# A Study of Quasi-static Problem by SPH Method

Tatsuo Sakakibara, Dr.Eng.
Toru Tsuda, Dr.Eng.
Ryo Ohtagaki
*ITOCHU Techno-Solutions Corporation*
*4-1-3,Kyutaro-machi,Chuo-ku, Osaka, Japan*

## Abstract

The SPH method is well known for the hydrodynamics and recently SPH is applied for the impact and the penetration behavior into solid materials. SPH is completely mesh free so that it seems to be appropriate to simulate crash behaviors of concrete and rock materials. However, the applicability and the effects of the parameter of SPH for quasi-static problems are not clear completely.

In this paper, in order to verify the applicability and the accuracy of SPH on quasi-static problems, a number of unconfined uni-axial compression simulations are performed by SPH. The specimens of different number of elements are used and the parameter sensitivities of SPH are examined. The influences of formulation type of SPH in LS-DYNA® are also investigated. The results of SPH are compared with the Laglangian result.

From the results, the effects of the parameter for SPH are made clear. Besides, it is found that the renormalized formulation is efficient to get the accurate results at boundary. As the result, it is conformed that SPH is applicable for the quasi-static problems.

## Introduction

SPH is recently well-known particle method as Lagrangian mesh free and it is applicable to high velocity and large deformation analysis like impact and penetration behaviors. SPH has some advantages, one is that it is easy to built up simulation model even though very complicated geometry because of completely mesh less method. Another advantage is that SPH is not need to take into account any distortion so that SPH is applicable to analyze large deformation and fracture behavior. On the other hand, the accuracy of the results by SPH depends on the particle density or smoothing length. In specially, it is necessary to pay attention to decrease the accuracy at boundaries. In case of the problem like penetration behaviors, the affects by the boundaries are not so large. However, in case of quasi-static problems, the high accuracy in terms of stresses and strains are required. A large number of simulations on fluid dynamics and impact problems have been performed using SPH method so far, but the number of applications to the quasi-static problem and the case study for the investigation of accuracy by SPH is not many because of mainly cost efficiency compare to continuum method.
In this paper, uni-axial compression simulations to the hard sand stone by SPH were performed and investigated the applicability on the quasi-static problems by SPH. The results of SPH were compared with the experiments and FEM result. The version we used is LS-DYNA Ver.971.

## Analytical Condition

Figure 1 shows a cylindrical specimen with 10mm in diameter and 10mm in height. We used four type of the SPH models and a Lagrangian model as shown in Figure 2. FEM model is used to compare with SPH results. Each SPH model has different particle density. SPH-10 means that the 10 particles line up in the direction of the diameter. Therefore, 40 particles line up in SPH-40. The number of particles and the distance between particles of simulation models are summarized in Table 1. Those of FEM in Table 1 mean the number of elements and minimum element size respectively. The constant loading velocity is applied to the particles on the top surface. As boundary condition, the particle displacements to the Z-direction on the bottom surface are fixed. The material used in this study is assumed to be elasto-plastic as shown Figure 3 defined by MAT 3 in LS-DYNA. Material properties are shown in Table 2.

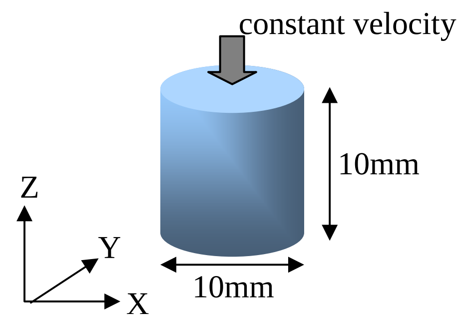



**Figure 1 Specimen**

**Table 1 Summary of Models**

| Model Name | number of particles | distance between particles |
|---|---|---|
| SPH-10 | 880 | 1.0 mm |
| SPH-20 | 6636 | 0.5 mm |
| SPH-30 | 21480 | 0.33 mm |
| SPH-40 | 50560 | 0.25 mm |
| FEM | 960(1) | 1.0 mm(2) |

