Preliminary Results for an Isogeometric Shell

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Abstract

Piecewise continuous Lagrangian polynomials are the traditional interpolation functions used in the finite element method. They work well for many applications, but they also have shortcomings for many important applications. For example, in metal forming, the dies are designed using CAD programs and their geometry is defined in terms of NURBS (non-uniform rational B-splines) which can not be exactly replicated with a piecewise continuous Lagrangian polynomial in all cases. Therefore, there is a geometric incompatibility between the desired shape and the kinematic range of the blank modeled with traditional finite elements. This paper presents initial results for a shell element formulation based on NURBS.

Introduction

Geometry is the foundation of analysis yet modern methods of computational geometry have until recently had very little impact on analysis. The reason may be that the Finite Element Method (FEM), as we know it today, was developed in the 1950's and 1960's, before the advent and widespread use of Computer Aided Design (CAD) programs, which occurred in the 1970's and 1980's. Many difficulties encountered with FEM emanate from its approximate, polynomial based geometry, such as, for example, mesh generation, mesh refinement, sliding contact, flows about aerodynamic shapes, buckling of thin shells, p-methods, etc. It would seem that it is time to look at more powerful descriptions of geometry to provide a new basis for analysis. An attempt to generalize and improve on the finite element analysis in the area of geometry modeling and representation has led to the introduction and development of *isogeometric analysis*.

Different approaches with this spirit are being developed. Among those that have demonstrated considerable potential over typical FEM basis functions are subdivision surfaces, NURBS, and T-Splines. NURBS-based isogeometric analysis methods have been successfully applied to fluids, structures, fluid-structure interaction, and phase-field modeling of phase separation.

This paper focuses on the initial results for isogeometric shell elements that were implemented in LS-DYNA via the new *DEFINE_ELEMENT_SHELL_TYPE feature. The ultimate objective of the NSF grant funding our research is to use isogeometric shell elements in sheet metal stamping.

A Brief Summary of NURBS-Based Isogeometric Analysis

Non-Uniform Rational B-Splines (NURBS) are a standard tool for describing and modeling curves and surfaces in computer aided design and computer graphics (see Piegl and Tiller [1] and Rogers [2]). The aim of this section is to introduce them briefly and to present an overview of isogeometric analysis, for which an extensive account has been given in Hughes, Cottrell and Bazilevs [3].

B-splines are piecewise polynomial curves composed of linear combinations of B-spline basis functions. The coefficients are points in space, referred to as *control points*. A *knot vector*, Ξ is a set of non-decreasing real numbers representing coordinates in the parametric space of the curve:

$$\Xi = \{\xi_1, \ldots, \xi_{n+p+1}\}$$

where p is the order of the B-spline and n is the number of basis functions (and control points) necessary to describe it. The interval $[\xi_1, \xi_{n+p+1}]$is called a patch.

Given a knot vector, Ξ, B-spline basis functions are defined recursively starting with $p=0$ (piecewise constants):

$$N_{i,0}(\xi) = \begin{Bmatrix} 1 & if & \xi_i < \xi < \xi_{i+1} \\ 0 & otherwise & \end{Bmatrix}$$

For p>1:

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi).$$

In Figure 1 we present an example consisting of n=9 cubic basis functions generated from the open knot vector Ξ={0,0,0,0,1/6,1/3,1/2,2/3,5/6,1,1,1,1}.

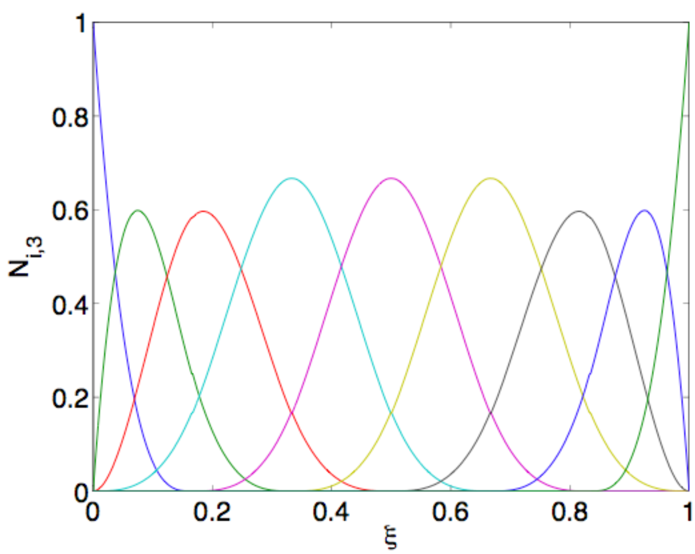

Figure 1: Cubic basis functions formed from the open knot vector Ξ={0,0,0,0,1/6,1/3,1/2,2/3,5/6,1,1,1,1}.

