Verification of a New Fracture Criterion Using LS-DYNA

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Abstract

Several fracture models are available in the material library of LS-DYNA. This paper is concerned with a newly developed constitutive model that covers the full range of plasticity till the onset of fracture. It is understood that the fracture initiation in uncracked solids is an ultimate result of a complex damage accumulation process. Such damage is induced by plastic deformations. A new damage model is proposed to incorporate the pressure sensitivity and the Lode angle dependence through a nonlinear damage rule using a reference fracture strain on a restricted loading path. The onset of fracture is predicted by integrating incremental damage along the actual loading path. In this cumulative fashion, fracture can be predicted for complex loading paths, which are not limited to the restricted loading in which the pressure is constant. This modified model also incorporates the coupling between the damage and the strain hardening function.

The new fracture model is implemented to LS-DYNA as a user defined material subroutine. A series of benchmark tests and simulations have been performed to verify this model. The loading situations of these tests cover a wide range of standard laboratory testing, which include uniaxial tension of a round bar, uniaxial tension of a hollow bar and the three-point bending of a rectangular bar. A remarkable agreement between the experimental and numerical results is achieved.

Introduction

In many industrial applications, the fracture of ductile metals occurs after large plastic deformations. These situations include vehicle crashworthiness, ship collision, sheet metal forming, ballistic impact etc. To prevent a catastrophic failure or enhance the quality of products, a robust fracture criterion is essential in the design process.

A sound physical understanding of ductile fracture is the key in the modeling of the fracture phenomenon. In reality, the fracture is influenced by many factors and more sophisticated models have to be used to obtain a satisfactory result. Various assumptions and simplifications were made to make the problem of fracture mathematically tractable. Several ductile fracture criteria of different complexity have been proposed from macroscopic or microscopic point views. For instance, the Johnson-Cook model [1] which incorporates the stress triaxiality effect, the thermal effect and the strain rate effect is often used in the high-velocity impact problems where the rate dependence and adiabatic heating become important. In fact, in the last four decades, continuous effects are made in the development of ductile fracture models that are suitable for the military and industrial applications.

It is well-known that the classic plasticity theory is suitable in the range of small and moderate plastic deformation. This theory may be not sufficient in fracture prediction because the fracture site usually undergoes extensive plastic deformation. Now, the question is:

What happens beyond “moderate” plasticity?

The conventional continuum mechanics assumes the microstructure of the solids to be unchanged. The strength of material is often simplified as a scalar problem where the resistance of material is characterized as a function of the equivalent plastic strain only. This simplification works well for many of the practical problems where only small plastic deformation is involved, especially for the design purpose of many structures carrying dead load. However, for the extreme loading applications, the mechanical response of the material in the full range of its deformability is critical and desired. In such conditions, the neglect of the change of microstructure is over simplified. Some type of damage has to be included to characterize the material deterioration. In the present paper, we present a new constitutive model and demonstrate the predictive power through several simulations. Simple geometries are used in these simulations and, yet, the underlying physical consideration is revealed.

Constitutive modeling

A new constitutive model that characterizes the full range of plasticity up to the fracture point is recently proposed by Xue [2]. In this model, the strength of material is treated as a four dimensional problem.

Firstly, the macroscopic response of the material is separated from the matrix. The strength of the matrix material is considered to be a basic property of the material, which does not change along the deformation path. A scalar quantity – the damage - is used as an internal variable to calculate the magnitude of material deterioration. This is the material dimension. Secondly, the damage accumulation along the loading path is a three-dimensional problem in which the pressure, the Lode angle and the equivalent stress affact the damage accumulation rate in different ways. The Lode angle is an azimuthal angle in the octahedral plane, which can be used to distinguish various deviatoric stress states.

(1) Material softening

When ductile metals is subjected to large plastic deformation, the micro structure changes due to the extensive slip around inclusions or micro defects where stress concentrates. Micro cracks initiate and propagate; micro voids nucleate and grow. These material deterioration are no longer negligible when the deformation is beyond moderate plasticity. This impairs the continuum assumption of classic plasticity theory. Therefore, some weakening function is adopted to account for this softening effect. Meanwhile, the material is still considered as a continuum.

Lemaitre uses the concept of effective load carrying area to describe the damage effect on the macroscopic strength [3]. He relates the damage with the relative reduction of actual cross-sectional area. The macroscopic strength σ_{eq} can be conveniently expressed by

$$\sigma_{\mathrm{eq}} = \left(1 - D_s\right)\sigma_{\mathrm{M}} \tag{1}$$

where D_s is the relative loss of loading carrying area, σ_{M} is the matrix strength.