(1) number of elements (2) minimum element size

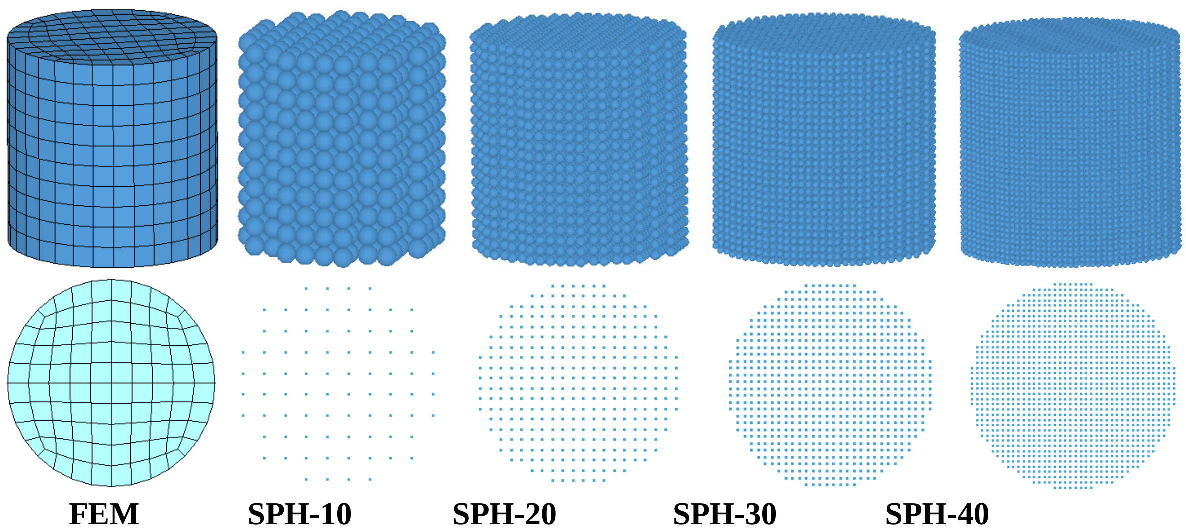



**Figure 2 SPH particle distributions and FEM model**

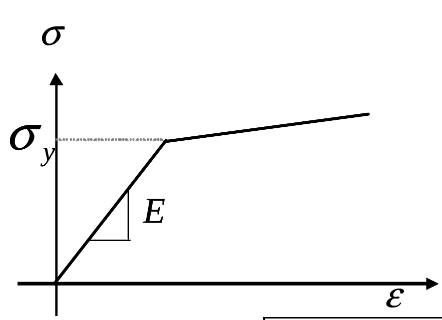



**Table 2 Material properties**

| | | |
|---|---|---|
| Elastic modulus | MPa | 82 |
| poisson's ratio | | 0.32 |
| Density | g/cm3 | 1.5 |

| | | |
|---|---|---|
| compressive strength | MPa | 4.3 |

**Figure 3 Stress-Strain Correlation**

## Analytical Results

### *Effect of loading velocity*

During the uni-axial compression test, the loading rate must be slow enough to ensure that the specimen remains in quasi-static equilibrium throughout the test. In terms of the calculation time, the loading velocity is required faster. So a number of tests with several loading speeds from 0.1 mm/sec to 1000 mm/sec were examined. Figure 4 shows the relationship between force and displacement at the top surface. The results with the speed from 0.1 mm/sec to 100 mm/sec are identical completely. In case of the loading speed of 1000 mm/sec, excessive energy is going into the kinetic energy. Hereafter the loading velocity of 10 mm/sec was used in all subsequent tests.

