Isogeometric Analysis in LS-DYNA

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Introduction

- Isogeometric analysis: finite element analysis performed using the same basis functions as in computer aided design (CAD).
- CAD basis functions:
  - NURBS: accepted standard for many years.
  - T-Splines: newcomer with advantages.
  - Subdivision surfaces: from animation industry. Future in CAD and analysis unclear.
  - It is clear that basis functions are a very active area of research for both the CAD and computer animation industries.
- Implementing elements for specific basis functions is
  - Extremely time consuming.
  - Software may quickly become obsolete as new basis functions are introduced.
- Desire an ability to rapidly prototype new elements.

B-Spline Basis Functions

- Piecewise polynomials in space.
- Degree determined the the knot vector:

$$\Xi = \{\xi_1, \ldots, \xi_{n+p+1}\}$$

- Coefficients of polynomials are points in space, referred to as *control points*, B_i
- Basis functions are generated recursively using the knot vector starting at *p*=0 (piecewise constants).

$$N_{i,0}(\xi) = \begin{cases} 1 \text{ if } \xi_i \leq \xi < \xi_{i+1} \\ 0 \text{ otherwise.} \end{cases}$$

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi).$$

B-Spline Basis Functions

- Each increase in degree typically increases the continuity too:
  - Linear B-spline: C_0
  - Quadratic B-spline: C_1
  - Cubic B-Spline: C_2
- Example: Euler-Bernoulli beams require C_1 continuity.
  - Conventional FEM: Cubic Hermitian polynomials.
  - B-spline: Quadratic w/o rotational DOF

B-Spline Basis Functions

- 1-D: For n elements with degree p polynomials and a continuity of c, then number of basis functions N is

$$N = n \cdot (p+1) - (c+1) \cdot (n-1)$$

- Example: 10 quadratic ($p=2$) elements
  - Lagrange polynomial: 10(2+1)-(0+1)(10-1)=21
  - B-spline: 10(2+1)-(1+1)(10-1)=12

B-Spline Basis Functions

- Fewer basis functions means fewer integration points ➔ cheaper higher order elements.
- Continuation of 1-D Example:
  - Lagrangian: 21/10 ~ 2 points/element.
  - B-Spline: 12/10 ~ 1 point/element.
- Multi-D:
  - 2x2x2 for quadratic B-Spline solids.
  - 2x2 for quadratic shells.

Properties of B-Splines

- B-splines sum to 1 like Lagrange interpolation functions.

$$\sum_{i=1}^{n} N_{i,p}(\xi) = 1 \ \forall \xi$$

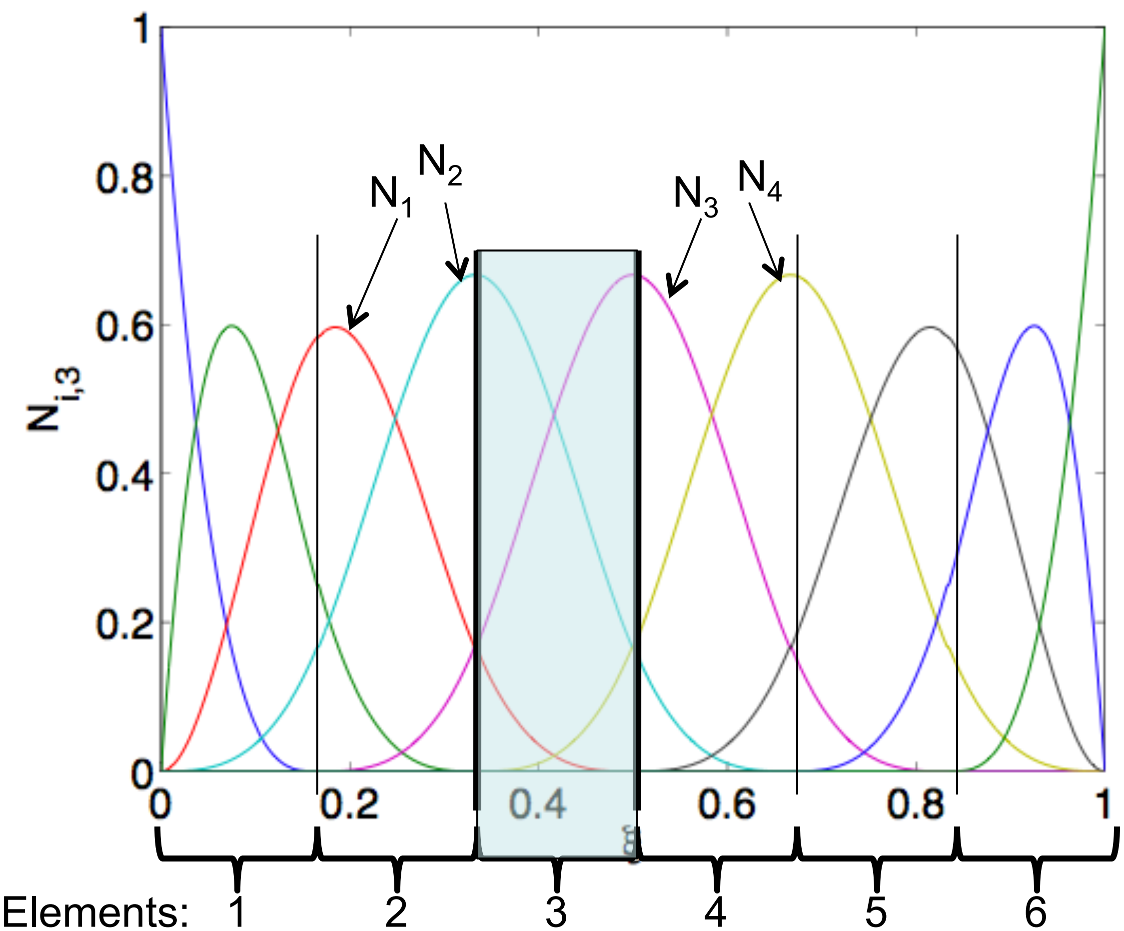
- The support of each $N_{i,p}(\xi)$ compact and contained in the interval $[\xi_i, \xi_{i+p+1}]$ similar to Lagrange interpolation polynomials.
- B-spline basis functions are non-negative:

$$N_{i,p}(\xi) \geq 0 \ \forall \xi$$

(in contrast to higher order Lagrange polynomials).

Cubic B-Spline Basis Functions

$$\Xi = \{0,0,0,0,1/6,1/3,1/2,2/3,5/6,1,1,1,1\}$$



B-Spline Surfaces and Solids

- Surfaces and solids are described in terms of tensor products of one-dimensional basis functions as is standard with Lagrange interpolation functions in standard FEA.

$$\mathbf{S}(\xi,\eta) = \sum_{i=1}^{n}\sum_{j=1}^{m} N_{i,p}(\xi) M_{j,q}(\eta) \mathbf{B}_{i,j} \quad \text{Surface}$$

$$\mathbf{S}(\xi,\eta,\zeta) = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{l} N_{i,p}(\xi) M_{j,q}(\eta) L_{k,l}(\zeta) \mathbf{B}_{i,j,k} \quad \text{Solid}$$

NURBS Basis Functions

- Non-Uniform Rational B-Splines (NURBS)
- Control points are homogenous coordinates.

$$(\mathbf{B}_i)_j = \frac{(\mathbf{B}_i^w)_j}{w_i}, \; j = 1, \ldots, d \qquad w_i \text{ =weights}$$

- Basis functions: $R_i^p(\xi) = \dfrac{N_{i,p}(\xi) w_i}{\sum_{\hat{i}=1}^{n} N_{\hat{i},p}(\xi) w_{\hat{i}}}$

- Curve: $\mathbf{C}(\xi) = \sum_{i=1}^{n} R_i^p(\xi) \mathbf{B}_i$

- Previous comments about B-splines apply to NURBS

Intro to Generalized Elements

KEY IDEAS:

1. Elements are formulated in terms of *generalized coordinates*.
2. Software implementation is independent of the basis functions.
3. New basis functions can be used immediately, permitting rapid prototyping of elements.
4. Instantiations of the generalized elements are defined in the input data by specifying the values of the integration weights, the basis function values and their derivatives.

Kinematics & Semi-discrete Equations

- Similar to the standard FE formulation.

$$x(s,t) = \sum_{A=1,N} N_A(s) q_A(t) \qquad X(s) = x(s,0) = \sum_{A=1,N} N_A(s) q_A(0)$$

$$\dot{x}(s,t) = \sum_{A=1,N} N_A(s) \dot{q}_A(t) \qquad u(s,t) = x(s,t) - X(s)$$

$$\ddot{x}(s,t) = \sum_{A=1,N} N_A(s) \ddot{q}_A(t) \qquad \delta x(s) = \sum_{A=1,N} N_A(s) \delta q_A$$

- Everything else follows similarly.

$$\sum_B M_{AB} \ddot{q}_B + \int_V B_A^T \sigma dV = \int_S h N_A dS + \int_\Omega b N_A dV$$

Basis Functions for Shells & Solids

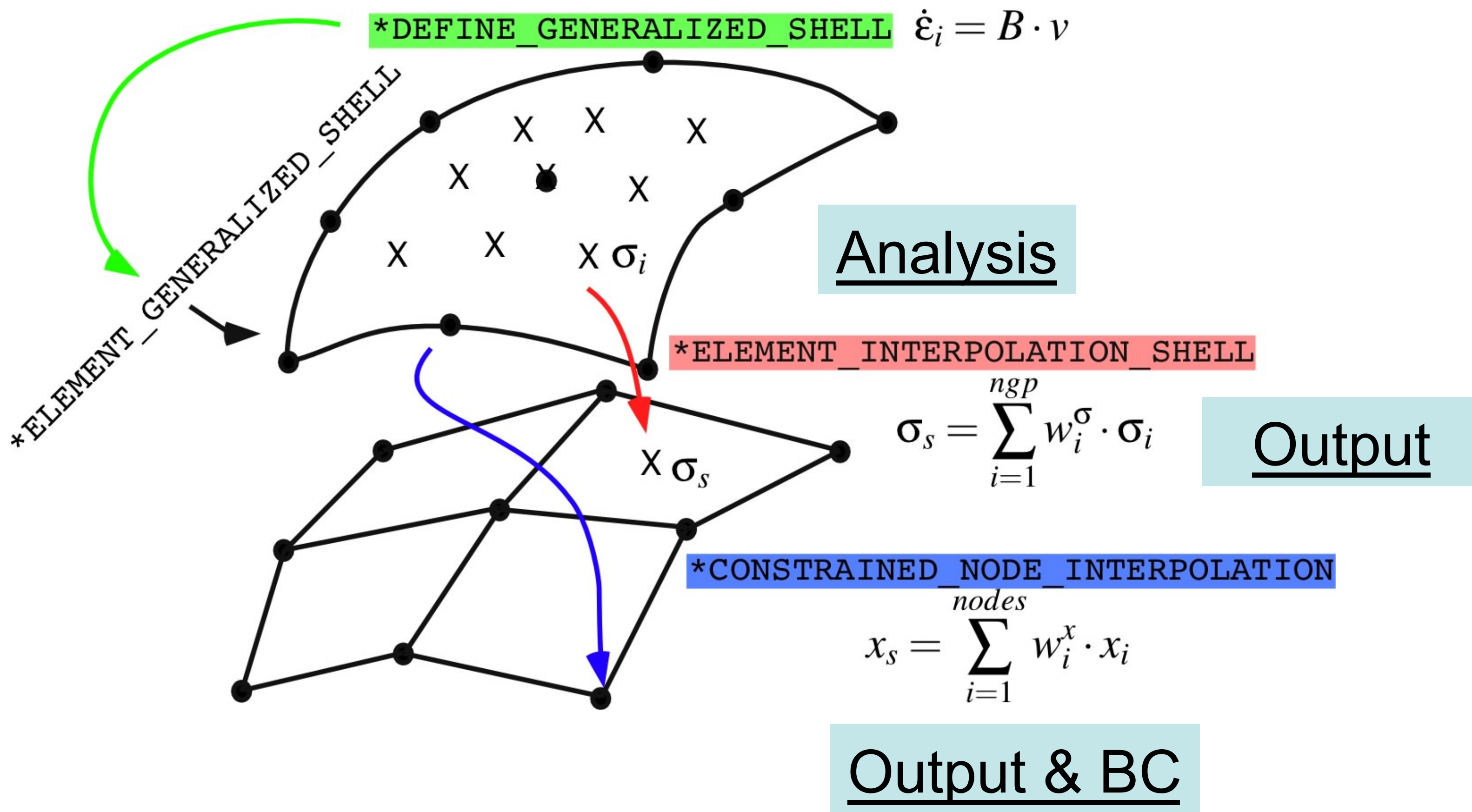
- Valid for:
  - NURBS
  - T-Splines
  - Lagrange polynomials (standard FEM)
  - Subdivision surfaces
  - X-FEM (in collaboration with Belytschko)
  - Normal modes
  - & More…
- All LS-DYNA isogeometric (NURBS & T-Spline) calculations currently performed with generalized elements.

