# Simple Input Concrete Constitutive Models: An Illustration of Brick Wall & Concrete Cylinder Perforation

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## Abstract

*Analysts faced with making predictions that involve uncharacterized materials need to bound their results in a rational manner. For concrete materials, LS-DYNA® includes three simple input concrete material models that provide strength envelopes ranging from low, to intermediate, and high strength. Using a range of concrete strengths in numerical simulations is rational when little or no other information about the concrete is available.*

*The typical system response quantity of interest (SRQ) in perforation studies is the exit velocity of the projectile. The simple input concrete models provide a range of concrete resistance to penetration that in turn produces a corresponding range of projectile exit velocities. Quantifying this possible range of exit velocities provides the decision maker with an assessment of what to expect in the field or laboratory for uncharacterized concrete targets.*

*The use of multiple simple input concrete models to provide a range of exit velocities is demonstrated for a simple model of a brick wall, where no experimental data was available. To provide the reader with some indication how this approach can be used to compare predictions with laboratory data, the experimental and numerical results for a set of concrete cylinder perforations is provided.*

## Introduction

The increased emphasis on urban combat has created a need to numerically assess the effectiveness of existing and new weapons against common urban structures. These are existing structures constructed of unknown or uncharacterized brick masonry and concrete. The analyst is faced with selecting from a wide variety of appropriate constitutive models, available in most general purpose commercial and proprietary analysis software, and the more daunting task of determining a large number of constitutive model inputs with little or no guidance.

The selection of a consistent set of concrete constitutive model parameters for an unknown concrete is difficult-to-impossible, even for concrete modeling experts. The difficulty lies in the large number of model parameters, often exceeding a dozen, and the complex relations among parameters. As a partial answer to this problem, a recent trend in concrete modeling is to incorporate *simple input* options for sophisticated concrete models. The required simple input typically consists of primarily the concrete unconfined compressive strength. While these simple input concrete models offer the analyst a consistent set of model input parameters, there is no guidance as to how well they represent the specific concrete for the application of interest.

An obvious approach to quantifying the uncertainty in the structural response, due to the unknown concrete properties, is to vary the unconfined compressive strength of the concrete using simple input models. Another approach is to consider more than one simple input concrete model, using the same concrete strength. The most comprehensive approach to quantifying the uncertainty in the response is to vary both the concrete strength and the simple input constitutive model.

In the present work, three simple input concrete models, available in LS-DYNA, are used in modeling the perforation of a brick masonry wall of known unconfined compressive strength. Although no experimental data are available for model assessment and evaluation, the comparison of the three model results illustrates a large difference in projectile residual velocities and the importance of performing more than one calculation when the material (concrete) is uncharacterized.

## Model Geometry and Discretization

This section describes the geometry, and its discretization, used to model the target wall and projectile.

### Brick Wall Target

The target wall consists of two components included in the model: the bricks and associated mortar joints. The specified overall dimensions of the brick wall are 72 inches square with one layer of brick through the thickness. The width of the modeled wall is 1954 mm (76.9 inches) and includes 9.5 bricks in each course (layer), as shown in Figure 1. The height of the modeled wall is 1200 mm (47.2 inches) but only the upper half of the wall height is modeled as a horizontal plane of symmetry is assumed.