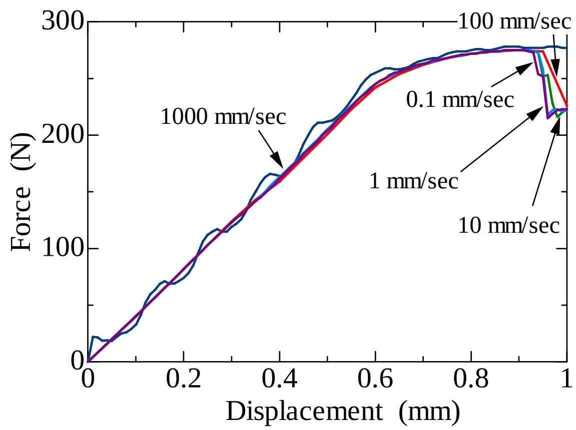



**Figure 4 Effect of loading velocity**

### *Effect of particle density*

Particle density of SPH method is very important factor. Because, an accuracy of the result depends on number of particles which are used in a unit volume. To clarify the effect of particle density, four models shown as Figure 2 were examined. SPH-10 is the coarsest and SPH-40 is the finest. Figure 5 shows the relationship between force and displacement at the top surface. The results of SPH-30 and SPH-40 are in good agreement with the result of FEM. On the other hand, the model of SPH-10 and SPH-20 seems to be less stiff than the others. From this result, at least, number of particles as many as SPH-30 on horizontal surface is necessary to get the accurate result.

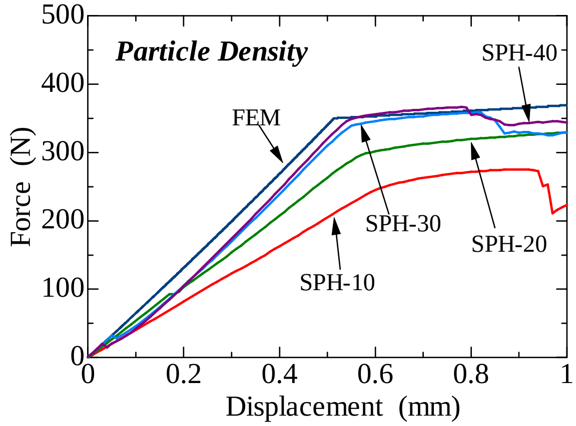
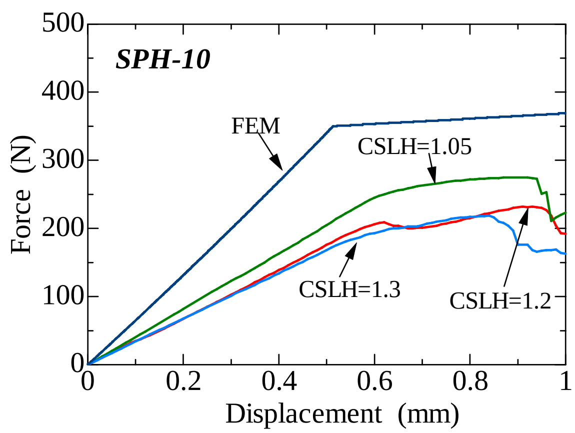