Notes on Visualization

- NURBS control points don't live on shell surfaces or solid volumes.
- LS-PREPOST only visualizes elements with linear basis functions.
- Interpolation elements: linear elements generated to visualize isogeometric results.
- Interpolation nodes: nodes defined for interpolation elements. Motion is linear function of control points.
- Usually represent isogeometric element of degree P with P x P patch of linear elements.
  - Quadratic: 2 x 2 patch of linear quadrilaterals.
  - Cubic: 3 x 3 patch of linear quadrilaterals.

LS-DYNA Input Structure

Generalized Shell Elements



Input for the generalized solid elements is similar.

Intro to Generalized Elements:
Applications to Solids

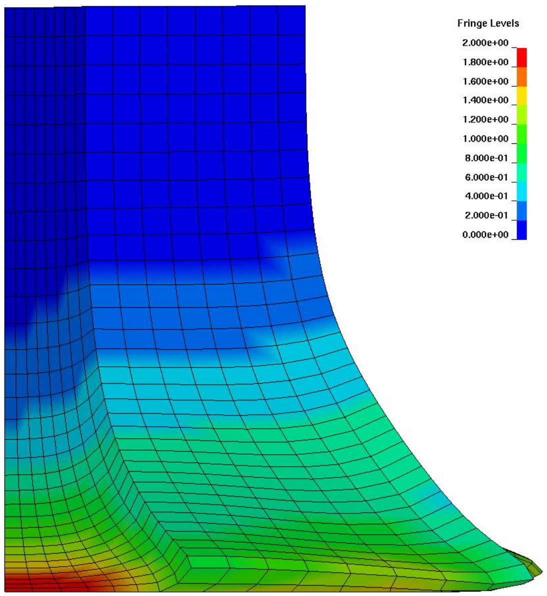
- Geometry defined in terms of
  - Parametric coordinates on domain, s
  - Generalized coordinates in time, $q_A(t)$
  - Basis functions, $N_A(s)$

$$x(s,t) = \sum_{A=1,N} N_A(s) q_A(t)$$

- Generalized coordinates are not assumed to be interpolatory.
- Formulation is isoparametric and spatially isotropic.

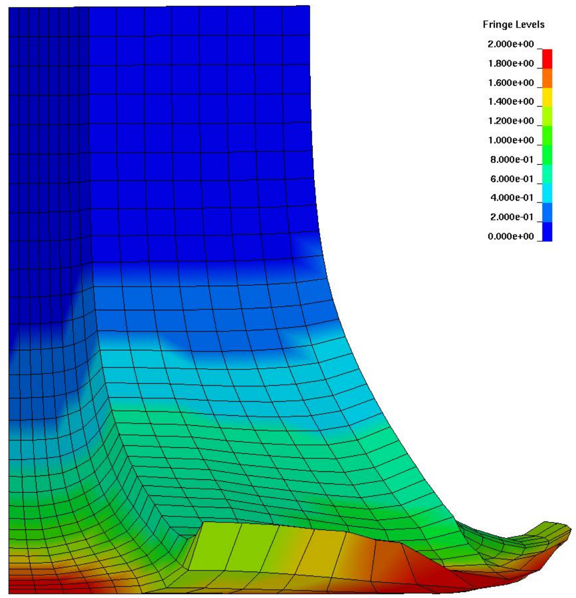
Square Taylor Bar Impact

1-Pnt Hex



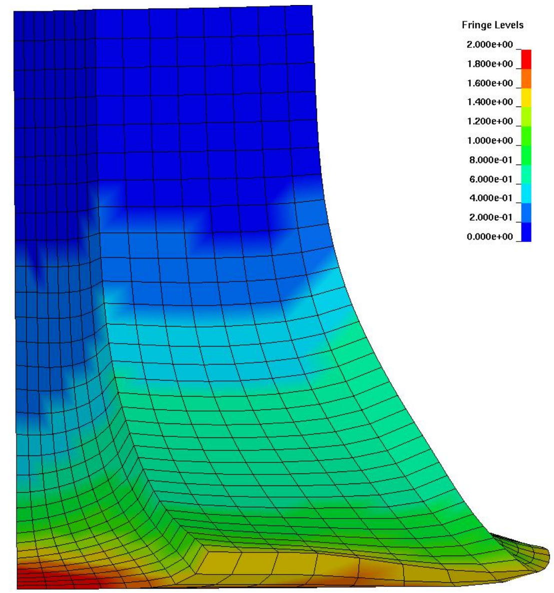
Standard LS-DYNA element

27 Node Quadratic



Generalized Element

Quadratic NURBS



Generalized Element

Formulation	# Nodes/ CP	Peak Plastic Strain	# Time Steps
1-Pnt Hex	2677	2.164	**2136**
Quad. Lagr.	2677	2.346	**3370**
Quad. NURBS	648	2.479	**954**

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Square Taylor Bar Impact

Type	Degree	Integ. Points	Nodes	Elements	$\bar{\epsilon}^p_{\max}$	Time Steps	CPU Seconds
Lagrange	1	1	81	32	1.34628	314	7.325e-2
Lagrange	1	1	425	256	1.8642	872	3.726e-1
Lagrange	1	1	1225	864	2.04989	1500	1.535
Lagrange	1	1	2673	2048	2.16408	2136	4.6027
Lagrange	2	27	81	4	1.6764	609	.58415
Lagrange	2	27	425	32	1.91551	1293	2.1039
Lagrange	2	27	1225	108	2.20551	2436	10.918
Lagrange	2	27	2673	256	2.34649	3370	35.509
NURBS	2	27	54	4	1.50409	229	.10276
NURBS	2	27	160	32	1.93467	465	.6906
NURBS	2	27	648	256	2.47937	954	9.2623
NURBS	2	8	54	4	1.57547	200	7.739e-2
NURBS	2	8	160	32	1.85617	432	.28568
NURBS	2	8	648	256	2.41749	1051	3.878
NURBS	3	27	160	4	1.79937	293	.2098
NURBS	3	27	648	32	2.02539	702	1.8819
NURBS	3	27	3400	256	2.32435	1974	37.472
NURBS	4	64	350	4	1.86798	380	.7969
NURBS	4	64	1664	32	2.09956	1015	10.17
NURBS	4	64	9800	256	2.48212	2550	204.8

Type 1

Full Integration

Reduced Integration

Square Taylor Bar Impact Cost Comparisons

Element	8-Node Hex	27-Node NURBS	64-Node NURBS
Integration	1-Point	2x2x2	4x4x4
Cost/ Element	$1.05x10^{-6}$	$14.14x10^{-6}$	$74.0x10^{-6}$
Cost/Node	$0.31x10^{-6}$	$0.53x10^{-6}$	$1.16x10^{-6}$
Cost Ratio/ Node	1.0	4.0	8.85
Node Ratio	1.0	3.375	8.0

Cost per node scales roughly linearly with the number of nodes in element.

Isogeometric X-FEM

- Higher order linear static fracture analysis in collaboration with Ted Belytschko.
- Enriched degrees of freedom treated as additional nodes.
- Enrichment functions are not spatially isotropic, therefore constraints are added to eliminate unwanted enrichment contributions.

Enrichment Functions

Ventura, Gracie, and Belytschko, *IJNME*, 2009

$$u(x) = u^{\mathrm{IsoGeo}}(x) + \sum N_J(x)\,[H(y) - H_J]\,a_J + K_I[u^{\infty}_{K_I}(x) - \sum_L N_L(x) u^{\infty}_{K_I,L}]$$

Anisotropic enrichment field violates generalized element isotropy assumption.

$$u^{\infty}_{K_I}(x) = \begin{Bmatrix} u^{\infty}_x \\ u^{\infty}_y \end{Bmatrix} = \frac{\sqrt{r}}{2\sqrt{2\pi}\mu} \begin{Bmatrix} (-1/2+\kappa)\cos(\frac{\theta}{2}) - 1/2\cos(\frac{3\theta}{2}) \\ (1/2+\kappa)\sin(\frac{\theta}{2}) - 1/2\sin(\frac{\theta}{2}) \end{Bmatrix}$$

y = level set distance function from crack $\qquad \kappa = 3 - 4\nu$

Generalized Element Format

$$u(x) = \sum_A N_A q_A + \sum_J N'_J a_J + N''_{K_{Ix}} K_{Ix} + N''_{K_{Iy}} K_{Iy}$$

$$N'_J = N_J(x)[H(y) - H_J]$$

$$N''_{K_{Ix}} = u^{\infty}_{K_{Ix}}(x) - \sum_L N_L(x) u^{\infty}_{K_{Ix},L}, \quad N''_{K_{Iy}} = u^{\infty}_{K_{Iy}}(x) - \sum_L N_L(x) u^{\infty}_{K_{Iy},L}$$

q_A, a_J, K_{Ix}, K_{Iy} : Nodal variables have dimension 3

$$K_{Ix2} = K_{Ix3} = 0, \quad K_{Iy1} = K_{Iy3} = 0$$

Nodal constraints to account for anisotropic enrichment.

