

## Visualization of Pareto Optimal Fronts for Multiple Objectives with D-SPEX

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### Abstract

*Most engineering optimization problems require multi-objective optimizations that have no unique optimum because the objectives often conflict. LS-OPT 3.3 offers the capability to simultaneously compute many Pareto optimal solutions using a multi-objective genetic algorithm. The optimization post processing tool D-SPEX provides advanced features to visualize these Pareto optimal solutions and to approximate the Pareto optimal front using the moving least squares method. In addition, the moving least squares method may now be used in D-SPEX as an additional option for building response surfaces as well as for the computation of virtual histories.*

## Introduction

Most engineering optimization problems require multi-objective optimizations that have no unique optimum because the objectives often conflict, e.g. the power and the consumption of a vehicle. So no single design can be considered optimal with respect to all objectives, instead the optimum is a set of solutions, the so-called Pareto optimal solutions. For a Pareto optimal solution, there is no design that provides better solutions for any objective without worsening another objective.

LS-OPT 3.3 offers the possibility to compute multiple Pareto optimal solutions for multi criteria optimization problems using a genetic algorithm ([1]). Usually they are calculated meta model based.

In order to aid the engineer in understanding different trade-offs, the optimization post processing tool D-SPEX provides some features to visualize the Pareto optimal solutions.

## Introduction to D-SPEX

D-SPEX is a Matlab<sup>®</sup> based post processing tool that specializes in the visualization of optimization data provided by LS-OPT. It has been developed in cooperation with AUDI AG for the occupant safety department but is currently used in various disciplines by AUDI, Daimler and other clients. Installation packages for D-SPEX are available for Windows and Linux. Although D-SPEX is a Matlab based tool, Matlab is not needed in order to run D-SPEX.

D-SPEX is intended to provide features that are not currently implemented in the LS-OPT viewer. Therefore, it is a complement to the visualization capabilities of LS-OPT. Its primary focus is on the visualization of meta-models although it also provides features to visualize stochastic results. D-SPEX is also thought of as a testing platform for new features that might evolve in LS-OPT. For more information on the features of D-SPEX, see [4]. In the following, the latest development of D-SPEX, the visualization of Pareto optimal solutions, is presented.

## Visualization of Pareto Optimal Solutions

The first approach to visualize the Pareto optimal front in D-SPEX is to visualize the Pareto optimal solutions computed by LS-OPT. If the optimization problem has more than three objectives, all dimensions may not be visualized at once. Hence, 2D or 3D section planes have to be selected for visualization (Figure 1), according to the visualization of response surfaces. The objectives that are not visualized are set to fixed values. In contrast to response surfaces, the Pareto optimal solutions data set provided by LS-OPT may not have continuity among objectives. If only the points that lay exactly on the section plane are selected, only a few or no points will be available for plotting. Therefore, a set of points within a small distance  $\varepsilon$  to the selected section is plotted. The bandwidth  $\varepsilon$  may be selected by the user as a percentage of the design space (Figure 1). To provide an impression of the proximity of the selected points to the section plane, the points are color-coded from black to white depending on the distance to the plane. The closer the point is to the plane, the darker the color is, see Figure 4 and Figure 5.