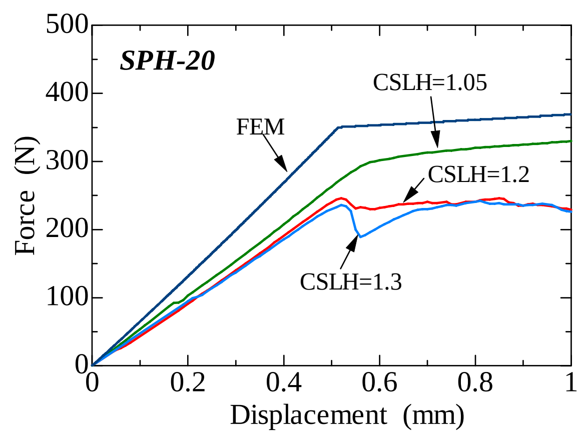

**Figure 5 Effect of particle density**

***Effect of Smoothing length***

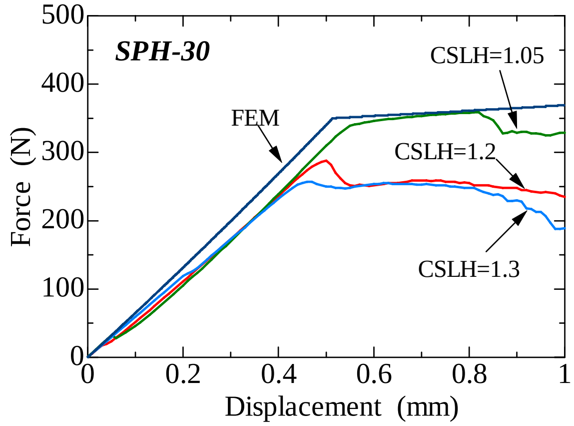
Smoothing length is a one of specific parameter of SPH. In LS-DYNA, the smoothing length can be varied by CSLH keyword in *SECTION_SPH card. CSLH is the constant applied to the smoothing length of the particles. According to the LS-DYNA manual, the default value 1.2 of CSLH is recommended and values between 1.05 and 1.3 are acceptable. In order to make sure this recommendation is useful even thought the quasi-static problem, three case of the smoothing length are applied to each SPH model shown as Figure 2. Figures 6 show the correlation between force and displacement on the top surface with the different smoothing length. The case of CSLH 1.05 is the most accurate in all models. Even if the particle density is very high, in case of smoothing length larger than 1.2 makes model weaker. That means a yield stress decrease than the value specified as input parameter. From these results, the smoothing length should be 1.05 on the quasi-static problem.