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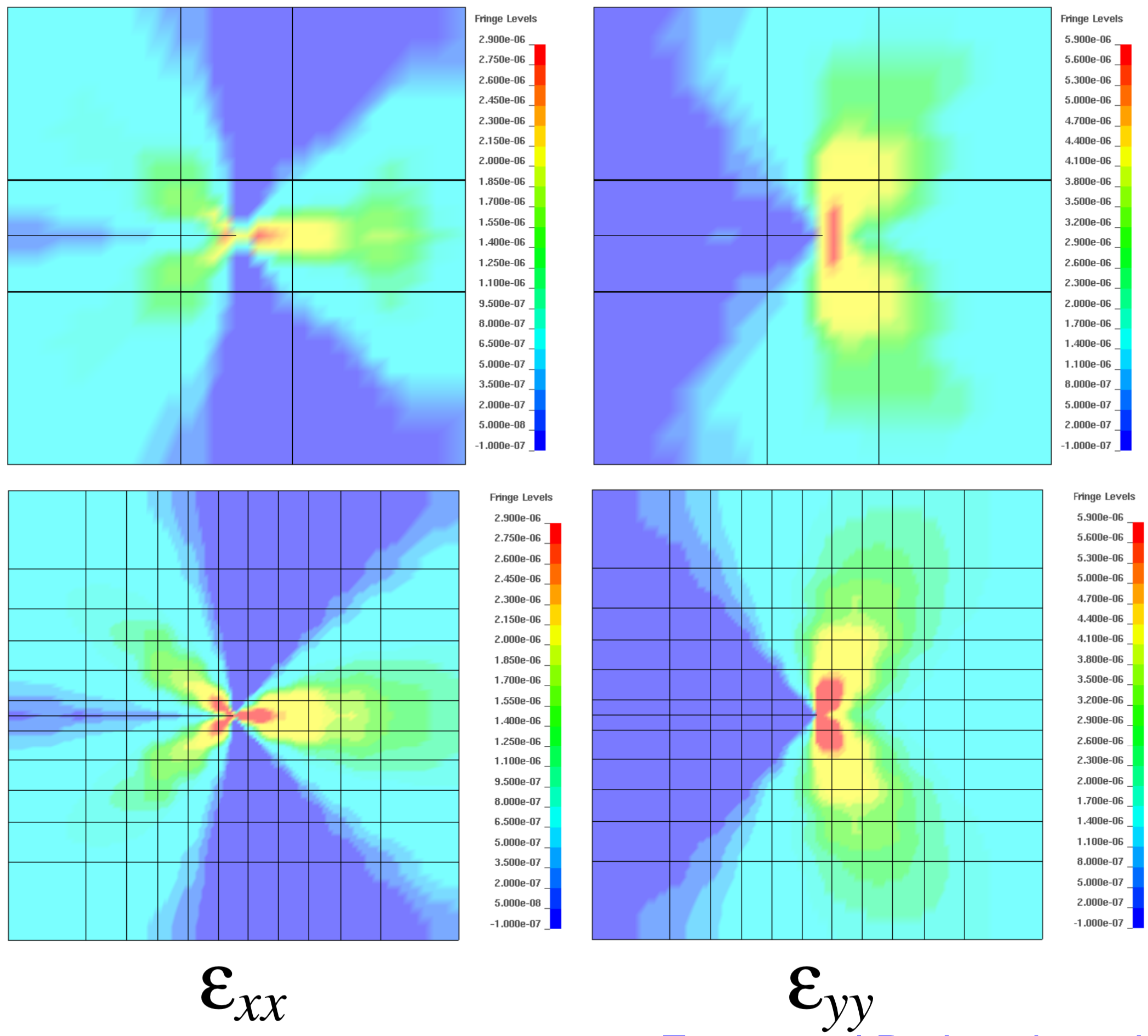
X-FEM + Isogeometric for Linear Fracture
Exact K_I=100

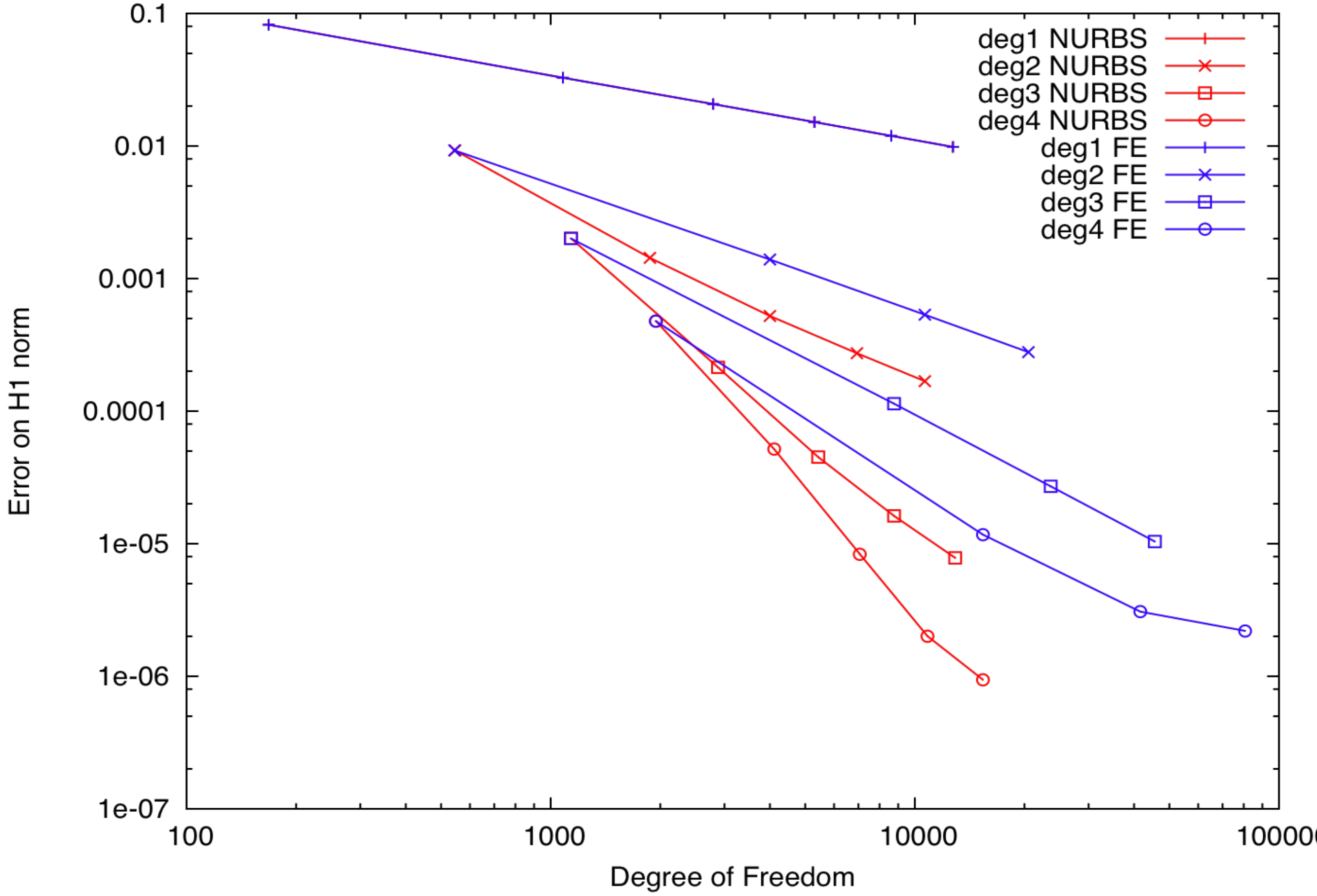
3x3 Mesh
Quintic
Isogeometric
Elements
K_I=97

11x11 Mesh
Quintic
Isogeometric
Elements
K_I=99.49



Emmanuel De Luyckyer, UCSD

X-FEM + Isogeometric Convergence in H_1 Norm



Generalized Shells

- Shear deformable and thin shell theories have been implemented.
- Shear deformable implementation is a hybrid of two formulations:
  - Degenerated solid of Hughes-Liu for basic kinematics.
  - Use normal vector instead of fiber vector as in Belytschko-Tsay to avoid ambiguities at shell intersections and to enhance the robustness for explicit calculations.

Shell Formulation With Rotations

- Geometry: $$x(s,t) = \sum_A N_A(s)\left(q_A + \frac{h}{2}s_3 n_A\right)$$
- Velocity field:

$$\dot{x}(s,t) = \sum_A N_A(s)\left(\dot{q}_A + \frac{h}{2}s_3 \omega_A \times n_A\right)$$

- Definition of normal:

$$n_A = \frac{p}{|p|}, \quad p = \frac{\partial x}{\partial s_1} \times \frac{\partial x}{\partial s_2}$$

- Current input restricted to constant thickness shells, but not a theoretical limitation.

Thin Shell Formulation Without Rotations – Formulation 1

- Geometry: $x(s,t) = \sum_A N_A(s) \left(q_A + \frac{h}{2} s_3 n_A \right)$
- Velocity field:

$$\dot{x}(s,t) = \sum_A N_A(s) \left(\dot{q}_A + \frac{h}{2} s_3 \dot{n}_A \right)$$

- Definition of normal:

$$n_A = \frac{p}{|p|}, \quad p = \frac{\partial x}{\partial s_1} \times \frac{\partial x}{\partial s_2}$$

- Current input restricted to constant thickness shells, but not a theoretical limitation.

Thin Shell Formulation Without Rotations – Formulation 2

- Geometry: $x(s,t) = \sum_A N_A(s) q_A + \frac{h}{2} s_3 n(s)$
- Velocity field:

$$\dot{x}(s,t) = \sum_A N_A(s) \dot{q}_A + \frac{h}{2} s_3 \dot{n}(s)$$

- Definition of normal:

$$n = \frac{p}{|p|}, \quad p = \frac{\partial x}{\partial s_1} \times \frac{\partial x}{\partial s_2}$$

- Current input restricted to constant thickness shells, but not a theoretical limitation.

Rotation Free Shell Formulations

- Formulation 1:
  - Requires only 1st derivatives of the basis functions.
  - Sensitive to location of evaluation.
- Formulation 2:
  - Requires the 2nd derivatives of the basis functions.
- Approximately equal accuracy and costs *provided Formulation 1 derivatives evaluated correctly.*

Rotations versus No Rotations

- Rotational DOF are simpler to implement: $\dot{n}_A = \omega_A \times n_A$ versus

$$\dot{n}_A = \frac{1}{\sqrt{p_A \cdot p_A}} (I - n \otimes n) \sum_B \frac{\partial p_A}{\partial q_B} \cdot \dot{p}_B$$

$$\dot{p}_B = \sum_C \frac{\partial N_C}{\partial s_1} \dot{q}_C \times \sum_D \frac{\partial N_D}{\partial s_2} q_D + \sum_C \frac{\partial N_C}{\partial s_1} q_C \times \sum_D \frac{\partial N_D}{\partial s_2} \dot{q}_D$$

- No rotations:
  - Half the DOF in implicit.
  - True thin shell approximation.

Implementation

- Strain rate evaluation through thickness.

$$L = \left[\frac{\partial \dot{x}}{\partial x} \right] = \left[\frac{\partial s}{\partial x}(s_1^\ell, s_2^\ell, s_3^t) \right] \left\{ \left[B^m(s_1^\ell, s_2^\ell)\dot{q} \right] + s_3^t \left[B^b(s_1^\ell, s_2^\ell)\dot{q} \right] \right\}$$

- Force evaluation at lamina integration point.

$$R^f = \int_{-1}^{+1} \sigma \frac{\partial s}{\partial x} J ds_3, \quad R^m = \int_{-1}^{+1} \sigma \frac{\partial s}{\partial x} s_3 J ds_3$$

$$F^\ell = [B^m]^T R^f, \quad M^\ell = [B^b]^T R^m$$

Contributes to forces in rotation free formulations.

Available in LS-DYNA

- Analysis capabilities:
  - Implicit and explicit time integration.
  - Eigenvalue analysis.
  - Other capabilities (e.g., geometric stiffness for buckling analysis) implemented but not yet tested.
- LS-DYNA material library available in solids and shells (including user materials).
- Some boundary conditions implemented via interpolation elements.
  - Contact doesn't have underlying smoothness of NURBS.
  - Pressure distribution is not exactly integrated.
- Time step control: maximum *system* eigenvalue.
  - D. J. Benson,*Stable Time Step Estimation for Multi-material Eulerian Hydrocodes*,CMAME,191--205 (1998).