Using tensor products, B-spline sufaces can be constructed starting from knot vectors $\Xi = \{\xi_1, \ldots, \xi_{n+p+1}\}$and $\mathrm{H} = \{\eta_1, \ldots, \eta_{m+q+1}\}$ and an n x m net of control points $x_{i,j}$. One-dimensional basis functions $N_{i,p}$ and $M_{j,q}$ (with *i=1,...,n* and *j=1,...,m*) of order *p* and *q*, respectively, are defined from the knot vectors, and the B-spline surface is constructed as:

$$S(\xi,\eta)=\sum_{i=1}^{n}\sum_{j=1}^{m}N_{i,p}(\xi)M_{j,q}(\eta)x_{i,j}.$$

The patch is the domain $[\xi_1,\xi_{n+p+1}]\otimes[\eta_1,\eta_{m+q+1}]$. Identifying the logical coordinates (*i*,*j*) of the B-spline surface with the traditional notation of a node, *A*, and the Cartesian product of the associated basis functions with the shape function, $N_A(\xi,\eta)=N_{i,p}(\xi)M_{j,q}(\eta)$, the familiar finite element approximation is recovered,

$$S(\xi,\eta)=\sum_{A=1}^{nm}N_A(\xi,\eta)x_A.$$

Non-uniform rational B-splines (NURBS) are obtained by augmenting the spatial coordinates of the control points with the homogenous coordinate w_A, then dividing through by *w*, giving the final spatial surface definition,

$$S(\xi,\eta)=\frac{\sum_{A=1}^{nm}N_A(\xi,\eta)w_A x_A}{\sum_{A=1}^{nm}N_A(\xi,\eta)w_A}=\sum_{A=1}^{nm}\tilde{N}_A(\xi,\eta,w)x_A$$

where placing *w* as an argument to the shape function is intended to emphasize that it is not treated as a solution variable, but is data from the original CAD definition of the surface. NURBS, in contrast to simple B-Splines, allow for exact representation of all conic sections. They are considered to be standard CAD technology. In the remainder of this paper, we will suppress the arguments of the basis functions for simplicity.

Shell Kinematics

As a reference formulation to facilitate the direct comparison of the isogeometric elements to the elements in LS-DYNA, a Reissner-Mindlin formulation based on a degenerated solid has been implemented. Associated with each control point is a unit fiber vector, f_A, a translational velocity, v_A, and an angular velocity ω_A. For simplicity, we assume a constant thickness *h*, although this is not an intrinsic limitation anymore than for the other shells in LS-DYNA. The velocity at a parametric point (ξ,η,ζ), where ζ is the parametric coordinate in the thickness direction, is

$$v=\sum_{A=1}^{nm}\tilde{N}_A(v_A+\tfrac{h}{2}\zeta\omega_A\times f_A).$$

Using this relation, the velocity gradient is obtained by differentiation, and from it, the B-matrix is extracted by gathering the coefficients associated with each control point's velocity. Standard Gauss quadrature is used to evaluate the internal force vector, $F^{\text{int}}=\int B^T\sigma dV$.

Results for a Model Problem

The pinched cylinder problem [3] is illustrated in Figure 3, left. A concentrated load is applied to a single control point and its displacement calculated. The problem has a high degree of symmetry, allowing only one-eighth of the domain to be modeled. This problem has been solved previously using isogeometric solids [3], and the response we obtained using the thin shell formulation with rotations is essentially identical (see Figure 3, right).

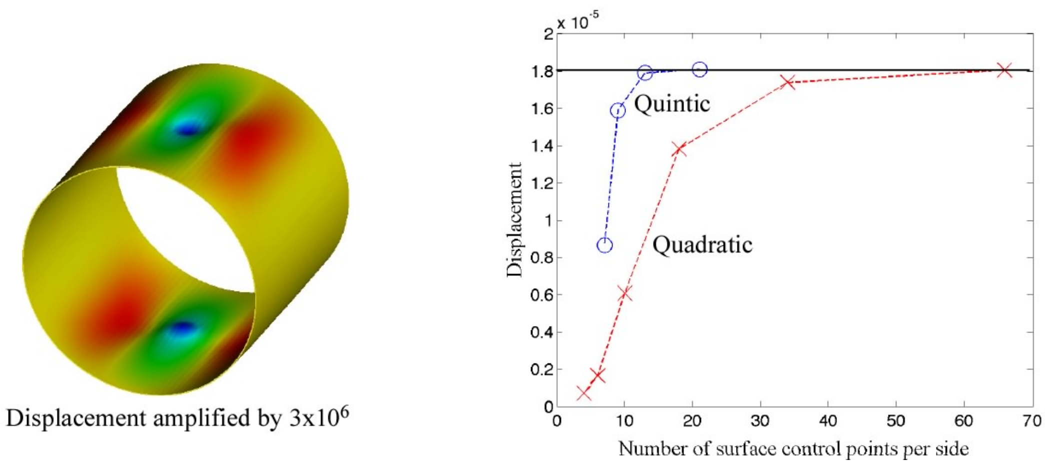

Figure 3: Pinched cylinder test. The number of surface control points refers to their number from the concentrated load to the edge of the cylinder.

References

[1] L. Piegl and W. Tiller. *The NURBS Book (Monographs in Visual Communication), 2nd ed.* Springer-Verlag, New York, 1997.

[2] D. F. Rogers. *An Introduction to NURBS With Historical Perspective*. Academic Press, San Diego, CA, 2001.

[3] T.J.R. Hughes, J.A. Cottrell, and Y. Bazilevs. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry, and mesh refinement. *Computer Methods in Applied Mechanics and Engineering*, 194:4135–4195, 2005.