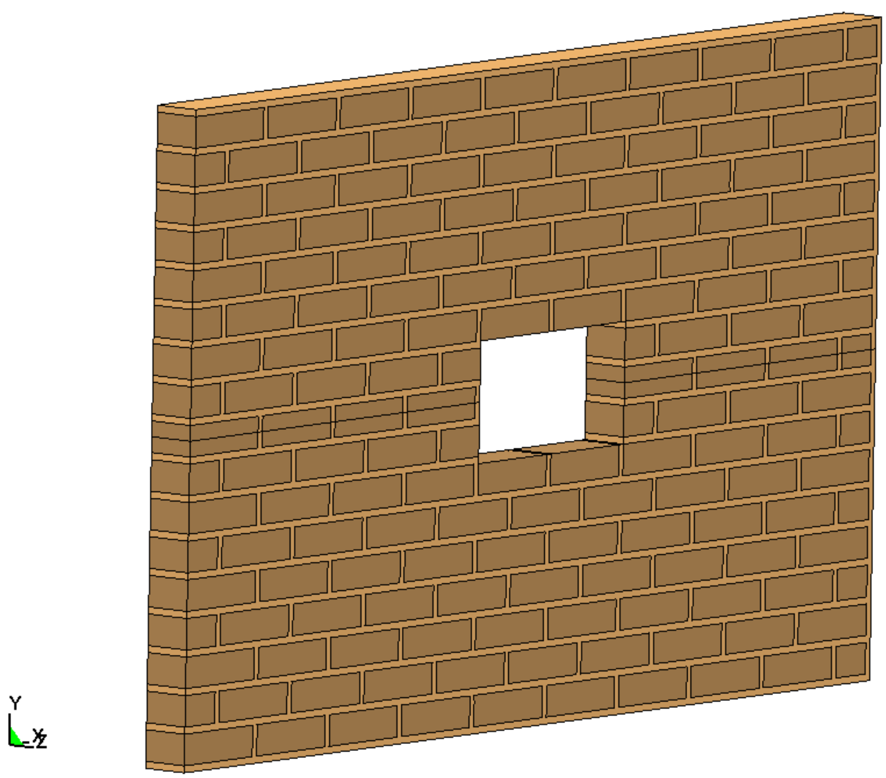


**Figure 1 Model brick wall with horizontal symmetry plane and removed bricks in the impact region.**

Bricks & Mortar

The nominal brick dimensions are specified as 3.5 x 2.25 x 7.625 inches and these are the brick dimensions used in the model. The mortar joint thickness was not specified, so a value of 0.5 inches was used, based upon measurements of a few brick walls. Each brick, and associated mortar joint, were discretized using the 0.5 inch mortar thickness as the baseline dimension, i.e. one solid element through the thickness of the mortar joint. The resulting brick discretization is 8 solid elements through the thickness of the brick, 4 solid elements along the brick height, and 16 solid elements along the brick length. The brick and mortar discretization is shown in Figure 2.

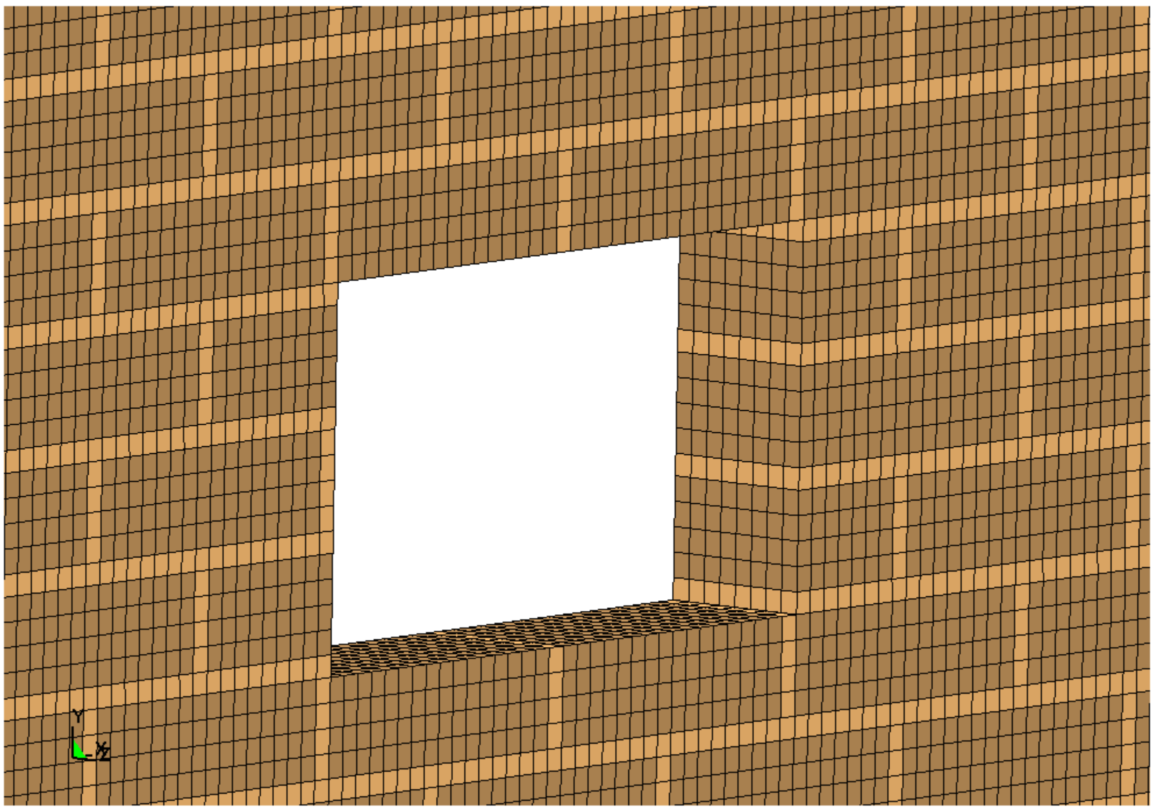

**Figure 2 Close up view of brick wall discretization in the vicinity of the impact region.**

Impact Region

The projectile impact region of the brick wall, see Figure 3, was modeled using the Lagrangian discretization known as Smooth Particle Hydrodynamics (SPH); one of a large class of so called meshfree methods. The SPH particles can be thought of as a node in the traditional node-and-element Lagrange discretization formulation, i.e. finite element method. All of the state variables, i.e. mass and motion, are calculated at the particles; the same as in the finite element formulation. Additionally, the strain and stress are also computed at the particles, which is different from the finite element method where strain and stress are computed at the Gauss integration points interior to the elements. The key difference between SPH particles and standard finite elements is in the displacement interpolation to determine the strain. In traditional finite elements the displacement at an element's associated nodes, typically 8 for a solid element, are interpolated to determine the element's strain. For SPH particles the notion of elements does not exist, so the strain is interpolated based upon the displacement of the particles in the vicinity (domain of influence) of each particle. This approach allows for very large distortion of SPH particles, and permits particles to move apart and thus no longer interact, i.e. outside the domain of influence. When neighboring particles no longer interact, the effect is the same as introducing a separation (crack/void) into the continuum approximation. Thus SPH particles have the ability to initially represent a continuum and evolve to a discontinuum as dictated by the physics of the problem. This is in contrast to a traditional finite element representation where crack/void are usually introduced via element erosion (removal) using *ad hoc* criteria.