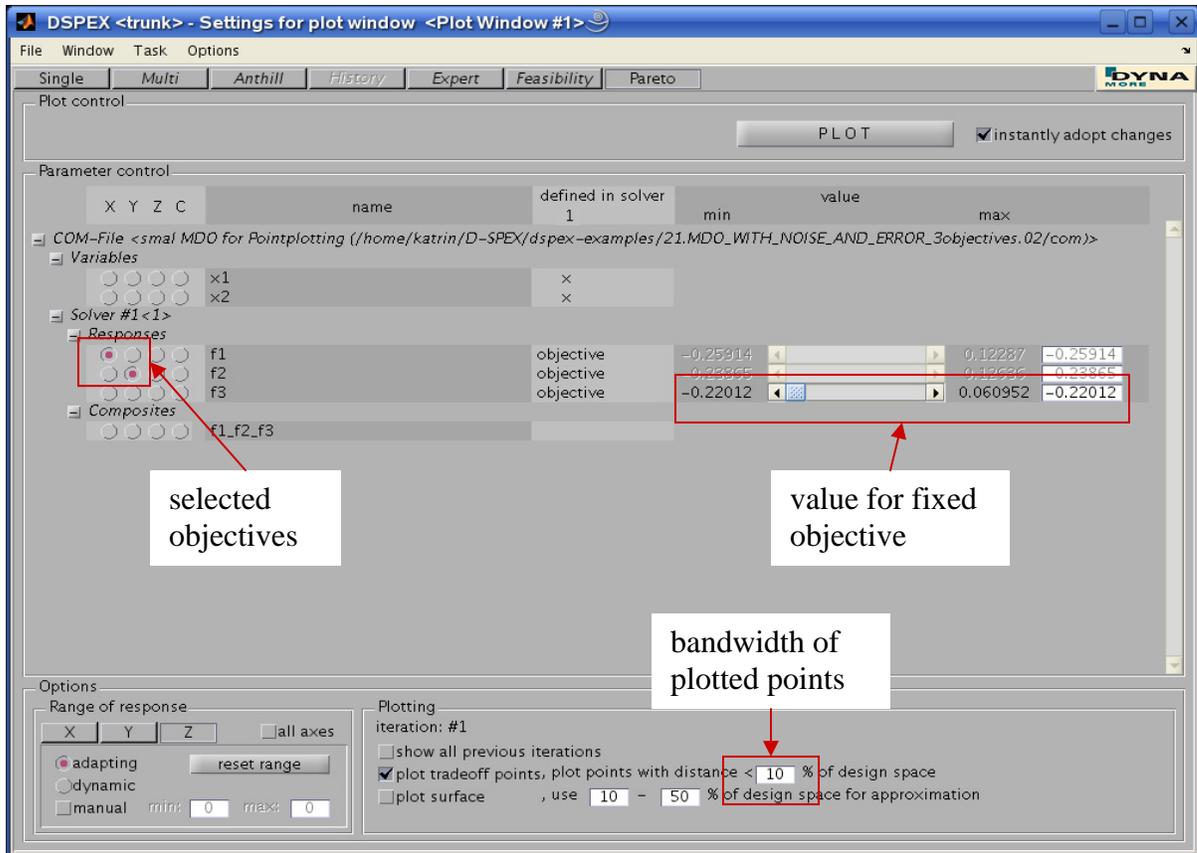


Figure 1: D-SPEX graphical user interface for visualization of Pareto optimal fronts

The D-SPEX visualization results are presented using a simple example with two variables  $x_1$  and  $x_2$ , and three objectives  $f_1$ ,  $f_2$  and  $f_3$ . The objectives are defined as

$$f_i = \sin\left(\frac{(x_1 - a_i)^2 + (x_2 - b_i)^2}{((x_1 - a_i)^2 + (x_2 - b_i)^2)}\right) \quad , \quad i = 1..3,$$

whereas  $a_i$  and  $b_i$  are real numbers that shift the function in  $x_1$  and  $x_2$  direction, respectively. Hence we get conflicting objectives. The objective  $f_1$  is displayed in Figure 2. The Pareto optimal solutions set for this example is obtained using LS-OPT and a three-dimensional visualization of this set is shown in Figure 3.

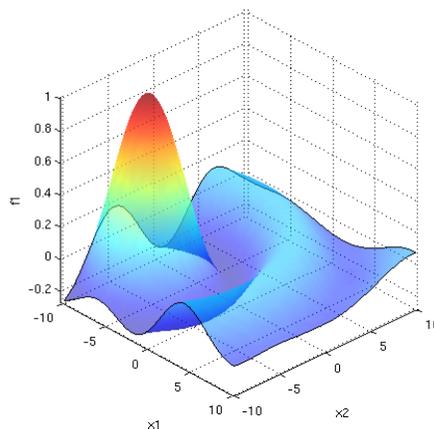


Figure 2: Objective  $f_1$

Figure 4 and Figure 5 show some 2D section planes. To give an example,  $f_3$  and  $f_2$  are set to a fixed value, respectively. Comparing these figures to Figure 3, where all dimensions of the Pareto optimal solutions are displayed, the visualization method of the sections becomes obvious. To see the influence of the bandwidth of the displayed points, both sections are displayed for a 10% and 1% bandwidth.

