frac{p}{|p|}, \quad p = \frac{\partial x}{\partial s_1} \times \frac{\partial x}{\partial s_2} \]

• Current input restricted to constant thickness shells, but not a theoretical limitation.
Thin Shell Formulation Without Rotations – Formulation 2

- Geometry: \[ x(s, t) = \sum_A N_A(s)q_A + \frac{h}{2}s_3 n(s) \]

- Velocity field:
  \[ \dot{x}(s, t) = \sum_A N_A(s)\dot{q}_A + \frac{h}{2}s_3 \dot{n}(s) \]

- Definition of normal:
  \[ n = \frac{p}{|p|}, \quad p = \frac{\partial x}{\partial s_1} \times \frac{\partial x}{\partial s_2} \]

- Current input restricted to constant thickness shells, but not a theoretical limitation.
Rotation Free Shell Formulations

• Formulation 1:
  – Requires only 1\textsuperscript{st} derivatives of the basis functions.
  – Sensitive to location of evaluation.

• Formulation 2:
  – Requires the 2\textsuperscript{nd} derivatives of the basis functions.

• Approximately equal accuracy and costs provided Formulation 1 derivatives evaluated correctly.
Rotations versus No Rotations

- Rotational DOF are simpler to implement: \( \dot{n}_A = \omega_A \times n_A \) versus

\[
\dot{n}_A = \frac{1}{\sqrt{p_A \cdot p_A}} (I - n \otimes n) \sum_B \frac{\partial p_A}{\partial q_B} \cdot \dot{p}_B
\]

\[
\dot{p}_B = \sum_C \frac{\partial N_C}{\partial s_1} \dot{q}_C \times \sum_D \frac{\partial N_D}{\partial s_2} q_D + \sum_C \frac{\partial N_C}{\partial s_1} q_C \times \sum_D \frac{\partial N_D}{\partial s_2} \dot{q}_D
\]

- No rotations:
  - Half the DOF in implicit.
  - True thin shell approximation.
Implementation

- Strain rate evaluation through thickness.

\[ L = \left[ \frac{\partial \dot{x}}{\partial x} \right] = \left[ \frac{\partial s}{\partial x} (s_1^\ell, s_2^\ell, s_3^t) \right] \left\{ [B^m(s_1^\ell, s_2^\ell)\dot{q}] + s_3^t [B^b(s_1^\ell, s_2^\ell)\dot{q}] \right\} \]

- Force evaluation at lamina integration point.

\[ R^f = \int_{-1}^{+1} \sigma \frac{\partial s}{\partial x} J ds_3, \quad R^m = \int_{-1}^{+1} \sigma \frac{\partial s}{\partial x} s_3 J ds_3 \]

\[ F^\ell = [B^m]^T R^f, \quad M^\ell = [B^b]^T R^m \]

Contributes to forces in rotation free formulations.
Available in LS-DYNA

• Analysis capabilities:
  – Implicit and explicit time integration.
  – Eigenvalue analysis.
  – Other capabilities (e.g., geometric stiffness for buckling analysis) implemented but not yet tested.

• LS-DYNA material library available in solids and shells (including user materials).

• Some boundary conditions implemented via interpolation elements.
  – Contact doesn’t have underlying smoothness of NURBS.
  – Pressure distribution is not exactly integrated.

• Time step control: maximum system eigenvalue.
Linear Vibration of a Square Plate
Simply Supported

Exact solution for thin plate theory:

\[ \omega_{ij} = C(i^2 + j^2) \quad 0 < i, j \]

\[ C = \pi^2 \sqrt{\frac{E}{\rho(12(1 - \nu^2))}} \frac{h}{L^2} \]

\[ \pi \approx 3.1415926535897932384626433832795 \]

\[ E = 10^7, \; \nu = 0.3, \; \rho = 1, \; L = 10.0, \text{ and } h = 0.05 \]
Linear Vibration of a Square Plate

Simply Supported

Error in frequency of first mode as a function of the number of nodes. Thin shell formulation without rotational DOF.
Linear Vibration of a Square Plate

Error as a Function of Frequency Number for Finest Meshes

Shear Deformable Formulations

Rotation Free Formulations

Frequency Error

Frequency Number
Impulsively Loaded Roof

$L = 12.56\text{ in}$

$l = 10.205\text{ in}$

$R = 3.0\text{ in}$

$r = 3.08\text{ in}$

$h = 0.125\text{ in}$

$E = 1.05 \times 10^7\text{ psi}$

$\nu = 0.33$

$\rho = 2.5 \times 10^{-4}\text{ lb-s}^2/\text{in}^4$

$\sigma_y = 4.4 \times 10^4\text{ psi}$

$V_0 = 5650\text{ in/s}$
Impulsively Loaded Roof

Reissner-Mindlin

B-T coarse: 224 el.
B-T fine: 4512 el.