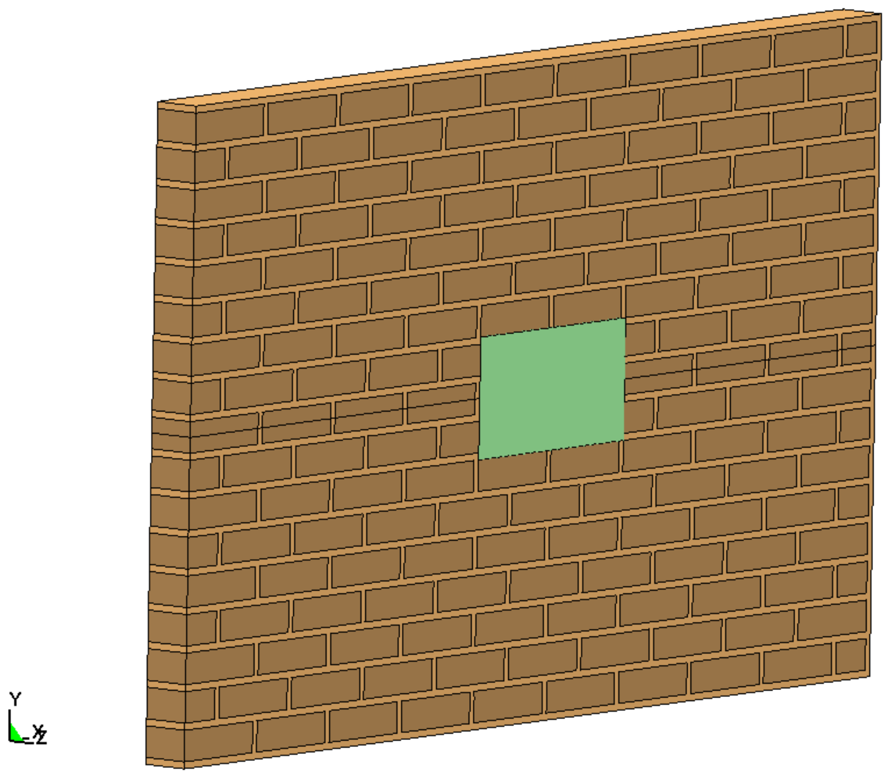


**Figure 3 Brick wall model including SPH particle continuum representation at the impact region.**

Just as with traditional finite element methods, the SPH discretization of the continuum affects the strain interpolation, and thus the representation of the stress in the material. A general rule-of-thumb when combining SPH particles and traditional Lagrange solid elements in the same simulations is to use at least 2 SPH particles along each face of a solid element. Since the brick and mortar solid elements have a nominal mesh spacing of 0.5 inches (12.7 mm), an initial SPH particle spacing of 7 mm, uniform in all three dimension, was used in the impact region of the wall. To assess the adequacy of this SPH particle spacing, some preliminary calculations were performed where the particle spacing was further reduced to 4 mm. Although no significant response difference was noted for the projectile, i.e. the exit velocity after perforating the target, the engineering decision was made to use the 4 mm particle spacing in future simulations to be within the cited rule-of-thumb. The SPH particle discretization is illustrated in Figure 4.

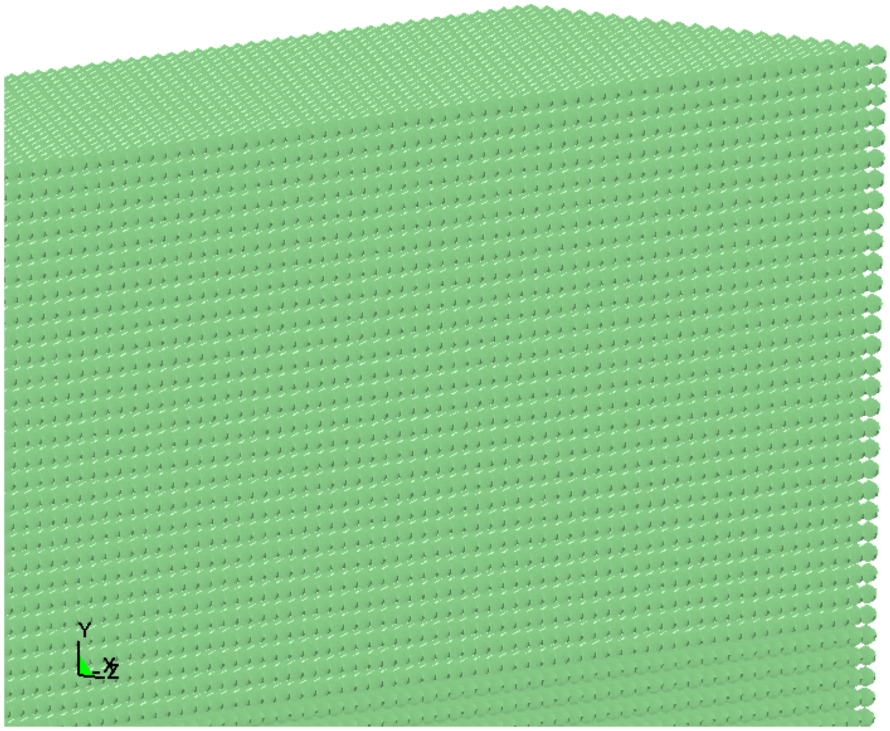


**Figure 4 Close up view of one corner of the SPH particles (4 mm spacing) used to model the impact region of the brick wall.**

## Projectile

Figure 5 shows the Lagrange solid element discretization of the simplified projectile. The projectile has a length of 20 inches (508 mm), a diameter of 2.6 inches (66 mm) and a weight of 2.2 lbf (1 kg mass).