So far, only the relation between different objectives are visualized, but if the engineer selects a suitable design, it is important to know which variable constellation leads to that design, hence D-SPEX offers two possibilities explained below to visualize the variable values in combination with the objectives.

1. As a further dimension, a variable, response or composite may be selected to color the points. Examples are displayed in Figure 6, Figure 7 and Figure 8. To be comparable to Figure 4 and Figure 5, the same sections are selected for Figure 7 and Figure 8. As explained above, a set of Pareto optimal points within a small distance  $\varepsilon$  to the selected section is plotted. Instead of the different gray values, the transparency of the points is used to visualize the distance from the selected section.
2. To visualize the relation between objectives and variables, responses or composites, it is also possible to plot objectives against variables, responses or composites in any combination.

In cases where enough Pareto optimal points are available, like in the shown example, the visualization of these points gives a sufficient impression of the Pareto optimal front. For other cases as are frequently encountered with the direct simulation based GA for engineering problem, a meta-model may be used to approximate the surface.

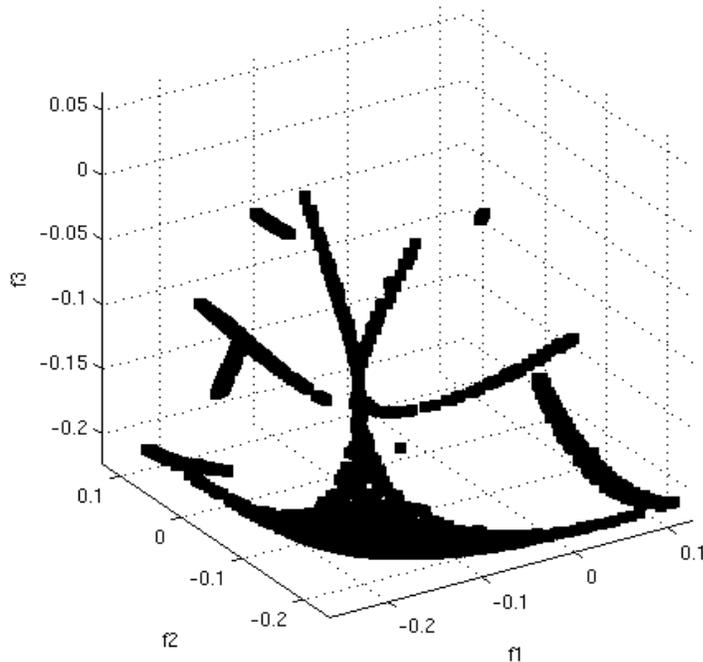


Figure 3: Pareto optimal points, 3 objectives

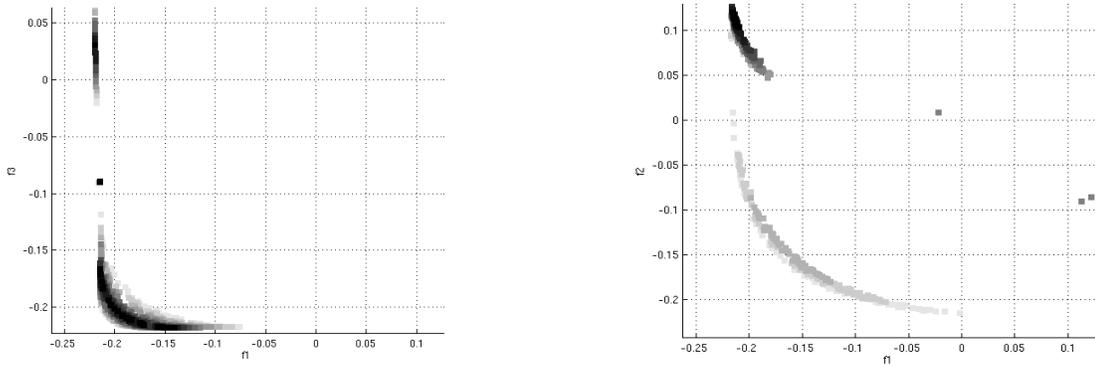


Figure 4: section planes  $f_3 = -0.22012$ , bandwidth 10% (left) and 1% (right)