As the damage affects the macroscopic material strength, meanwhile, the elastic moduli are considered to be damaged in the same fashion as the material strength.

(2) Damage rule

Extensive experiments reveal that the fracture strain is loading-path dependent. Therefore, a loading path is required to describe the fracture strain. A cylindrical coordinate system, whose base vectors are the pressure p, the Lode angle θ_L (or the relative ratio of the stress deviators χ, since there exists a one-to-one mapping) and the equivalent stress, is used to characterize the progression of the damage process. The damage is first considered on a restricted loading path, which is a deviatorically proportional path that has constant pressure and constant Lode angle. It has been shown that any incremental damage can be is projected to one of such restricted loading path [2]. The following fundamental hypothesis is proposed to quantify damage in a unified way.

Hypothesis: *At a fixed pressure, for all proportional deviatoric loadings, the damage processes are self-similar.*

In the present model, the damage is defined as the relative reduction of deformability, i.e. the damage is the reciprocal of the times that the material can survive on the same loading path before fracture occurs. Using Palmgreen-Miner's approximation, the damage rule can be derived from the Manson-Coffin's empirical law. It has been shown in Ref. [2] that this yields a power law damage rule, which can be written as

$$\mathrm{d}D = m\left[\frac{\varepsilon_{\mathrm{p}}}{\varepsilon_{\mathrm{f}}\left(p,\chi\right)}\right]^{m-1}\frac{\mathrm{d}\varepsilon_{\mathrm{p}}}{\varepsilon_{\mathrm{f}}\left(p,\chi\right)}, \tag{2}$$

where ε_{p} is the plastic strain, ε_{f} is a reference fracture strain on the restricted loading path that passes the current stress state and *m* is the damage exponent. The fracture criterion is the integral of the incremental damage reaches unity, i.e.

$$D = \int_0^{\varepsilon_{\mathrm{frac}}} \mathrm{d}D = 1, \tag{3}$$

where $\varepsilon_{\mathrm{frac}}$ is the actual fracture strain on the given loading path.

In the present model, we use the effect load carrying area concept to describe the material deterioration and assume the relative loss of load carrying area increases in the same manner as of the relative loss of deformability, i.e. $D_s = D$. At the onset of fracture, both D_s and D reach unity.

For the entire family of the restricted loading paths, we can construct a so-called *fracture surface* in the principal stress space [2]. This fracture surface can be equally represented in the space of the mean stress and the triaxial principal strain plane, as shown in Fig. 1.

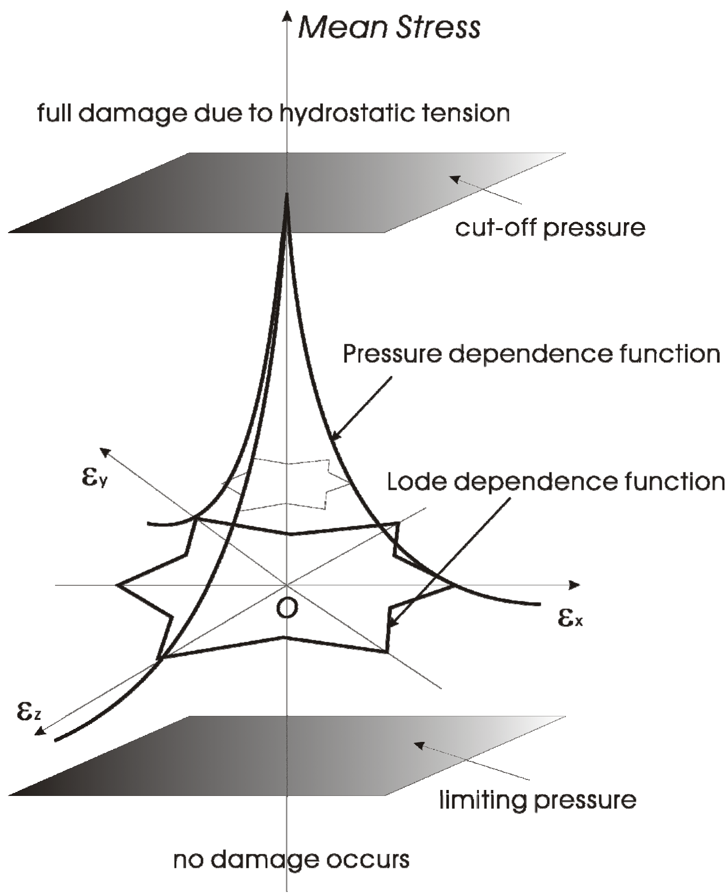


Figure 1. A three-dimensional fracture surface in the mean stress and plastic strain space.