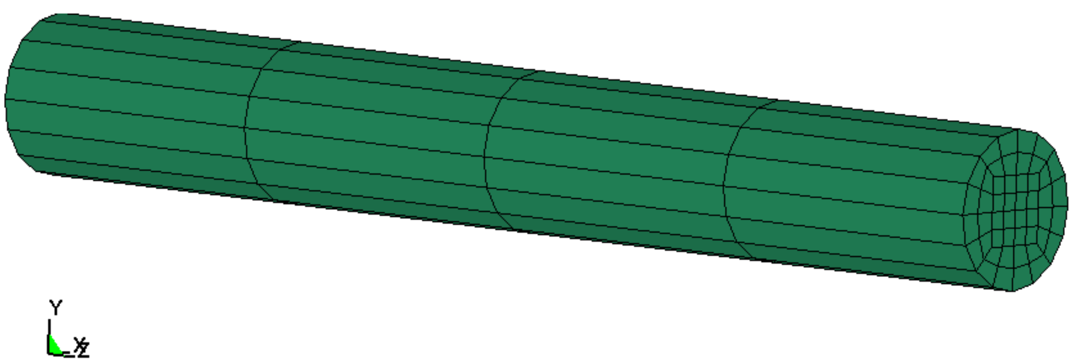


**Figure 5 Discretized cylindrical projectile model.**

## Material Modeling

This section describes the material models and parameters used to describe the constitutive response of the brick wall target and simplified projectile. Reference is made to material models described in more detail in the current LS-DYNA User's Manual (Version 971 May 2007).

### Brick Wall Target

The target wall consists of several courses (layers) of common construction bricks with associated mortar grouting between the bricks.

<u>Bricks</u>

Figure 6 shows the axial stress-strain response from an unconfined compression test for a typical brick. The only possible useful piece of information to be gleaned from this data is the unconfined compressive strength of the brick, approximately 4500 psi (31 MPa). Note: although Young's Modulus could be estimated from the slope of the stress-strain data, the strain at failure, i.e. 1.5%, is an order of magnitude larger than expected; it is possible the measured strain was incorrectly reported.

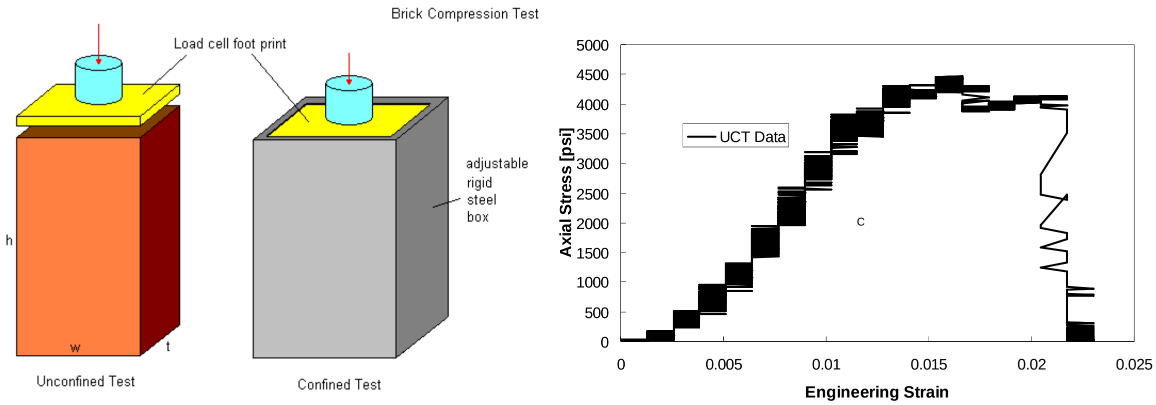


**Figure 6 Measurement of unconfined axial stress versus strain for a common house brick.**

The recommendation is to use the measured unconfined compression strength in conjunction with three LS-DYNA concrete models with a simple input option: *MAT_PSEDUO_TENSOR (MAT016), *MAT_CONCRETE_DAMAGE_REL3 (MAT072R3), and *MAT_CSCM_CONCRETE (MAT159). These material models use the unconfined compression strength to generate a consistent set of model input parameters that completely describe the shear failure and pressure-volume strain response for the equivalent strength *concrete*. The assumption is made that a laboratory characterization of the brick material would be similar to that of concrete.

The author advocates the practice of using all three simple input models to simulate any concrete structure when little (unconfined strength) or no information about the concrete's material characterization is available. While none of the results from these three model simulations may be correct, a comparison of the key system response quantities (SRQ) of interest will indicate if the concrete structure is sensitive to the material model, i.e. if the results are similar, then the SQR is insensitive to the concrete properties, and vice versa.

Although all three simple input concrete models use the same unconfined compressive strength, the resulting shear failure surfaces are quite different, see Figure 7. Each constitutive model uses a different internal algorithm to generate a full set of model parameters based only on the unconfined compressive strength, and thus the three different shear failure surfaces.