Linear Vibration of a Square Plate

Simply Supported

Exact solution for *thin* plate theory:

$$\omega_{ij} = C(i^2 + j^2) \quad 0 < i, j$$

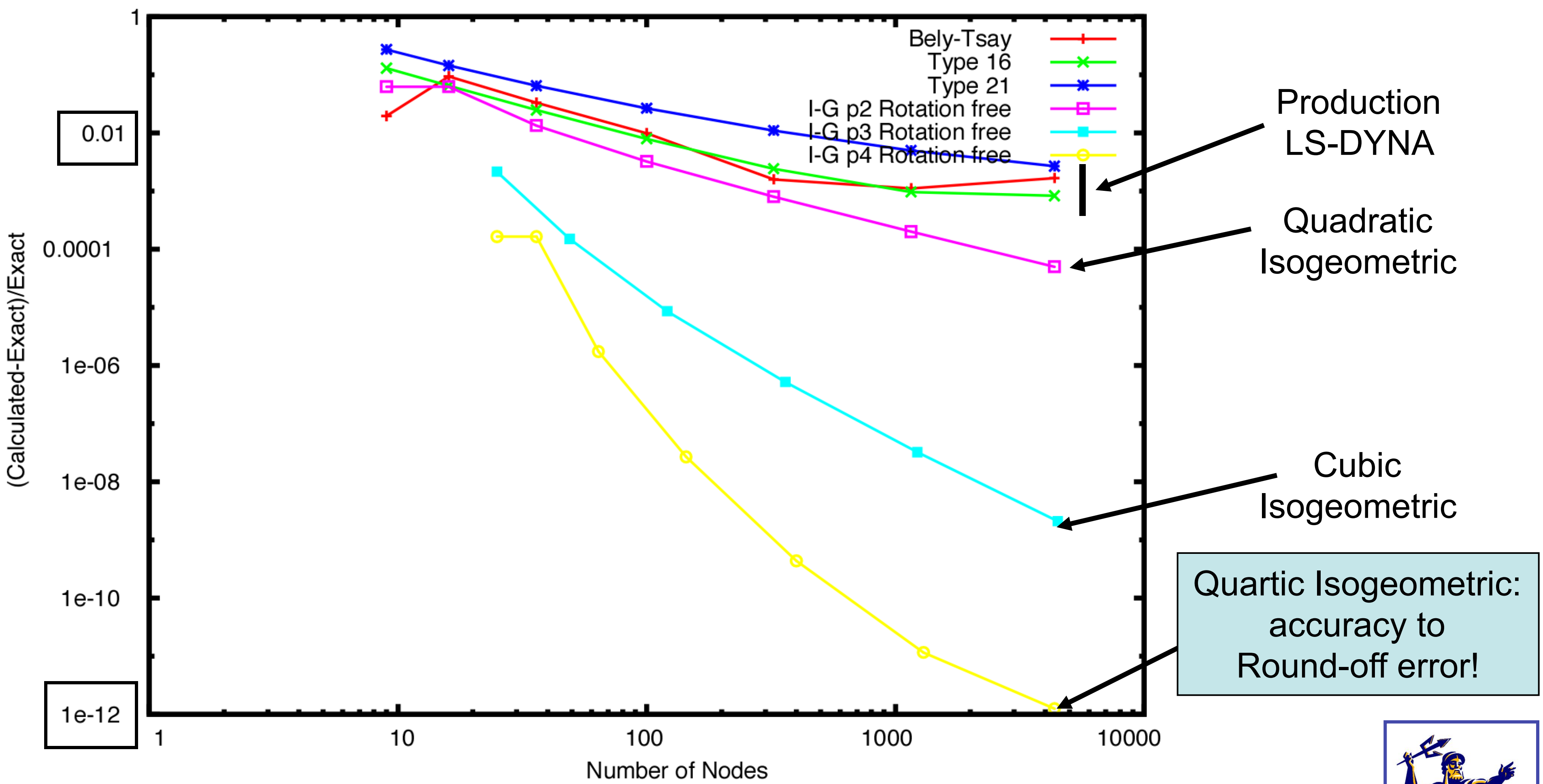
$$C = \pi^2 \sqrt{\frac{E}{\rho(12(1 - \nu^2))}} \frac{h}{L^2}$$

$$\pi \approx 3.14159265358979323846264338327950$$

$$E = 10^7,\ \nu = 0.3,\ \rho = 1,\ L = 10.0,\ \text{and}\ h = 0.05$$

Linear Vibration of a Square Plate

Simply Supported



Error in frequency of first mode as a function of the number of nodes.
Thin shell formulation without rotational DOF.

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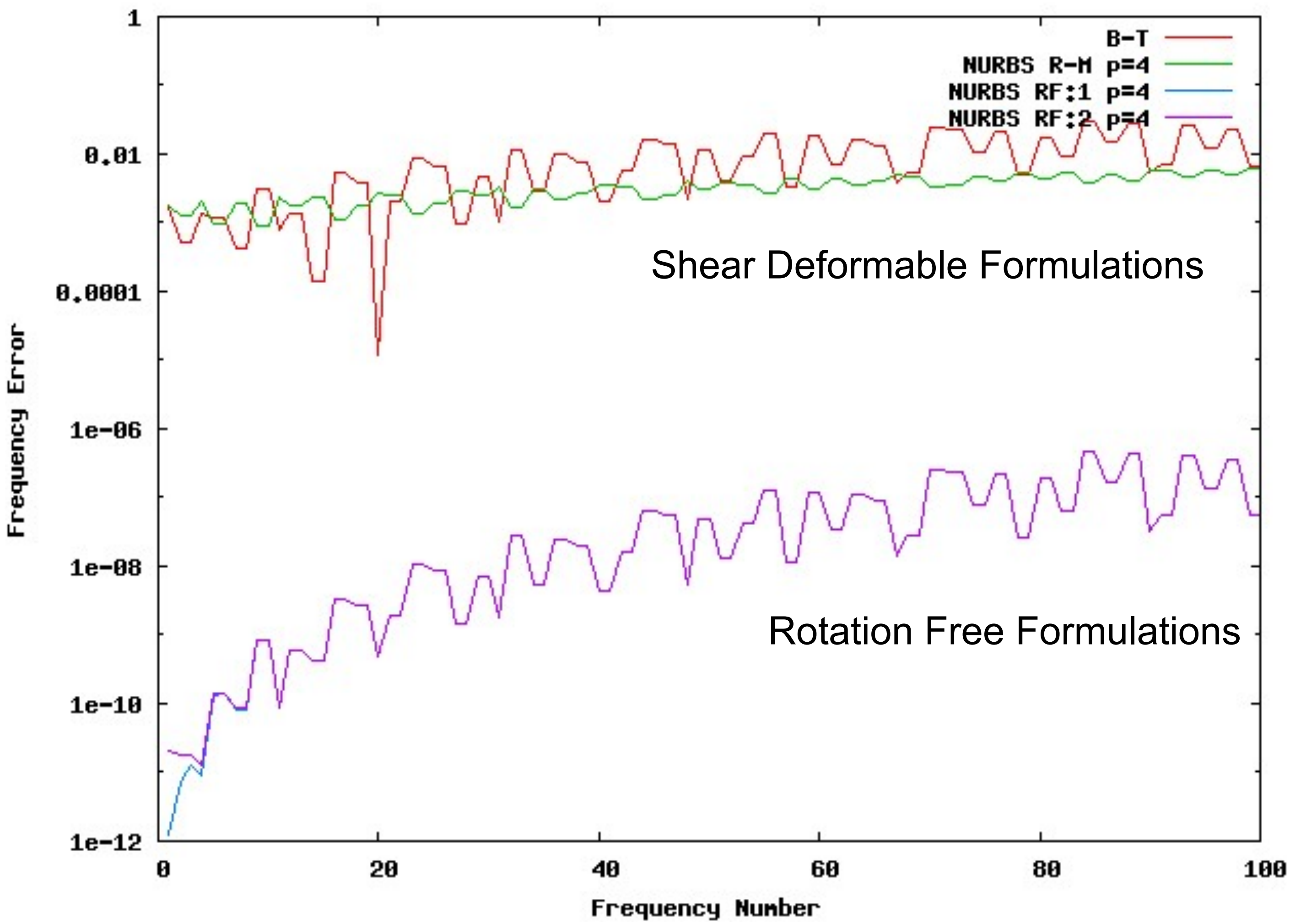
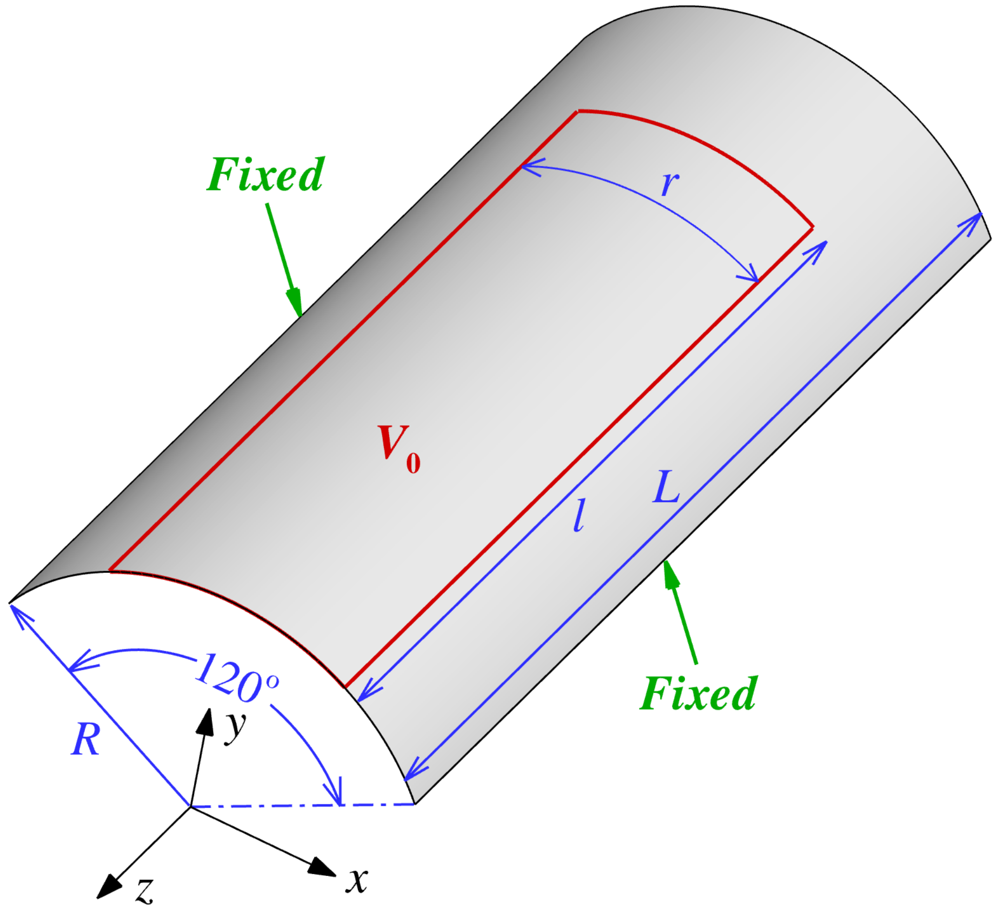
Linear Vibration of a Square Plate