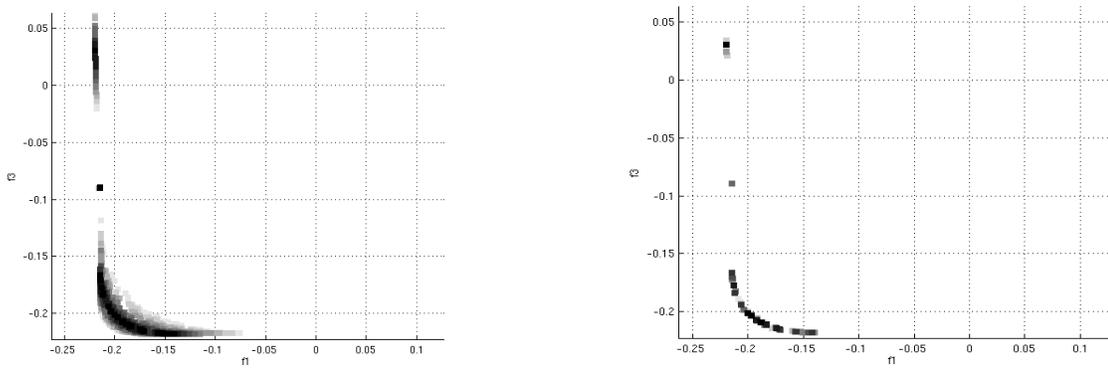


Figure 5: section planes  $f_2 = -0.16564$ , bandwidth 10% (left) and 1% (right)