The three shear failure surface shown in Figure 7 converge at 31 MPa (von Mises stress) in the lower left corner of Figure 7, but it is apparent at larger pressures the three concrete models have quite difference shear strengths. For example, at a confining pressure of 100 MPa, a reasonable confinement level in penetration simulations, the shear strengths are 80, 150, and 180 MPa for the three models, a factor of 2.25 between the low strength MAT016 and high strength MAT159.

Also, The wide variation in strength envelopes illustrated in Figure 7 may appear as physically unreasonable. However, laboratory material characterization of concrete, with similar unconfined compressive strengths, demonstrate an even wider range of strength envelopes and associated pressure volume strain characterizations. Thus it is quite possible to have two concrete targets with the same unconfined compressive strength produce different experimental results if their manufacturing process were different, i.e. the concrete 'recipes' were different.

The pressure-volume strain responses provided by the three models also differ widely, see Figure 8. Again, laboratory pressure-volume strain data show an even wider range of responses for concretes with similar, or identical, unconfined compressive strengths.

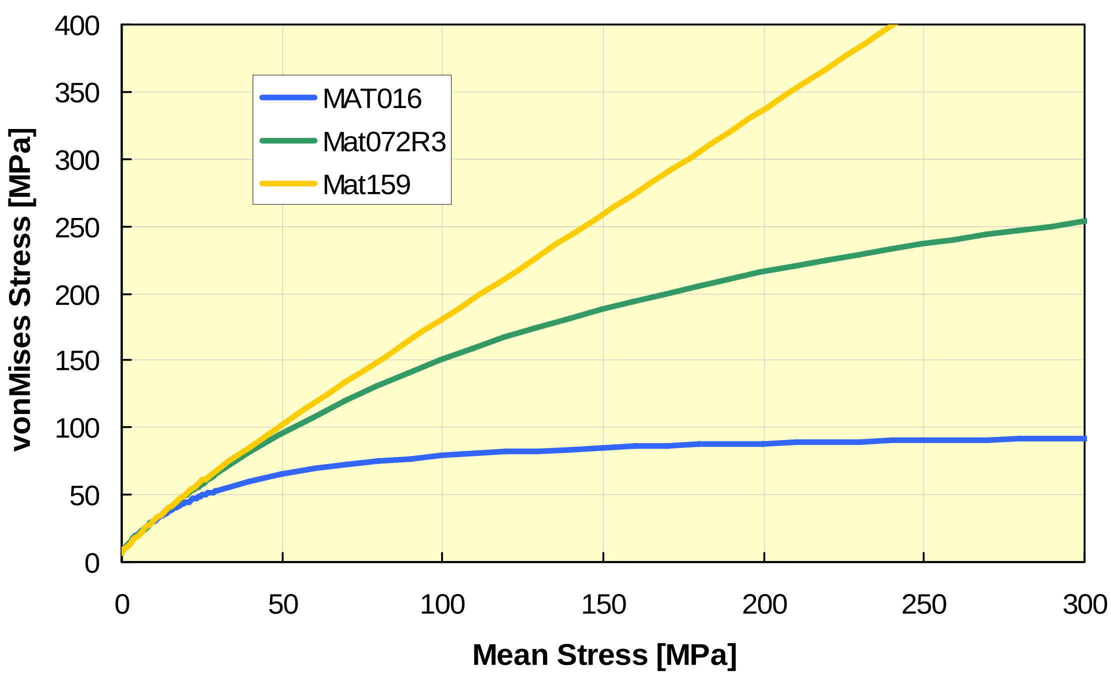


**Figure 7 Shear failure surfaces for three simple input concrete models with unconfined compressive strength of 31 MPa.**