(a) Mesh 1
(b) Mesh 2
Impulsively Loaded Roof

Rotation Free Formulation 1

![Graph showing displacement over time for different meshes and Belytshko-Tsay models.](image)
**Impulsively Loaded Roof**

*Rotation Free – Quadratic Elements*

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<td>B-T</td>
<td>191</td>
<td>224</td>
<td>$1 \times 1$</td>
<td>578</td>
<td>0.16</td>
<td>1.103</td>
</tr>
<tr>
<td>B-T</td>
<td>4656</td>
<td>4512</td>
<td>$1 \times 1$</td>
<td>2027</td>
<td>10.5</td>
<td>1.277</td>
</tr>
</tbody>
</table>

Costs of R-M and rotation free shells are approximately the same.

All calculations performed in double precision.
Impulsively Loaded Roof Element Cost Comparisons

• B-T Element
  – 4 nodes.
  – 1-point integration.
  – Geometry projected to flat plane.
  – $1.148 \times 10^{-6}$ s/element
  – $0.287 \times 10^{-6}$ s/node

• Quadratic NURBS
  – 9 control points.
  – 2x2 integration.
  – Doubly curved shell.
  – $8.340 \times 10^{-6}$ s/element
  – $0.927 \times 10^{-6}$ s/node

Cost Ratio/DOF $\sim 3$
Square Tube Buckling

- Standard benchmark for automobile crashworthiness.
- Quarter symmetry to reduce cost.
- Perturbation to initiate buckling mode.
- $J_2$ plasticity with linear isotropic hardening.
- Mesh:
  - 640 quartic (P=4) elements.
  - 1156 control points.
  - 3 integration points through thickness.
Square Tube Buckling
Quartic Isogeometric NURBS
Quartic Square Tube Buckling

square cross section for single surface
Time = 0
Contours of Effective Plastic Strain
min=0, at elem# 1001
max=0, at elem# 1001

Fringe Levels
1.000e+00
9.500e-01
9.000e-01
8.500e-01
8.000e-01
7.500e-01
7.000e-01
6.500e-01
6.000e-01
5.500e-01
5.000e-01
4.500e-01
4.000e-01
3.500e-01
3.000e-01
2.500e-01
2.000e-01
1.500e-01
1.000e-01
5.000e-02
0.000e+00

TRITONS
UC SAN DIEGO

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Metal Stamping

• NUMISHEET standard benchmark problem.

• Data:
  – Provided by R. Dick, Alcoa.
  – Benchmark solution uses $10^4$ type 16 shells.

• No changes made to input except to replace the blank with isogeometric shell elements.
NUMISHEET Benchmark Problem

UNTITLED
Time - 0, #nodes-2123, #elem-1958
Alcoa Benchmark Solution: Plastic Strain

max ipt. value
min=0, at elem# 2986
max=0.296051, at elem# 5301

10000 Type 16 shells
Comparison of Rotation-Free Shell to Reference Solution

<table>
<thead>
<tr>
<th>NURBS</th>
<th>240</th>
<th>1092</th>
<th>3840</th>
<th>7680</th>
<th>10000</th>
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</thead>
<tbody>
<tr>
<td>Bely.-Tsai</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
Alcoa Reference Solution: Z Disp.

S-RAIL Simulation
Contours of Z-displacement
min= - 0.1790e+0; at node# 5280
max= 40.0438, at node# 3354

10000 Type 16 shells
Isogeometric Solutions: Z Disp.
Rotation Free Shells

240 NURBS
1092 NURBS
3840 NURBS
7680 NURBS
10000 Bely.-Tsaiy

Wrinkling mode is the right shape but inverted in comparison to others.

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Design-to-Analysis With T-Splines

• Bumper modeled by Mike Scott with commercial T-Spline Inc. software.
• Eigenvalue analysis with generalized elements in commercial version of LS-DYNA.
• No constraints.
• Mesh data:
  – 876 generalized Reissner-Mindlin shell elements (cubic basis functions).
  – 705 control points.
• Material properties:
  – E=10^7.
  – Poisson’s ratio=0.3.
  – Thickness=1.0.
Bumper Model with Unstructured Mesh of Cubic T-Spline Elements

Interpolation elements displayed. Each generalized element depicted by 3x3 patch of interpolation elements.
Bumper: First Bending Mode
Summary

• Higher order accurate isogeometric analysis can be cost competitive even in explicit dynamics.
• Shell formulations without rotational DOF can be cost competitive to conventional formulations.
  – Cost competitive for explicit.
  – May be cost beneficial for implicit.
    • Fewer DOF.
    • Eliminate convergence problems with rotational DOF.
• Future implementations will only get faster.
• Accuracy is excellent.
• Robustness is excellent.