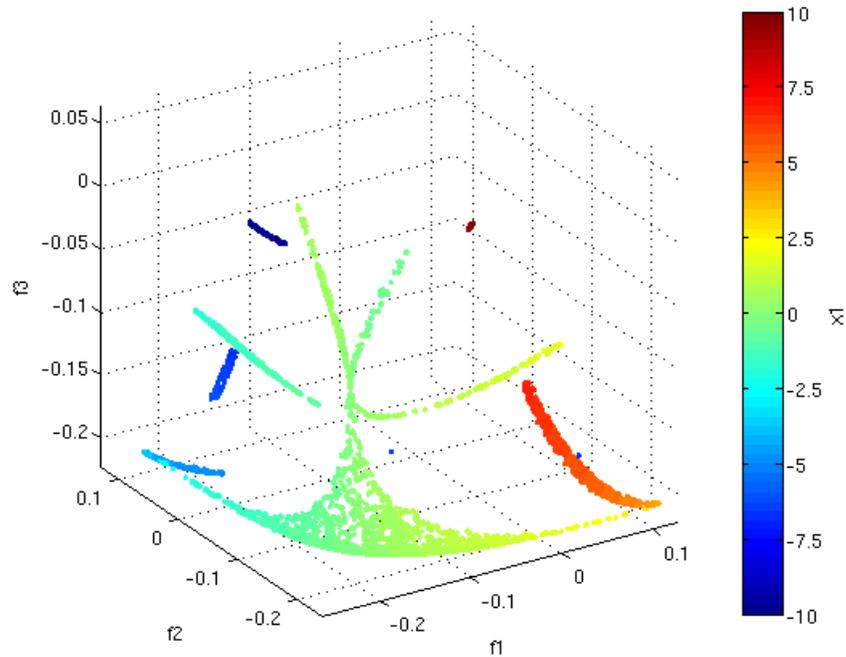


Figure 6: Pareto optimal points, 3 objectives, color variable  $x_1$

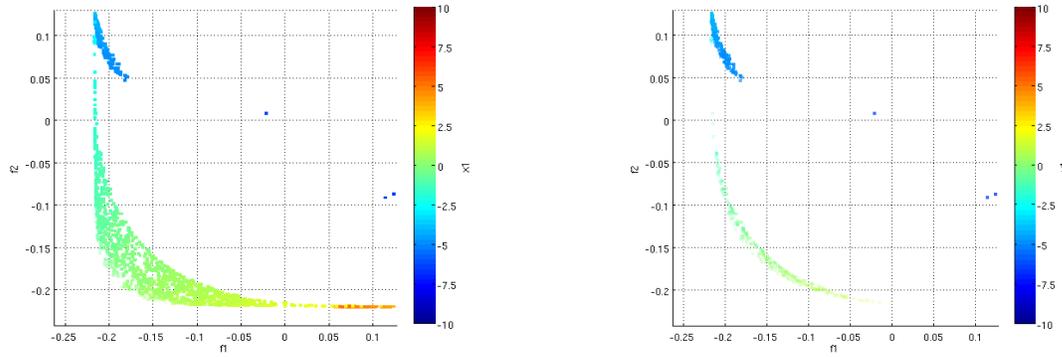


Figure 7: section planes  $f3 = -0.22012$ , bandwidth 10% (left) and 1% (right)

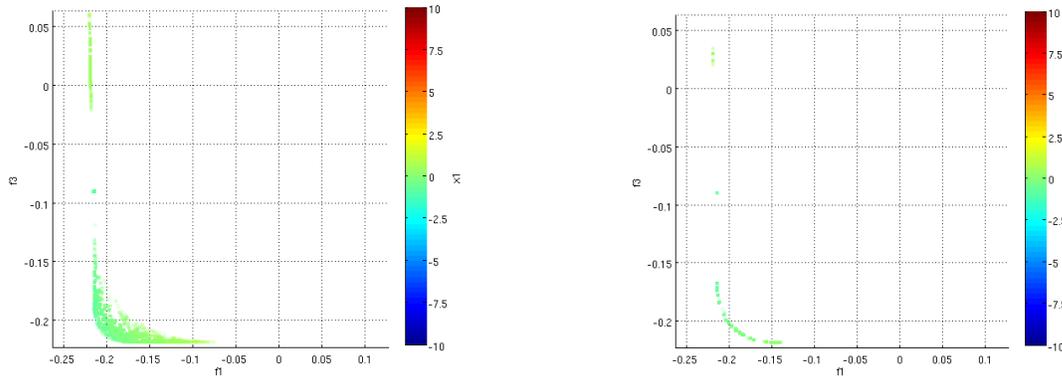


Figure 8: section planes  $f2 = -0.16564$ , bandwidth 10% (left) and 1% (right)

### Approximation of Pareto Optimal Points using Moving Least Squares

The meta-models used e.g. for the approximation of the response surfaces by LS-OPT may not be able to capture the Pareto optimal front information, because Pareto optimal fronts may be disconnected, as Figure 6 shows. Therefore, D-SPEX uses the moving least squares method to approximate the Pareto optimal fronts ([2]).

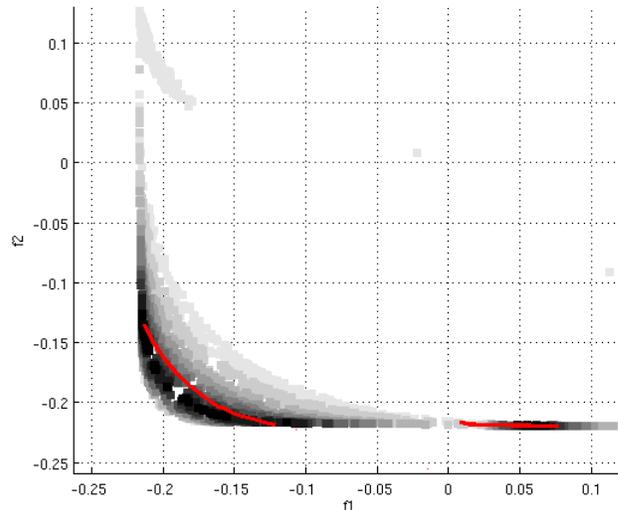


Figure 9: Pareto optimal points with approximated surface,  $f3 = -0.19202$

The idea of the moving least squares method is to compute a local approximation for each evaluation point using a weighted polynomial least squares approximation, e.g., a linear approximation. The points with small distance to the evaluation point have a large weight, the points with large distance have a small weight. If the weight is zero, the point is not considered. If enough points with non zero weights are not available, approximation values are not computed and disconnected surfaces may be found.