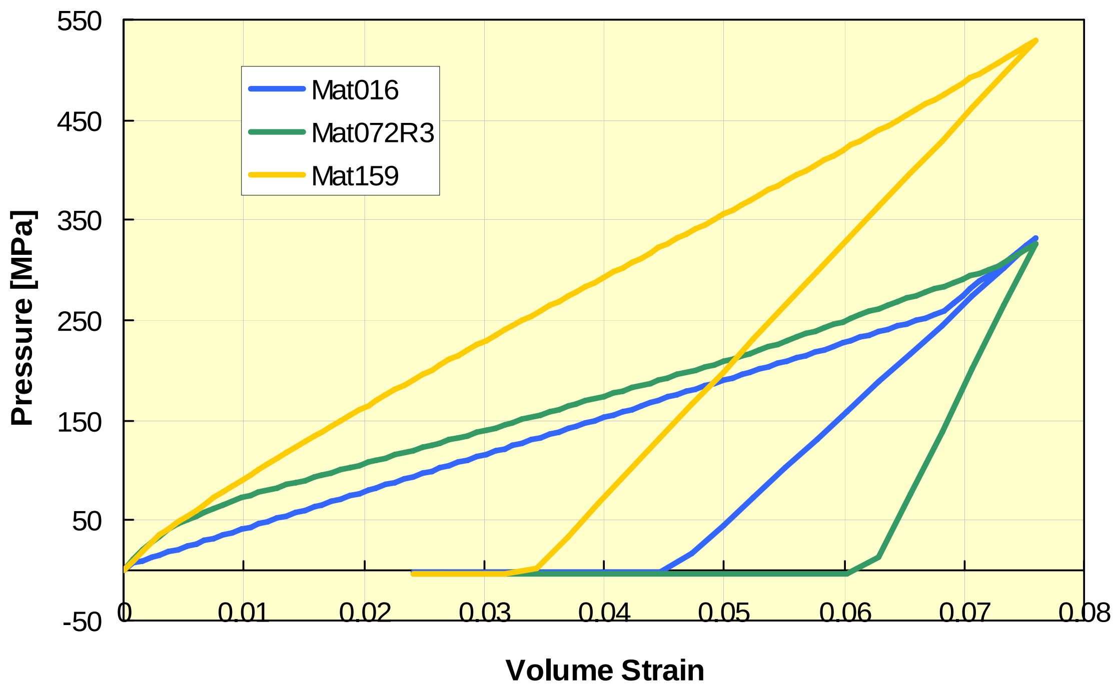


**Figure 8 Pressure-volume strain for three simple input concrete models with unconfined compressive strength of 31 MPa.**

<u>Mortar</u>

No material characterization data for the mortar was available. A brief search of the internet for typical mortar material characterization revealed one source[1] which cited:

> "Because Portland-cement-lime mortar is normally stronger than the brick, brick masonry laid with this mortar is stronger than an individual brick unit."

Based on the above statement, and lack of material characterization data, the assumption was made that the mortar would be modeled with the same constitutive model and parameters as the bricks. Other assumptions could have been made regarding the mortar strength and bonding to the bricks, but without data, or experience, to support these assumptions, it is more pragmatic to not make these baseless assumptions.

**Projectile**

The projectile was modeled as rigid *MAT_RIGID (MAT020).

[1] http://www.tpub.com/content/construction/14043/css/14043_232.htm

## Penetration Results

The typical system response quantity of interest (SRQ) in perforation studies is the exit velocity of the projectile. The simple input concrete models provide a range of concrete resistance to penetration that in turn produce a corresponding range of projectile exit velocities. Quantifying this *possible* range of exit velocities provides the decision maker with a better assessment of what to expect in the field or laboratory for uncharacterized concrete target.

The use of multiple simple input concrete models to provide a range of exit velocities is demonstrated for the above described brick wall, where no experimental data was available. To provide the reader with at least some indication how this approach can be used to compare predictions with laboratory data, the experimental and numerical results for a set of concrete cylinder perforations is provided.

### Brick Wall

In the present work the purpose of the analyses is to determine if the projectile perforates the target wall, and if so what is the predicted residual (exit) velocity. The projectile's initial velocity is prescribed to be 440 feet per second (134 meters/second) and impacts the center of the brick wall in two orientations: normal to the wall and at a 30 degree obliquity; see Figure 9.

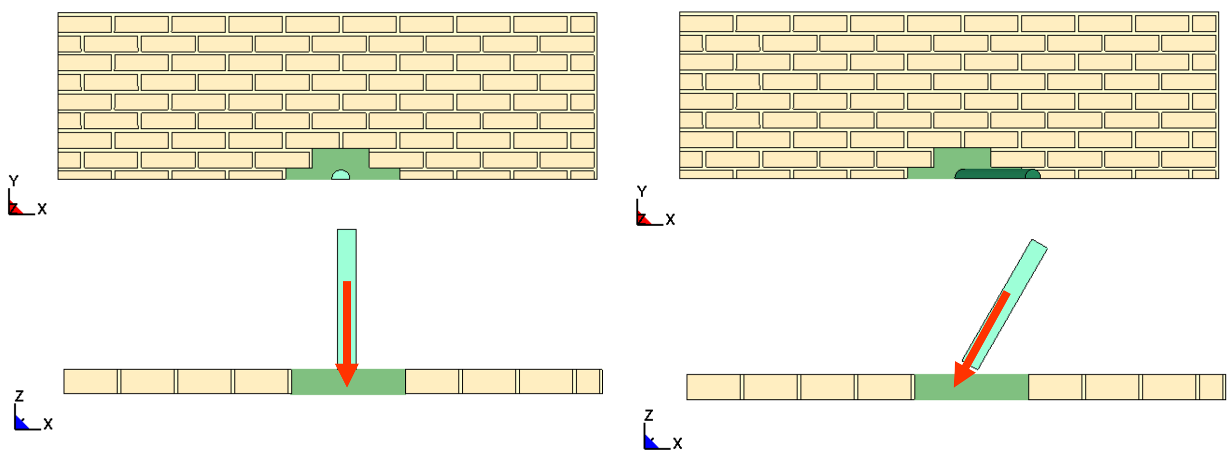


**Figure 9 Two impact configurations showing front (top row) and cutaway edge view (bottom row): normal to the target (left column) and 30 degree obliquity (right column).**