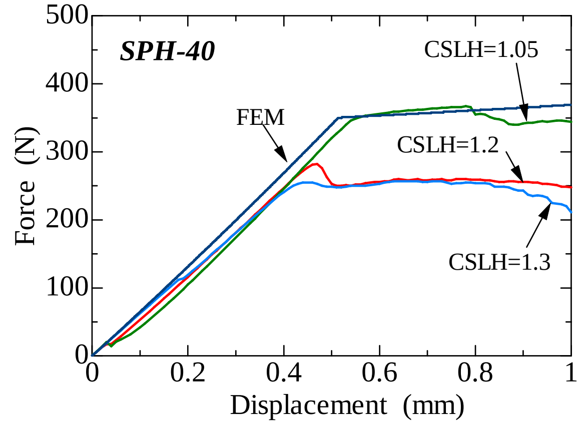

(a) SPH-10




(b) SPH-20




(c) SPH-30

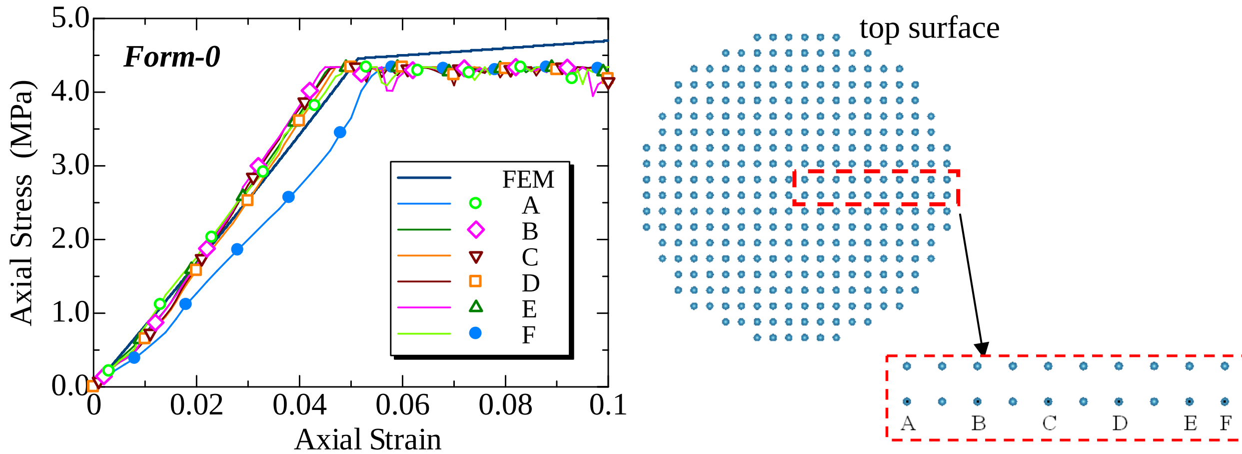



(d) SPH-40

**Figures 6 Effect of Smoothing length**

### *Effect of Renormalization*

As mentioned before, Calculation accuracy declines in specially when particle density is low. An influence of boundary elements is thought as one reason. The number of particles in the influence domain at boundary particles is smaller than that at inside particles. Figure 7(a) shows a relationship between axial stress and axial strain at selected elements on the top surface as shown Figure 7(b). The element F is located at boundary and stress of this element is lower than the other elements. This graph shows that particle on the boundary edge has inaccurate stress. To overcome this inaccuracy, the renormalized formulation technique is implemented in LS-DYNA. There are seven particle approximation theories which are specified by FORM keyword in *CONTROL_SPH card. In this study, the results of the renormalized formulation Form-1 are compared with the results obtained from default formulation Form-0. Figure 8 shows the comparison between the two formulations using SPH-20 model. The result of renormalized formulation is good agreement with FEM result. Therefore, the renormalized formulation is very effective on quasi-static problems.




(a) Stress Strain correlation (b) Selected elements

**Figure 7 Stress Strain correlation at SPH elements**

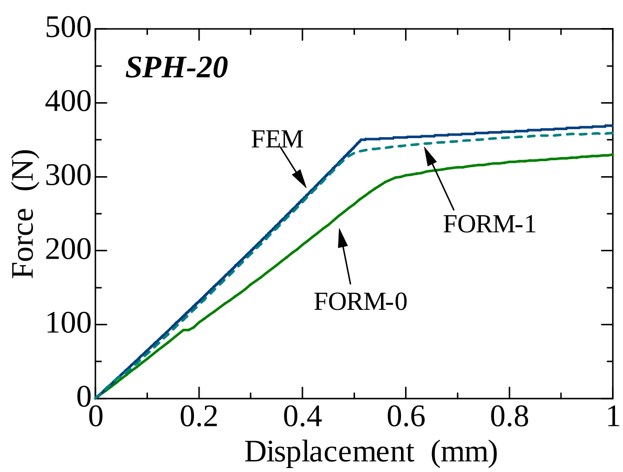



**Figure 8 Comparison between the two formulations**

***Comparison of computation time***

Figure 9 shows comparison of computation time between SPH and FEM. Abscissa axis means the number of particles or elements and vertical axis means computing time per one cycle. Computation time of SPH takes approximately three times as much as FEM.

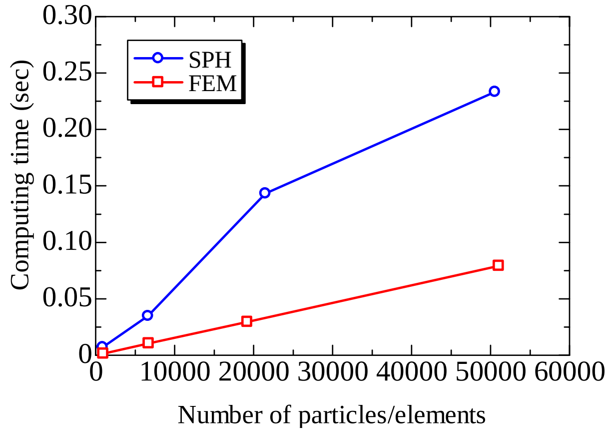



**Figure 9 Computation time per one cycle**

## Conclusions

In this study, the unconfined uni-axial compression tests were simulated by SPH to verify the the applicability and the accuracy on quasi-static problems. The conclusions obtained from the simulation results are summarized as follows.

The SPH method can be applied to quasi-static problems by an appropriate modeling.

1. Number of particles
   - Almost 30 x 30 particles on horizontal surface are necessary to get the accuracy compare to the FEM result.
   - The computation time increases when the particle increases.

2. Smoothing length
   - The case of CSLH 1.05 seems to be the most accurate.

3. Accuracy of boundaries
   - Less number of particles at boundary causes inaccurasy.
   - Renormalized formulation improves accuracy at boundary

### References

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