On the restricted loading path, the fracture surface is independent of the equivalent stress and dependent entirely on the pressure and the Lode angle. Further, it is assumed that the effect of the pressure and the Lode angle is independent of each other. Thus, the fracture surface takes the form of

$$\varepsilon_{\mathrm{f}} = \varepsilon_{\mathrm{f0}} \mu_{\mathrm{p}}(p) \mu_{\theta}(\chi) \tag{4}$$

where $\varepsilon_{\mathrm{f0}}$ is a reference strain which is determined from triaxial tension at constant zero pressure, $\mu_{\mathrm{p}}(p)$ is the pressure dependence function and $\mu_{\theta}(\chi)$ is the Lode dependence function, which will be discussed in the next two subsections.

(3) Pressure dependence

It has been shown experimentally that the material ductility increases as the pressure increases. Undoubtedly, this pressure effect is from the suppression of the initiation and propagation of micro cracks and voids. Probably, the most notable work in this connection has been conducted by Bridgman, who investigated the mechanical properties of various materials, including several types of armor steel [4]. Similar results are obtained by Pugh [5] and many other researchers (see a recent review paper by Lewandowski and Lowhaphandu [6]). It has also been shown that new bonds in ductile metals can also be created under high pressure. This property has been used in cold press welding from ancient time [7].

Upon these experimental observations, the dependence of fracture strain on the exerted pressure is nonlinear. A limiting pressure is proposed above which no damage occurs. It has been derived from experimental curve of the reduction of cross-sectional area versus pressure that the pressure dependence function is a logarithmic function, i.e. [8]

$$\mu_{\mathrm{p}} = \begin{cases} 1 - q \log\left(1 - \dfrac{p}{p_{\lim}}\right) & , \quad p < p_{\lim}; \\ \infty & , \quad p \geq p_{\lim}. \end{cases} \tag{5}$$

There immerges immediately a cut-off pressure from Eq. (5), when μ_{p} reduces to zero. This cut-off pressure is calculated as

$$p_{cutoff} = p_{\lim}\left[1 - \exp\left(1/q\right)\right] \tag{6}$$

which indicates when the material fails under isostatic hydrostatic tension. These two parameters $p_{\lim}$ and q should be determined from tests.

(4) Lode dependence

Comparing with the extensive work of pressure effect on ductile fracture, the Lode dependence effect attracted much less attention and is seemingly overlooked. the pressure dependence alone can not explain all fracture phenomena. One such example is that the fracture strain in torsion is less than the fracture strain in simple tension for some materials [9]. From pressure dependence, fracture strain in torsion should be larger than the uniaxial tension of a round bar due to the absence of mean stress. In another Lode angle related problem, in sheet metal forming industry, the forming limit diagram at fracture shows the equivalent fracture strain in shear is less than biaxial tension for some materials [10]. These experimental results indicate that the damage accumulates faster in shear than in tension. This Lode dependence of fracture may result from the internal necking of the ductile material [8]. Wilkins et al proposed the first Lode angle dependent fracture criterion based on the stress asymmetry, which also indicate the Lode angle [11].

Due to the scanty of experimental data, we use a heuristic way to construct a Lode dependence function. In the present model, the backward motion is assumed to create the same amount of damage as the forward motion [8]. Kinematically, a backward motion brings the current deformed configuration to the original configuration. Note that the backward motion should have the same pressure as the forward motion all the way along the loading path. Therefore, a simple compression is not the backward motion of a simple tension. However, a simple shear is the backward motion of the forward simple shear.

Based on the reverse hypothesis, the simplest Lode dependence model is a symmetric "*six-point star*" which just connect the alternating fracture strain in shear, tension and compression on the strain plane by straight lines [2].

We use the relative ratio of the stress deviators χ to denote the Lode angle, i.e.

$$\chi = \frac{s_2 - s_3}{s_1 - s_3} , \tag{7}$$

where s_1, s_2 and s_3 are the maximum, intermediate and minimum deviatoric stresses. For convenience, we define the ratio of the fracture strain in shear ($\chi = 0.5$) and in tension ($\chi = 0$) as a material parameter γ, i.e. $\gamma = \dfrac{\varepsilon_{\mathrm{f}}(\chi = 0.5)}{\varepsilon_{\mathrm{f}}(\chi = 0)}$. This parameter γ should be determined from a suitable set of tests.