**Figure 10** illustrates the deformed configuration of the brick wall for the two impact configurations.

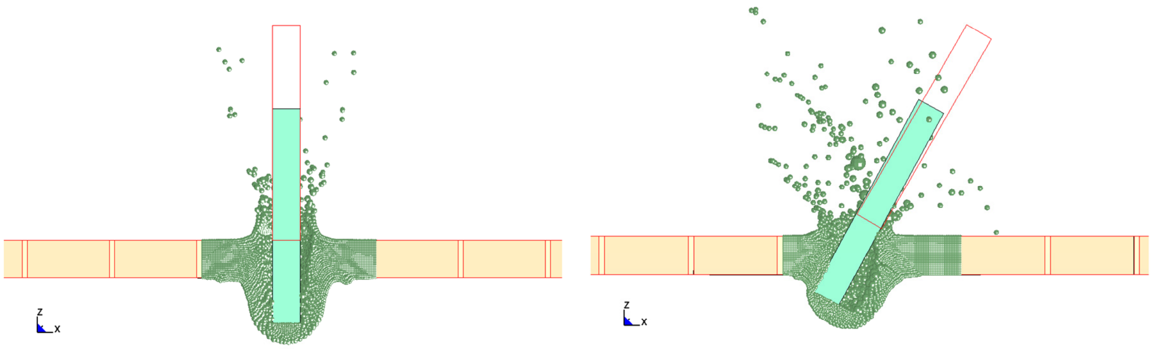

**Figure 10 Deformed configurations of the brick wall for normal (left) and 30 degree oblique (right) impacts; initial projectile position shown in outline.**

Figure 11 summarizes the projectile rigid body velocity for the six cases simulated. All six velocity histories indicate the projectile has reached a terminal velocity, i.e. constant velocity representing the residual (exit) velocity. The oblique impact velocity histories have about a 0.2 ms delay in their wave forms compared to the normal impact wave forms. This time delay is caused by the additional standoff of the projectile from the brick wall surface when the projectile was rotated 30 degrees.

As expected, the residual velocities for the weakest concrete material model MAT016 are larger than the intermediate strength concrete material model MAT072R3, which in turn are larger than the residual velocities for the strongest concrete material model MAT159. The residual velocities are independent of the projectile's initial orientation. This lack of dependence of the exit velocity on orientation is not a surprising result, as for thin targets the additional path length of the projectile through the target at 30 degrees is small.

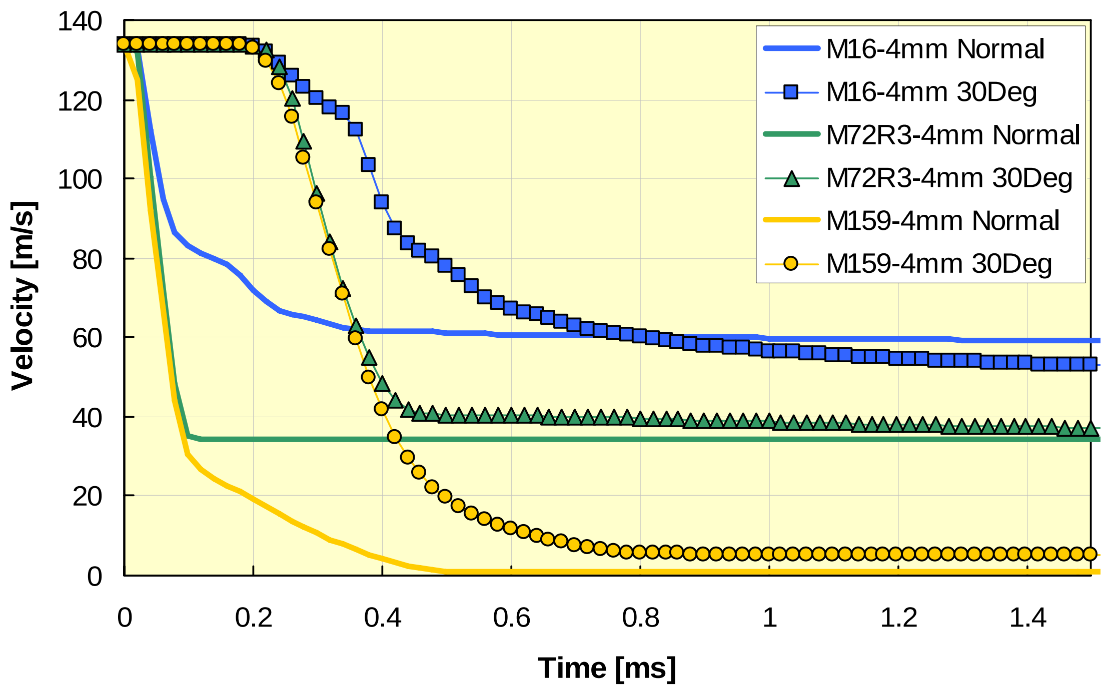


**Figure 11 Rigid projectile velocity histories for two impact configurations and three simple input concrete material models.**

The residual velocities for the normal impact configuration are 59.1, 34.2, and 0.71 m/s for the three simple input concrete models. These three velocities have an average value of 31.3 m/s and a standard deviation of 29.3 m/s. Thus to answer the question does the projectile perforate the target, the suggested response would be:

> "For a normal impact at a speed of 134 m/s (440 fps), the projectile's exit velocity is expected to be 31.3 ±29.3 m/s for a 31 MPa unconfined compressive strength target wall."

This form of the result statement allows the decision maker to decide if there is adequate 'margin' in the results, or if more resource should be expended to better characterize and model the brick and mortar.