An example of a Pareto optimal front approximated using the moving least squares method is displayed in Figure 9.

### The Moving Least Squares Method for Approximation

To assess the quality of the moving least squares (MLS) approximation, the method was implemented for response surfaces and compared to the feedforward neural network (FF) and the radial basis function network (RBF) used by LS-OPT. The SPRESS (Square root of the Predicted Residual Sum of Squares), which measures the prediction capability of any meta model, and the RMS error (square root of residual sum of squares) ([3]), were compared (Table 1) for FF, RBF and MLS approximations by means of a simple crash worthiness design with two load cases. Subject of this investigation is the hit of a car on a single pole.

In the first load case the car hits the pole in the middle of its front part. The thicknesses of the bumper, the hood, and the bottom are to be determined. For the second load case an antimetric scenario is investigated. In contrast to the first load case the hitting point is now moved from the middle to the right as shown in the figures below.

For both load cases the considered responses are

- ⊗ the minimal x displacement of two selected nodes (Intru1, Intru2),
- ⊗ the HIC value at a selected node and
- ⊗ the absolute intrusion that is calculated from the difference of Intru1 and Intru2.

The objective of both load cases is the respective HIC value which has to be minimized. For both load cases constraints are introduced for the absolute intrusion.

Response	SPRESS			RMS error		
	FF	RBF	MLS	FF	RBF	MLS
Intru2_mid	0.02	0.48	0.48	3.59e-3	0.20	0.20
Intru1_mid	0.11	0.31	0.27	2.40e-4	0.12	0.04
HIC_mid	23.00	31.56	29.87	0.26	12.25	4.19
Intrusion_absolute_mid	0.46	0.46	0.44	0.09	0.21	0.09
Intru2_right	0.37	0.47	0.44	0.15	0.15	0.06
HIC_right	6.12	7.72	7.15	2.49	2.48	1.87
Intru1_right	1.00	2.05	1.95	0.01	0.65	0.48
Intrusion_absolute_right	1.24	1.58	1.50	0.50	0.50	0.37

**Table 1: Prediction capability of Feed forward neural networks (FF), Radial Basis Functions (RBF) and Moving Least Squares (MLS), 5 points used for approximation.**

Judging from this example, it can be said that the moving least squares approximation has a comparable quality to the radial basis approximation and feed forward neural networks. Figure

10 and Figure 11 show examples of the surfaces. The upright black lines visualize the distance from the points to the surface.