Error as a Function of Frequency Number
for Finest Meshes



Impulsively Loaded Roof

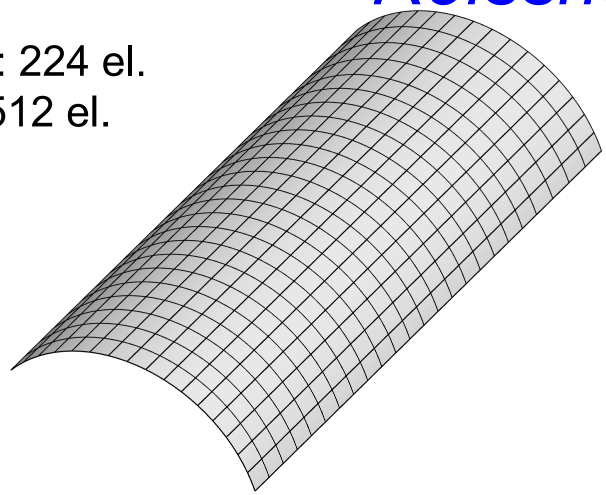


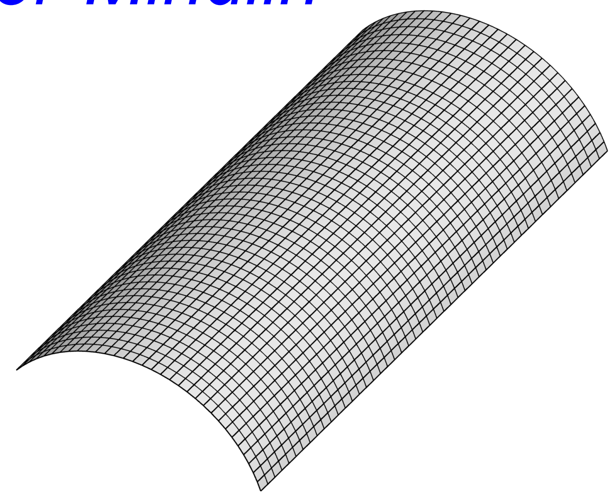
$$
\begin{aligned}
L &= 12.56 \text{ in} \\
l &= 10.205 \text{ in} \\
R &= 3.0 \text{ in} \\
r &= 3.08 \text{ in} \\
h &= 0.125 \text{ in} \\
E &= 1.05 \times 10^7 \text{ psi} \\
\nu &= 0.33 \\
\rho &= 2.5 \times 10^{-4} \text{ lb-s}^2/\text{in}^4 \\
\sigma_y &= 4.4 \times 10^4 \text{ psi} \\
V_0 &= 5650 \text{ in/s}
\end{aligned}
$$

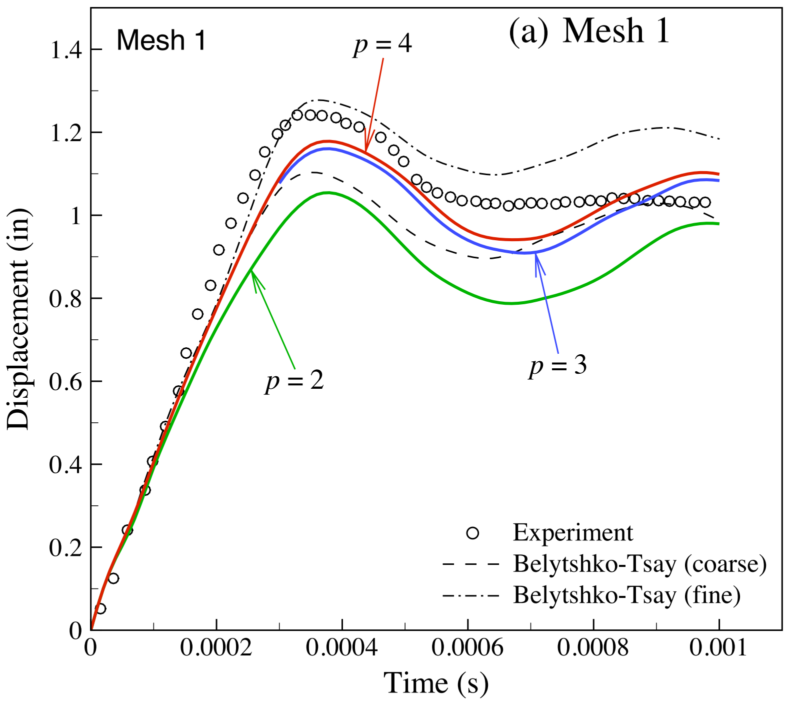
Impulsively Loaded Roof

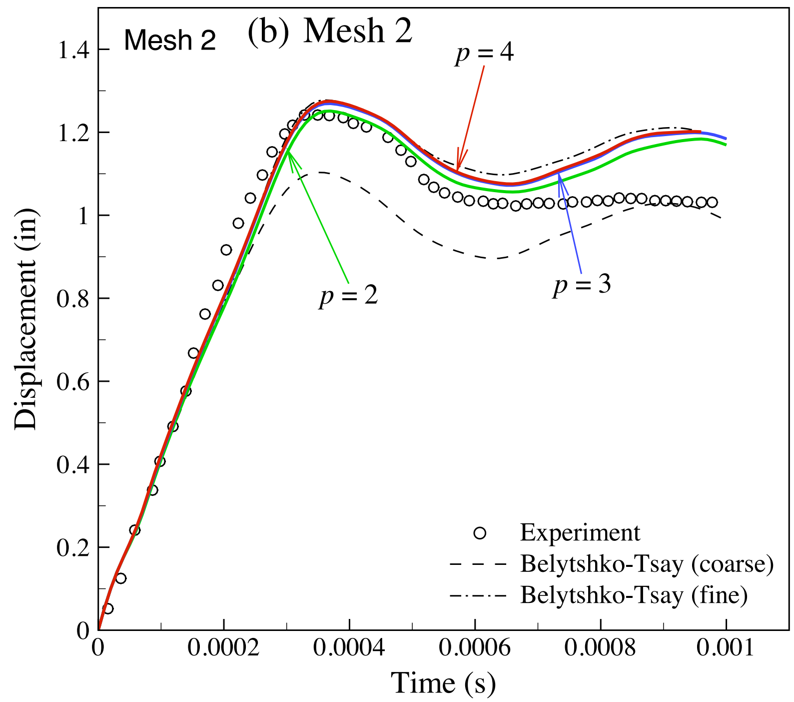
Reissner-Mindlin

B-T coarse: 224 el.
B-T fine: 4512 el.



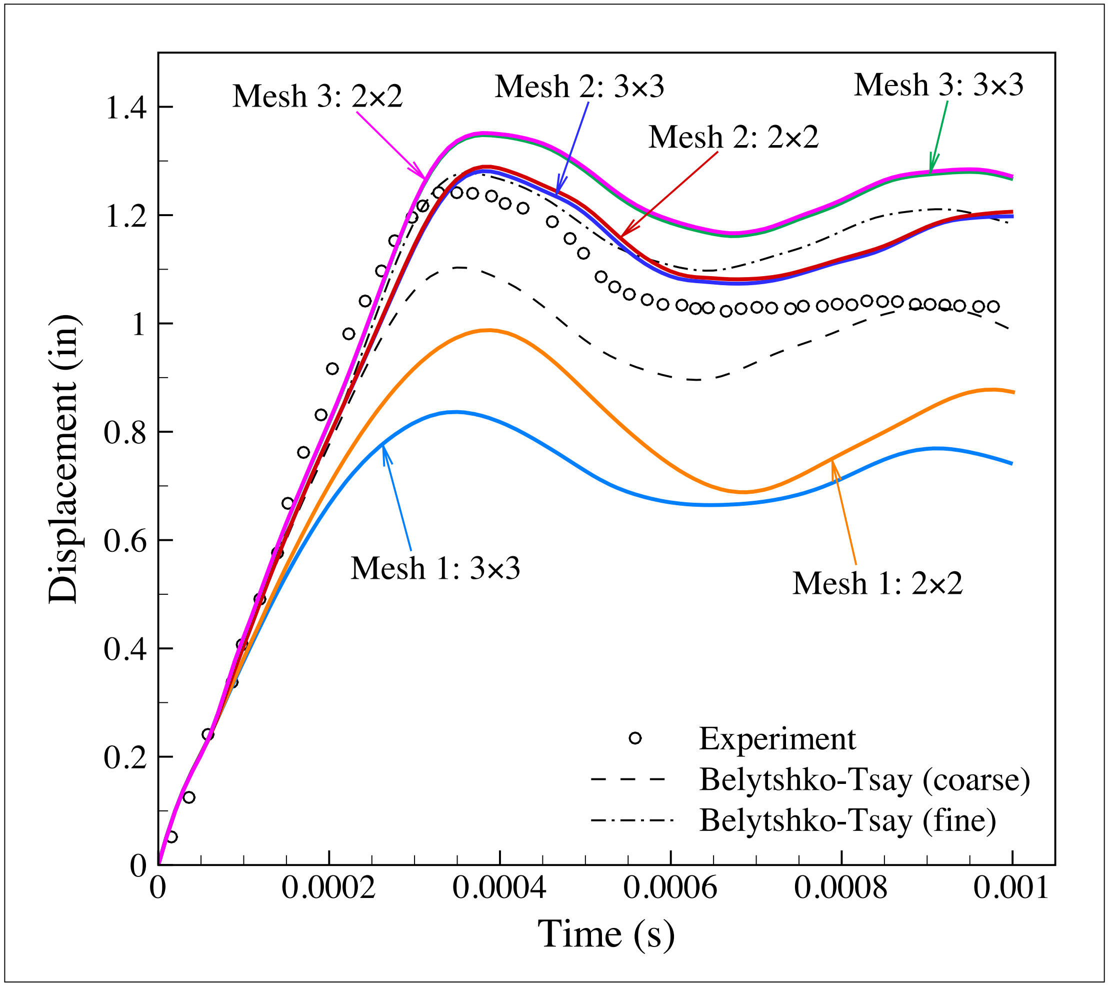






Impulsively Loaded Roof

Rotation Free Formulation 1



Impulsively Loaded Roof

Rotation Free – Quadratic Elements

Element Type	Number of Cntrl. Pnts.	Number of Elements	Integration Rule	Time Steps	CPU (seconds)	Maximum Displacement
NURBS	180	130	2×2	364	0.54	0.988
NURBS	180	130	3×3	367	0.81	0.836
NURBS	540	450	2×2	740	2.90	1.289
NURBS	540	450	3×3	743	5.28	1.281
NURBS	1836	1666	2×2	1502	20.87	1.351
NURBS	1836	1666	3×3	1502	36.92	1.348
B-T	191	224	1×1	578	0.16	1.103
B-T	4656	4512	1×1	2027	10.5	1.277

Costs of R-M and rotation free shells are approximately the same.