**Concrete Cylinders**

The approach of using multiple simple input concrete models to make penetration *predictions* was recently applied to concrete cylinders impacted by nearly rigid (non-deformable) projectiles. Figure 12 compares the measured projectile velocity ratio, i.e. residual to initial, with the corresponding CONWEP and LS-DYNA simulation results for various thicknesses of the concrete target. The symbols used in this comparison plot display the experimental observations as black filled squares, CONWEP results as grayed circles, and the LS-DYNA results as an open triangles representing the average result for the MAT072R3 and MAT159 concrete models, see the legend in the Figure 12. Additionally, the LS-DYNA results are associated with a range of velocity ratios with the lowest ratio obtained from the strongest concrete material model (MAT159) and the largest ratio from the intermediate strength concrete material model (MAT072R3). The results obtained from CONWEP[2] help to support the experimental observations and provide an of estimate the ballistic thickness limit for the concrete targets. The CONWEP results are empirical, obtained from fits to an extensive database of perforation results.

The results comparison shown in Figure 12 indicate the LS-DYNA predictions span the measurements, and most of the CONWEP results.

For the 1 inch thick target, the experimental observations, CONWEP results, and LS-DYNA predictions are all in close agreement. This is to be expected for relatively thin targets perforated by a robust (non-deformable) projectile.

For the 2 inch thick target, the two experimental observations show good repeatability. However, the CONWEP result has a significantly larger velocity ratio. The average of the two LS-DYNA predictions is in good agreement with the experimental observations, and the intermediate strength concrete material model, MAT072R3, result is nearly identical to the CONWEP result.

[2] ConWep is a collection of conventional weapons effects calculations from the equations and curves of TM 5-855-1, "Design and Analysis of Hardened Structures to Conventional Weapons Effects." ConWep performs a variety of conventional weapons effects calculations including an assortment of airblast routines, fragment and projectile penetrations, breach, cratering, and ground shock. https://pdc.usace.army.mil/software/conwep/

No experiments were performed with a 3 inch thick target, but again the intermediate strength concrete material model, MAT072R3, result is close to the CONWEP result.

For the 4 inch thick concrete target, the experimental observations and CONWEP results indicate the target will not be perforated. The LS-DYNA predictions indicate the intermediate strength concrete material model, MAT072R3, predicts target perforation with a significant velocity ratio of 28%, and the stronger concrete material model, MAT159 predicts no perforation of the 4 inch target.

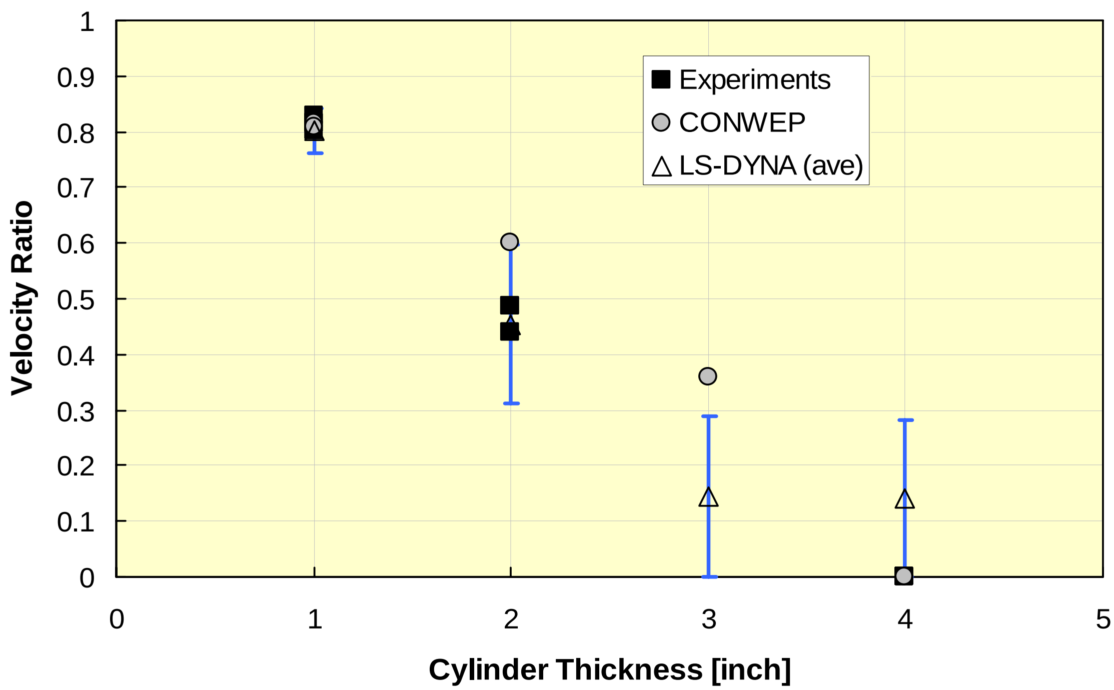


**Figure 12 Comparison of experiment and numerical projectile speed ratios for a projectile impacting at 270 m/s.**

## Summary

Using the three simple input concrete models available in LS-DYNA a range of exit velocities was generated for the perforation of two concrete targets. In the case where experimental data was available, i.e. cylinder perforation, the numerical *predictions* spanned the data. Spanning the data is the best an analyst can hope to achieve when the target material is uncharacterized.

The more typical case is the target material is uncharacterized and there is no experimental data. In this case, generating a range of simulation results, via three multiple plausible material models, allows a decision maker to decide if there is an adequate 'margin' in the results, or if more resources should be expended to better characterize and model the targets.

## Acknowledgement

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