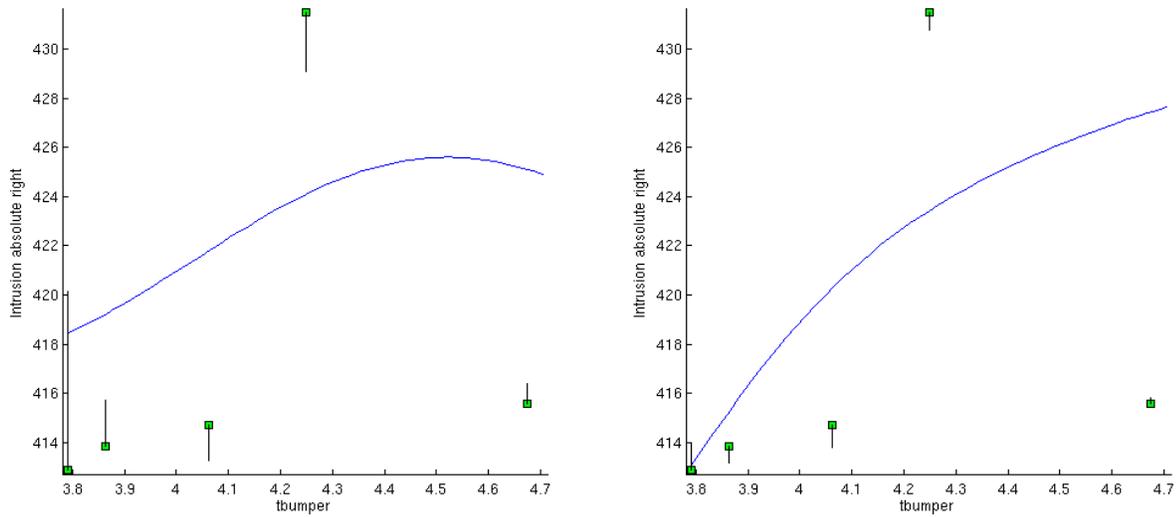


Figure 10: Feedforward Neural network (left) compared to Moving Least Squares (right)

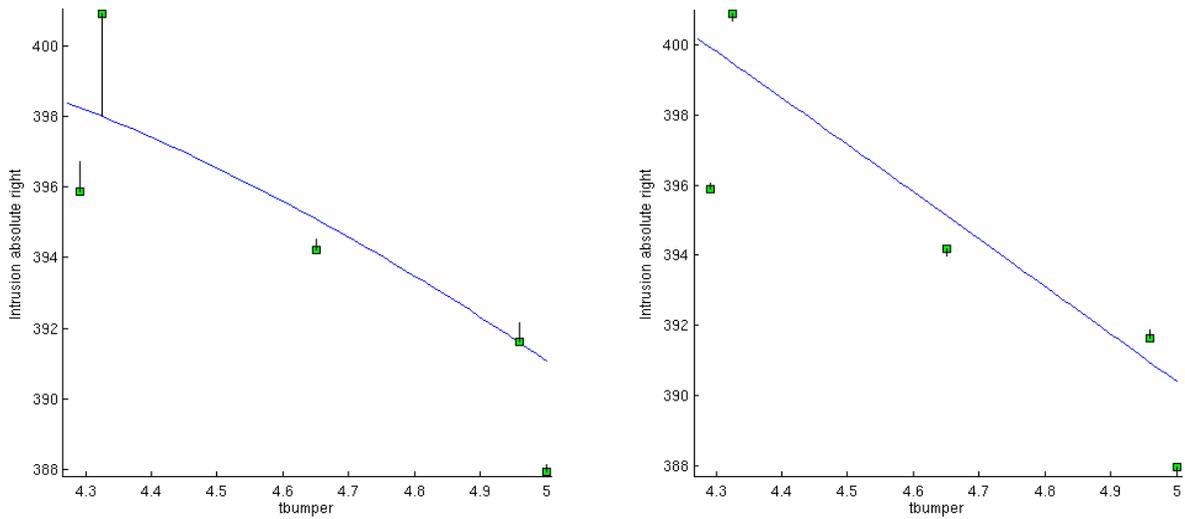


Figure 11: Radial Basis Functions (left) compared to Moving Least Squares (right)

### The Moving Least Squares Method for Virtual Histories

A virtual history is an estimate of a history for a specific parameter combination by extending the meta-model concept to histories. It means that meta-models are created at a number of discrete points in time of the histories. So the individual meta-models can be used to assemble “virtual histories”. So far, D-SPEX provided only linear meta models to approximate virtual histories, ([4]), but since the moving least squares approximations results in a good approximation, they were also applied to compute the virtual histories in D-SPEX to get better approximations.