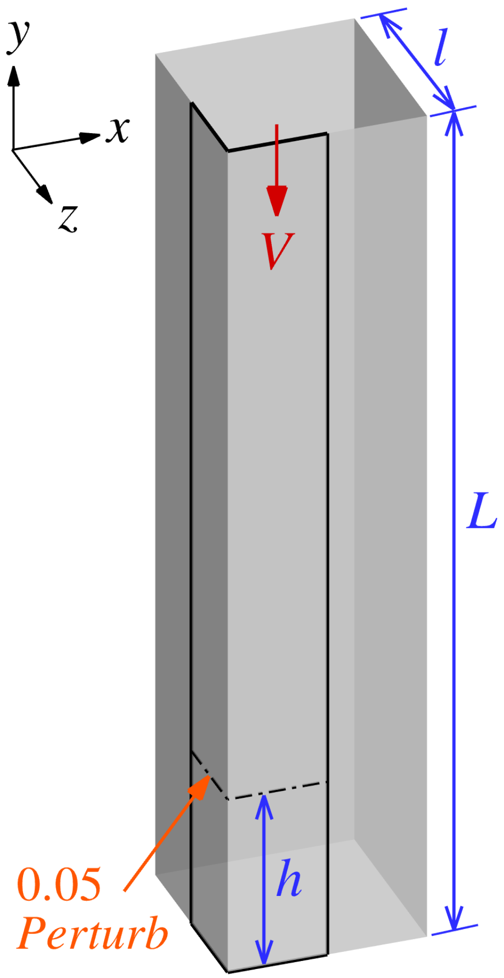
All calculations performed in double precision.

Impulsively Loaded Roof Element Cost Comparisons

- B-T Element
  - 4 nodes.
  - 1-point integration.
  - Geometry projected to flat plane.
  - 1.148×10^{-6} s/element
  - 0.287×10^{-6} s/node
- Quadratic NURBS
  - 9 control points.
  - 2x2 integration.
  - Doubly curved shell.
  - 8.340×10^{-6} s/element
  - 0.927×10^{-6} s/node

Cost Ratio/DOF ~ 3

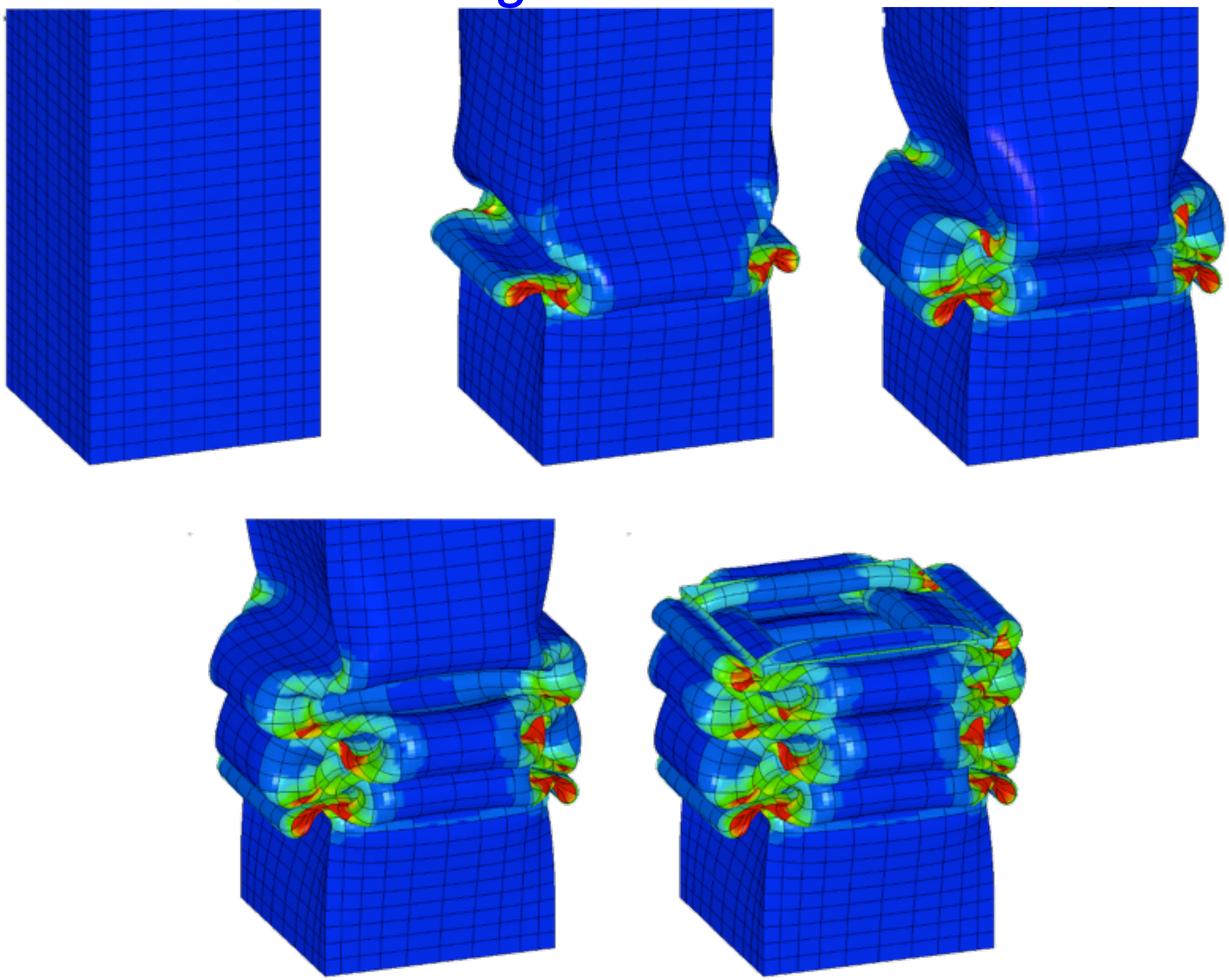
Square Tube Buckling



- Standard benchmark for automobile crashworthiness.
- Quarter symmetry to reduce cost.
- Perturbation to initiate buckling mode.
- J_2 plasticity with linear isotropic hardening.
- Mesh:
  - 640 quartic (P=4) elements.
  - 1156 control points.
  - 3 integration points throught thickness.

Square Tube Buckling

Quartic Isogeometric NURBS



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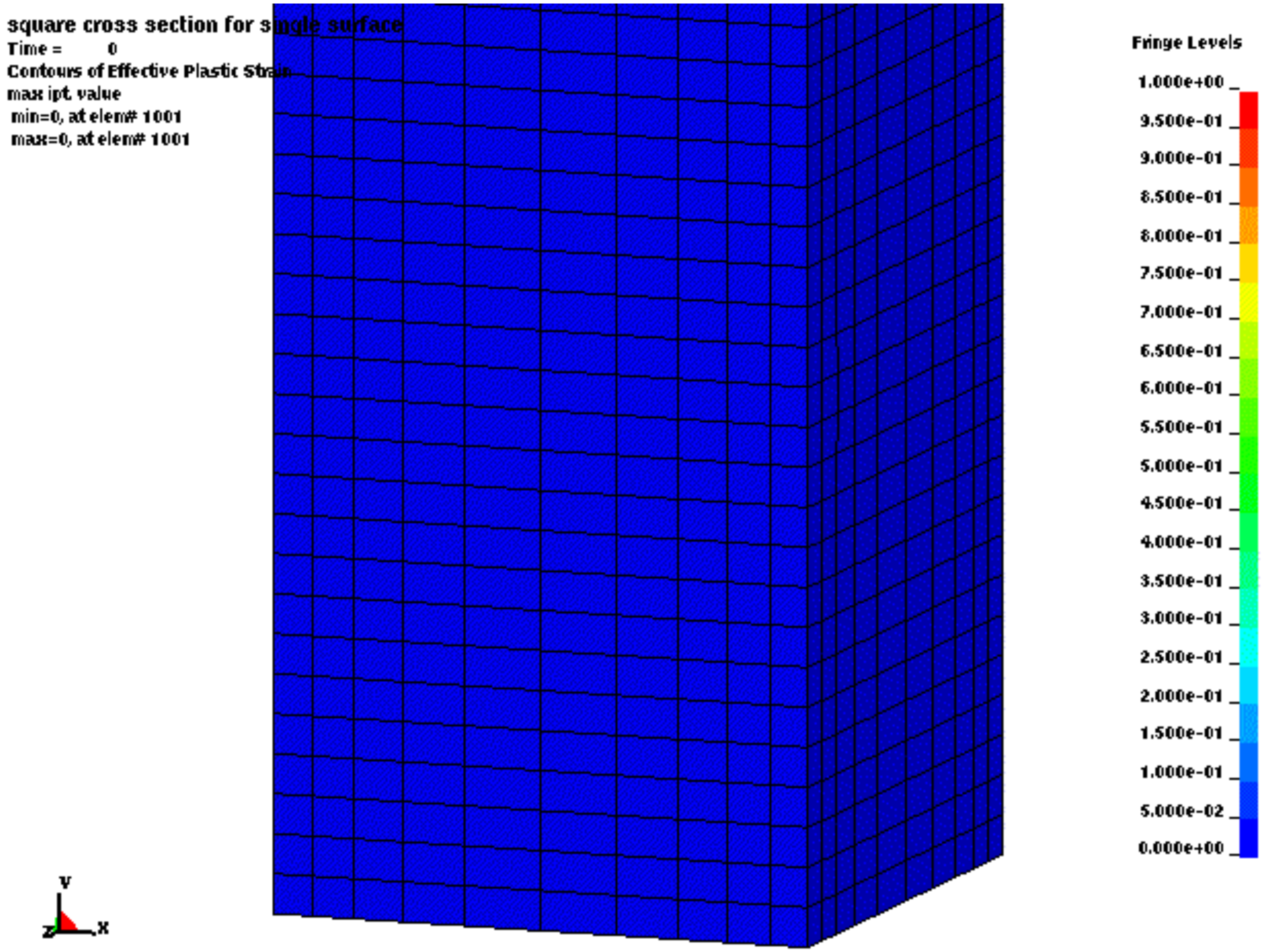
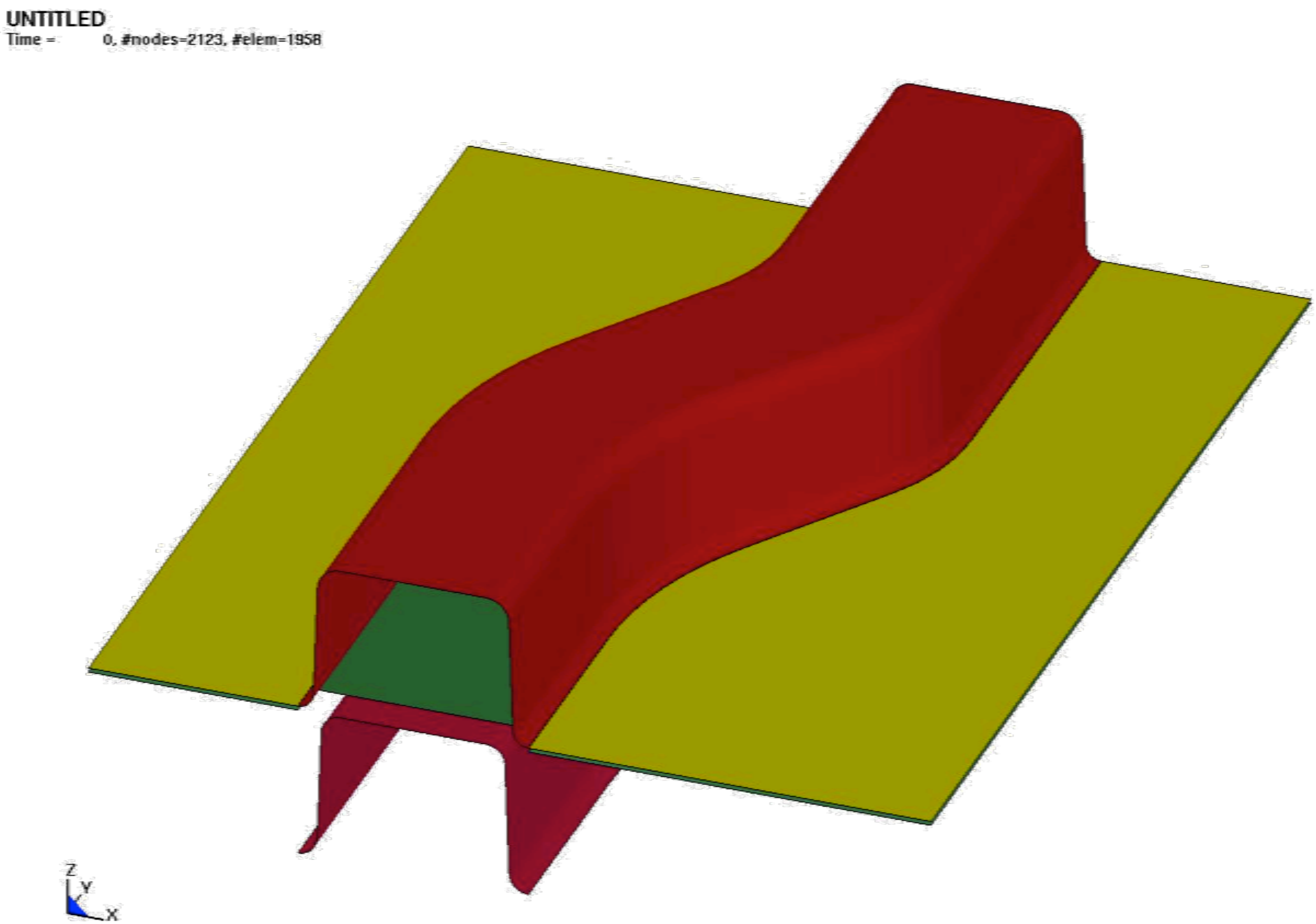
Quartic Square Tube Buckling



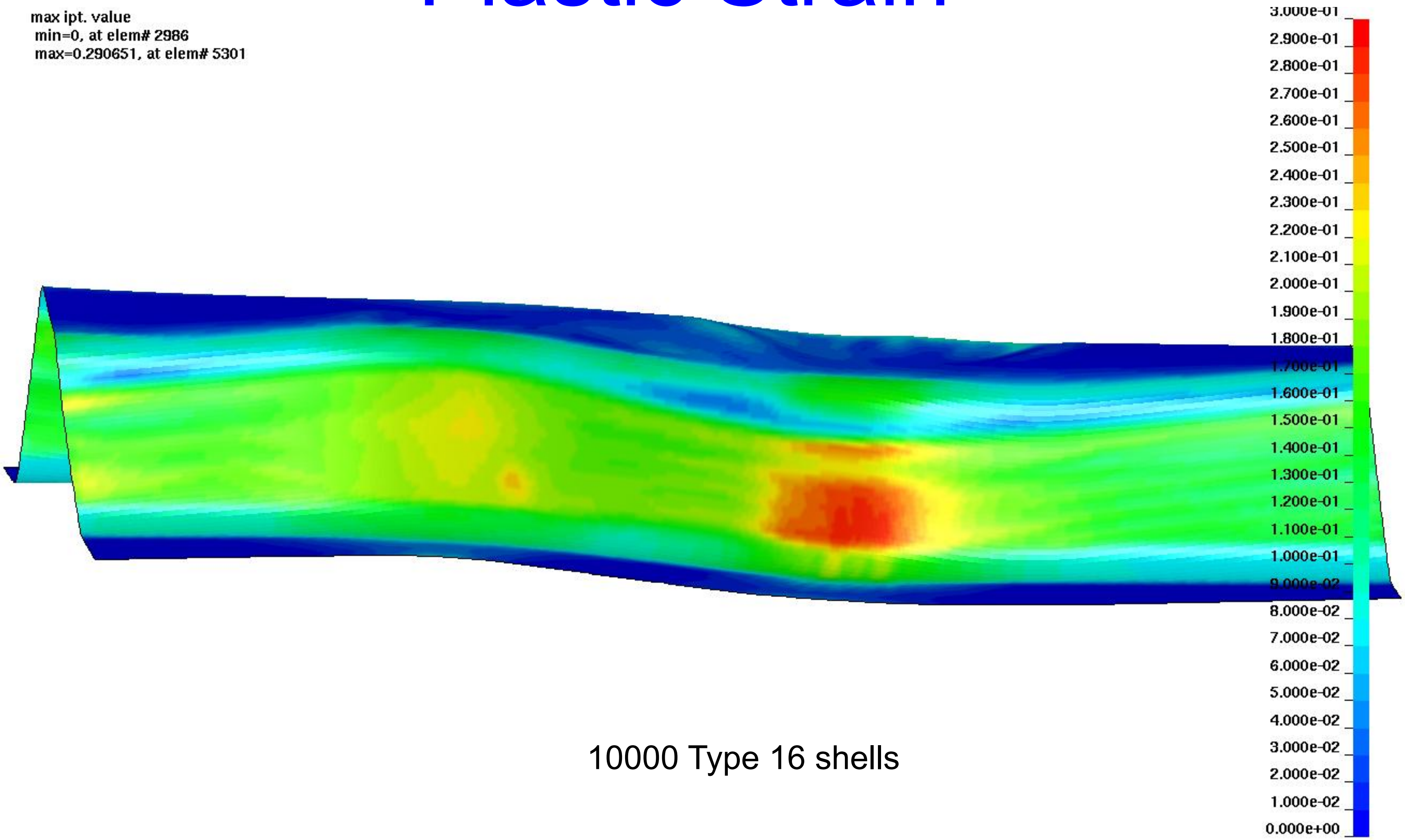
Metal Stamping

- NUMISHEET standard benchmark problem.
- Data:
  - Provided by R. Dick, Alcoa.
  - Benchmark solution uses 10^4 type 16 shells.
- No changes made to input except to replace the blank with isogeometric shell elements.