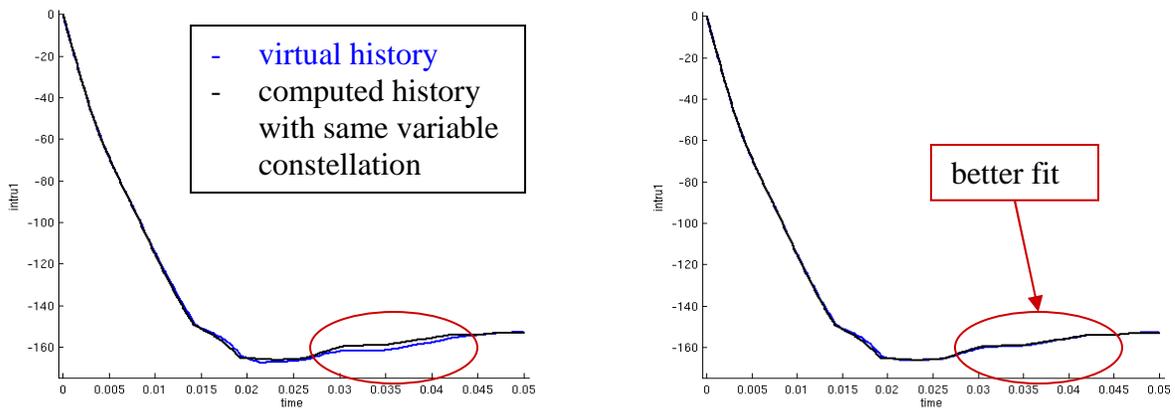


Figure 12: virtual history, linear approximation (left) and moving least squares approximation (right)

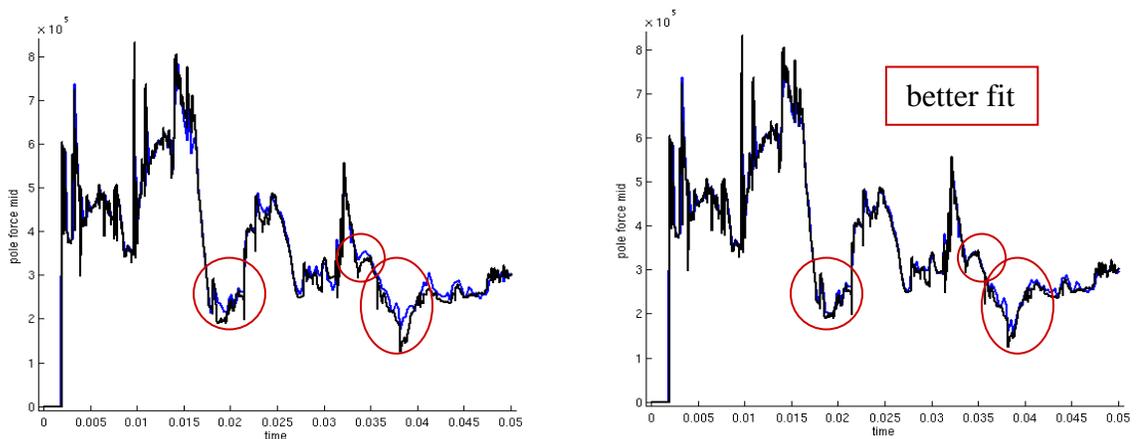


Figure 13: virtual history, linear approximation (left) and moving least squares approximation (right)

In Figure 12 and Figure 13, both approximations are compared using the example explained in the section above. Figure 12 shows the displacement of a node over time, Figure 13 shows the force at the pole. The black line is the computed history with the same variable constellation as the virtual history (blue curve). So the moving least squares approximation provides better results, at least for this example.

## Conclusions

In cases with two or three objectives, it is sufficient to visualize the Pareto optimal solution computed by LS-OPT. In cases with more objectives, there are often not enough points per dimension available to get an impression of the Pareto optimal front only by visualizing the points, so it is necessary to compute an approximating surface. The moving least squares method turned out to be suitable to approximate surfaces that may be disconnected. However further work will be necessary in order to provide a solution that is suitable for the engineering needs.

**References**

- [1] Goel, T., Stander, N., Multi-Objective Optimization Using LS-OPT, 2007
- [2] Kunle, M., Entwicklung und Untersuchung von Moving Least Square Verfahren zur numerischen Simulation hydrodynamischer Gleichungen, 2001
- [3] Stander, N., Roux, W., Eggleston, T., Craig, K., LS-OPT<sup>®</sup> User's Manual, Version 3.3, Livermore Software Technology Corporation, Livermore, 2008
- [4] Witowski, K., Thiele, M., Seer G., Mühlhuber, W., van den Hove, M., New Features in D-SPEX with Application, 2007