NUMISHEET Benchmark Problem

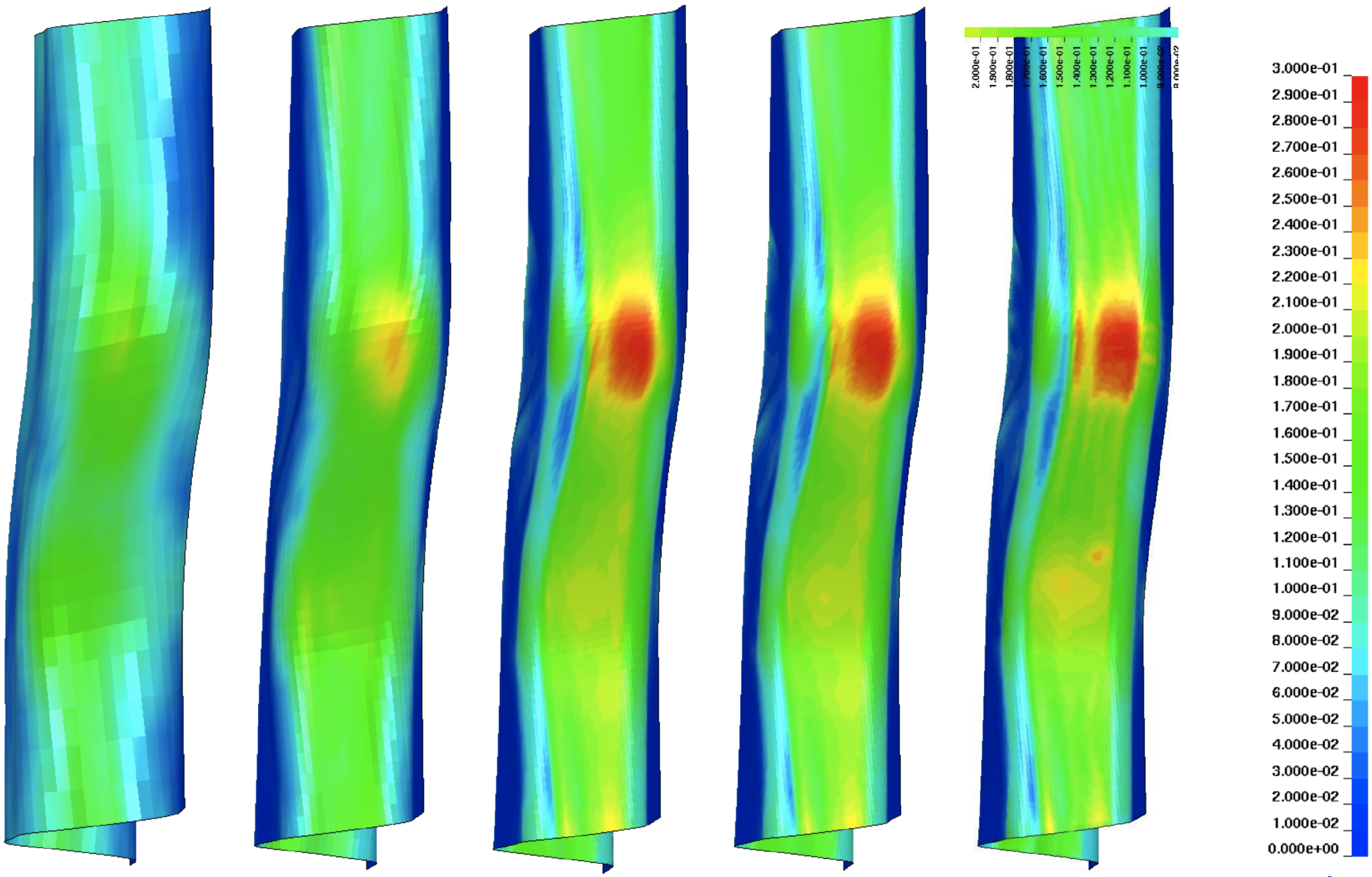


Alcoa Benchmark Solution: Plastic Strain



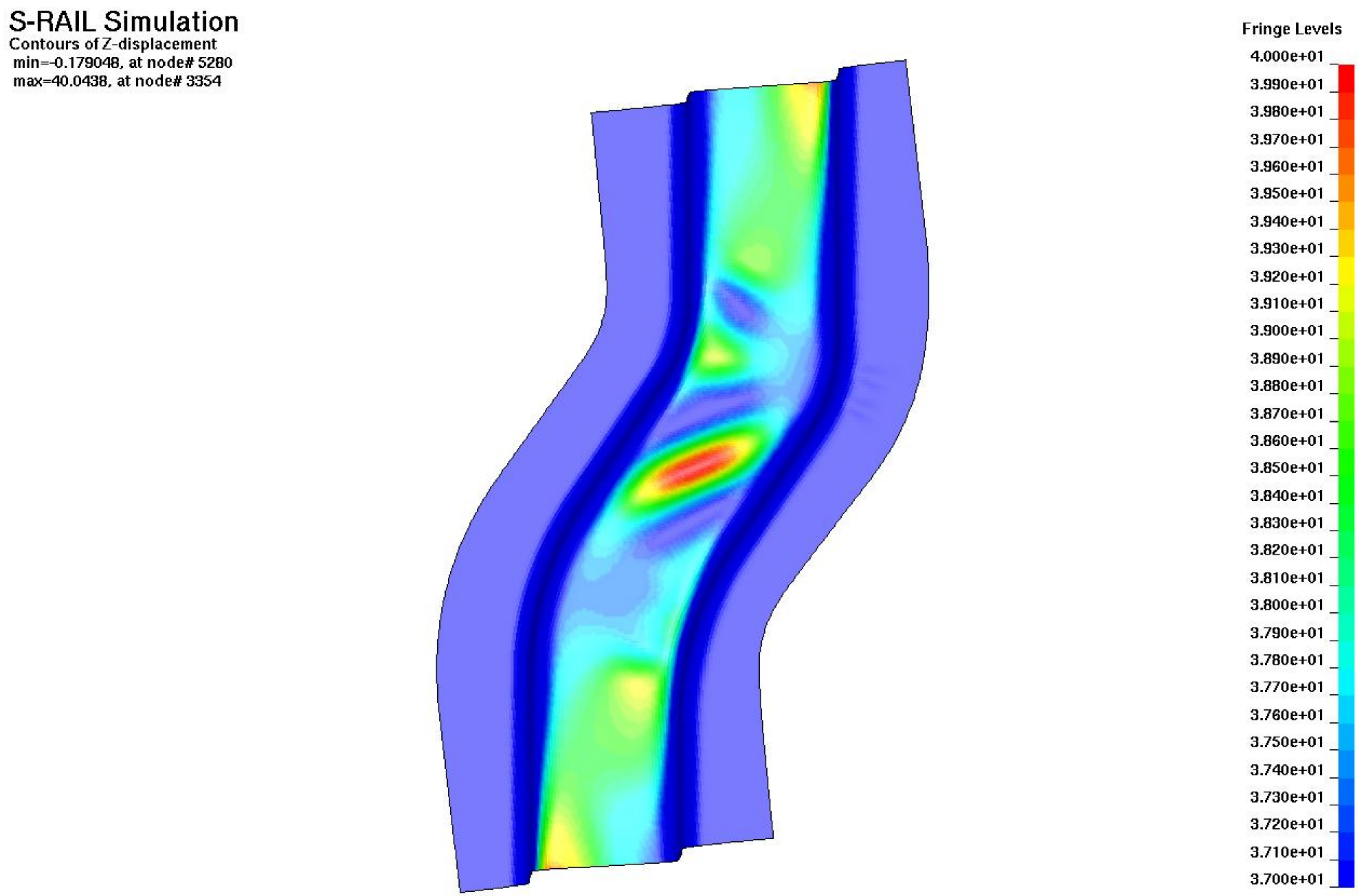
10000 Type 16 shells

Comparison of Rotation-Free Shell to Reference Solution



240 NURBS | 1092 NURBS | 3840 NURBS | 7680 NURBS | 10000 Bely.-Tsay

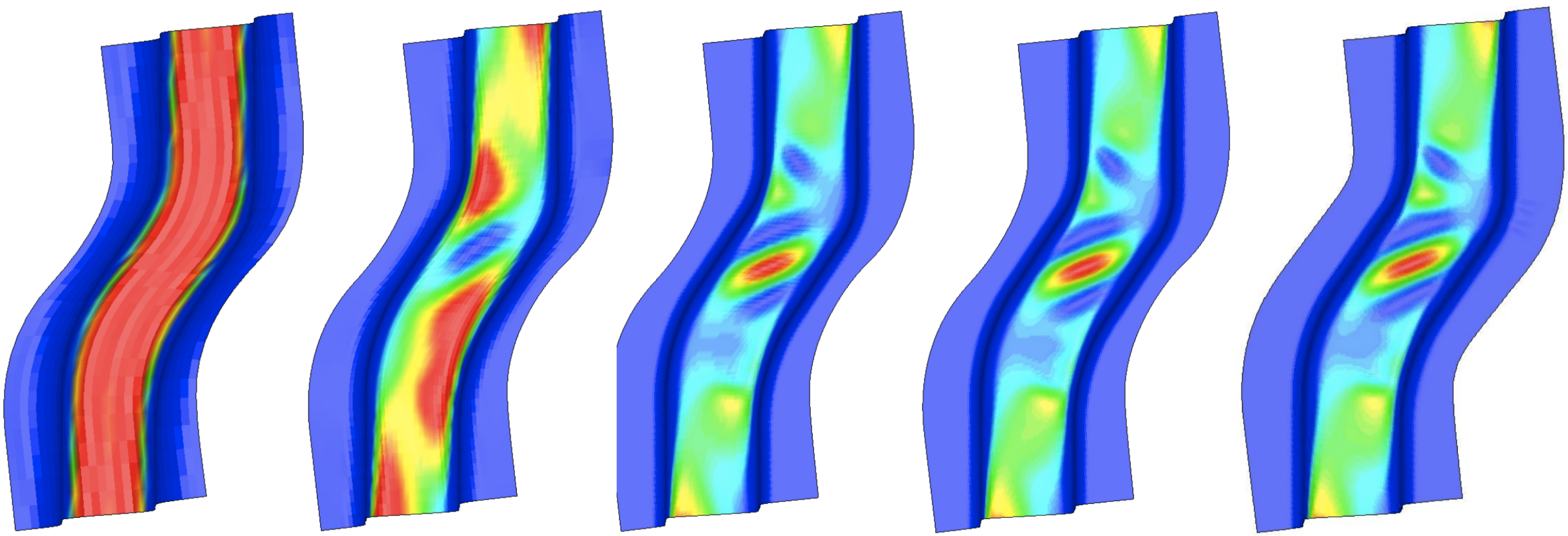
Alcoa Reference Solution: Z Disp.



10000 Type 16 shells

Isogeometric Solutions: Z Disp.

Rotation Free Shells



240 NURBS

1092 NURBS

3840 NURBS

7680 NURBS

10000 Bely.-Tsay

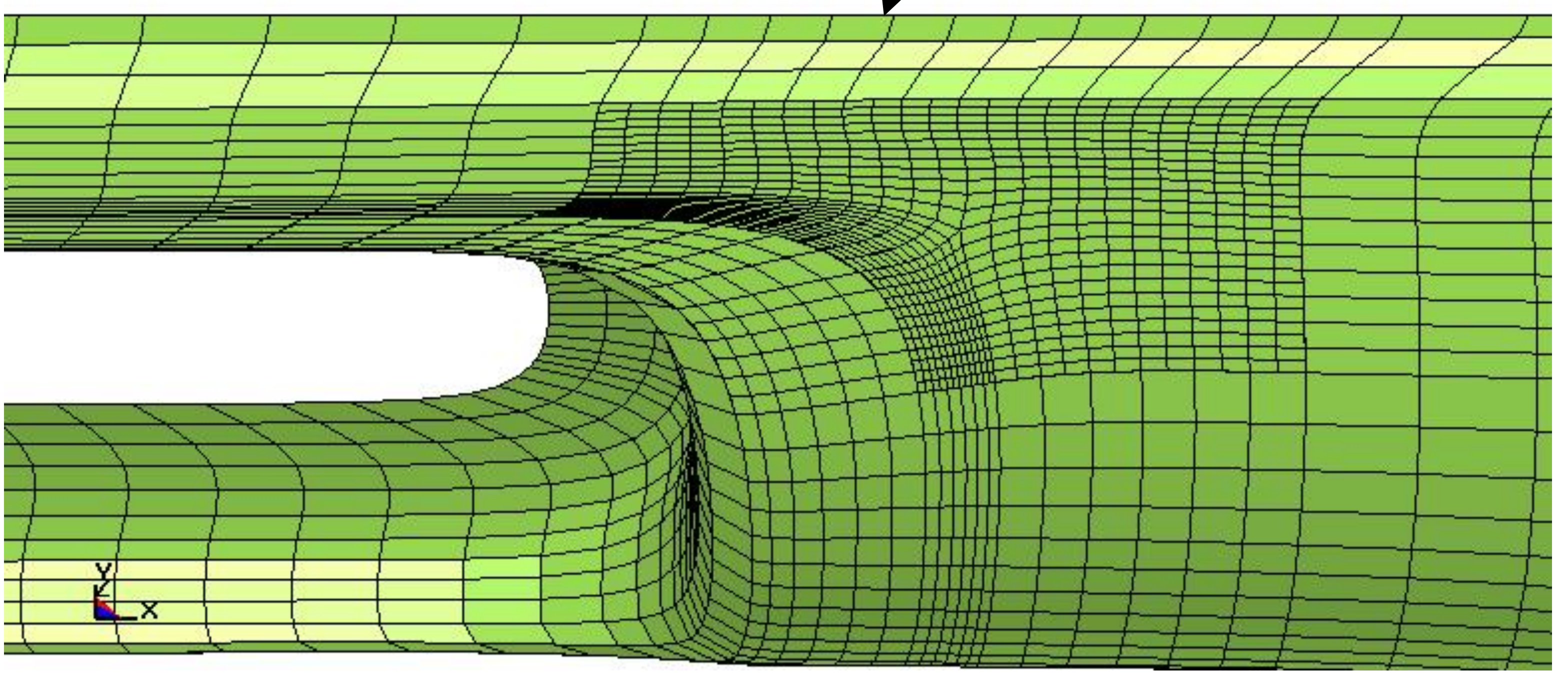
Wrinkling mode
is the right shape but
inverted in comparison
to others.

Design-to-Analysis With T-Splines

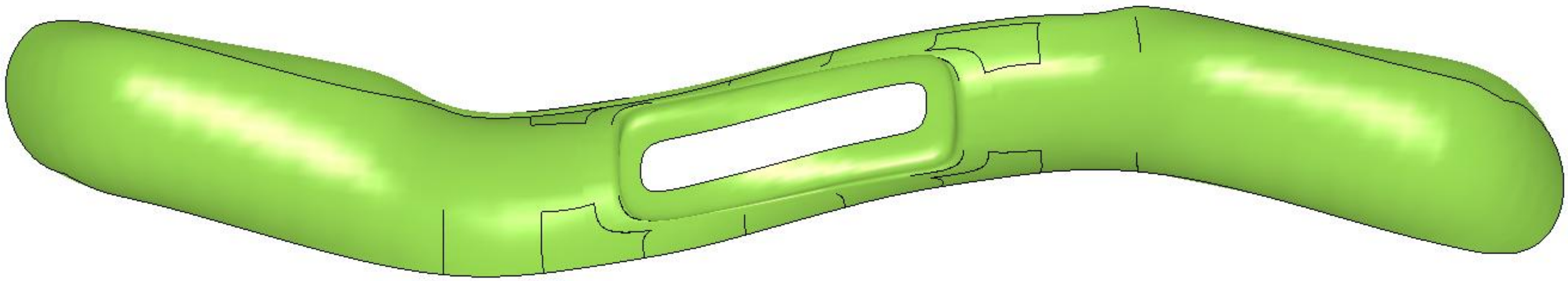
- Bumper modeled by Mike Scott with commerical T-Spline Inc. software.
- Eigenvalue analysis with generalized elements in commercial version of LS-DYNA.
- No constraints.
- Mesh data:
  - 876 generalized Reissner-Mindlin shell elements (cubic basis functions).
  - 705 control points.
- Material properties:
  - $E=10^7$.
  - Poisson's ratio=0.3.
  - Thickness=1.0.

Bumper Model with Unstructured Mesh of Cubic T-Spline Elements

Interpolation elements displayed.
Each generalized element depicted by 3x3 patch of interpolation elements.



Bumper: First Bending Mode



Summary

- Higher order accurate isogeometric analysis can be cost competitive even in explicit dynamics.
- Shell formulations without rotational DOF can be cost competitive to conventional formulations.
  - Cost competitive for explicit.
  - May be cost beneficial for implicit.
    - Fewer DOF.
    - Eliminate convergence problems with rotational DOF.
- Future implementations will only get faster.
- Accuracy is excellent.
- Robustness is excellent.