Using the relative ratio of principal stress deviators, this Lode dependence function can be written as

$$\mu_\theta = \begin{cases} \dfrac{\sqrt{\chi^2 - \chi + 1}}{1 + \left(\dfrac{\sqrt{3}}{\gamma} - 2\right)\chi} & , \quad 0 \le \chi \le 0.5; \\ \dfrac{\sqrt{\chi^2 - \chi + 1}}{1 + \left(\dfrac{\sqrt{3}}{\gamma} - 2\right)(1 - \chi)} & , \quad 0.5 < \chi \le 1. \end{cases} \tag{8}$$

This function degenerates to a right hexagon when $\gamma = \dfrac{\sqrt{3}}{2}$. Another family of Lode dependence function is the power function, which can written as

$$\mu_\theta = \gamma + (1 - \gamma)\left(\frac{|\theta_L|}{\pi/6}\right)^\beta , \tag{9}$$

where $|\,.\,|$ denotes the absolute value, θ_L is the Lode angle and β is the Lode dependence exponent. This family of functions degenerate to a right circle when $\gamma = 1$. Equation (9) is more general than the one given by Eq. (8) because the two parameters can fit the shape of Lode dependence function more accurately.

The Lode dependence functions can be plotted in the strain plane as shown in Fig. 2.

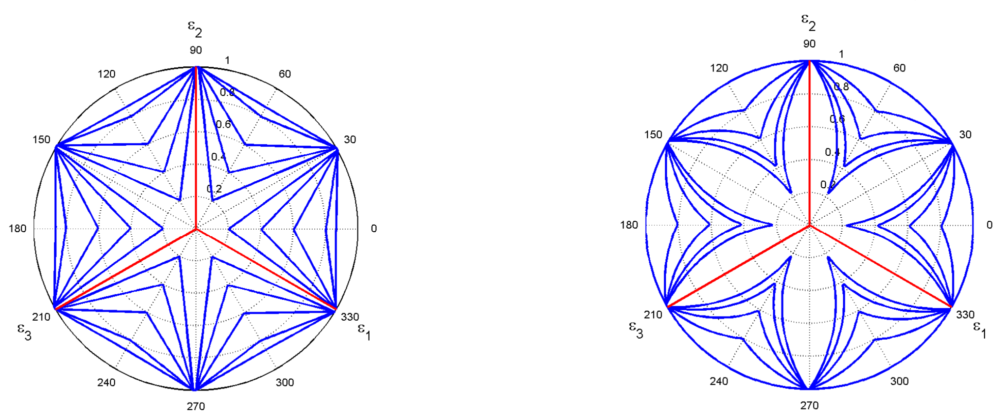


Figure 2. Two Lode dependence function for various parameter γ (Eq.(8) and Eq.(9) with $\beta = 1$).

Overview of existing models

By introducing an internal variable damage, the new constitutive model is complete. The onset of fracture is predicted by the integral of damage exceeding unity, meanwhile, the load carrying capacity drops to zero. This model is compatible with conventional plasticity theory. Four effects are included in the new model. The comparison of the present model with existing models is listed in Table 1.

Table 1. Comparison of existing models and the present model.

Models \ Effects	Pressure	Lode angle	Damage rule	Softening
Johnson-Cook [1]	●[1]		□	
Wilkins et al [11]	●	●	□	
Lemaitre [3]	●		●	●
Gurson [12]	●		●	●
Xue [2]	●	●	●	●

Numerical solutions

The present model has been implemented in LS-DYNA version 970 as a user defined material subroutine. The numerical scheme is presented in a separate paper [13]. We present here three examples to demonstrate the predictive power of the present model. This model is also used to predict the the shear crack in the transverse plane strain plate, the slant fracture of a tensile dog-bone specimen, the slant fracture of a compact tension specimen and similar crack propagation situations [2,14].

(1) Simple Tension of a round bar

Uniaxial tension of a round bar have been extensively studied due to the simplicity in geometry and in load condition. For many ductile materials, the experimental results show a cup-cone fracture pattern, in which the central portion of the crack is perpendicular to the load direction and a shear lip forms at the circumferential edge of outer surface at the neck. The fracture surface of the central portion of the cup is relatively rough which indicates void coalescence is important. On the contrary, the fracture surface of the shear lip is relatively smooth. The material is aluminum alloy 2024-T351, which has been tested by Bao and Wierzbicki [15] and calibrated by Xue [2]. The material constants are listed in Table 2. The geometry of the tensile round bar is shown in Fig. 3.

Table 2: Material constants for fracture characterization of 2024-T351 aluminum alloy.

Young's modulus E_0	Poisson ratio ν	ε_{f0}	p_{lim}	q	γ	m
70 GPa	0.3	0.70	925.7 MPa	0.97	0.296	2.04

[1] ● indicates a full dependence, • indicates a proportional linear relationship, which has some restriction. For Johnson-Cook model, the damage is assumed to be linearly related to the plastic strain on a proportional loading path. For Wilkins model, the damage is assumed to be linearly related to the proportional deviatoric loading path.

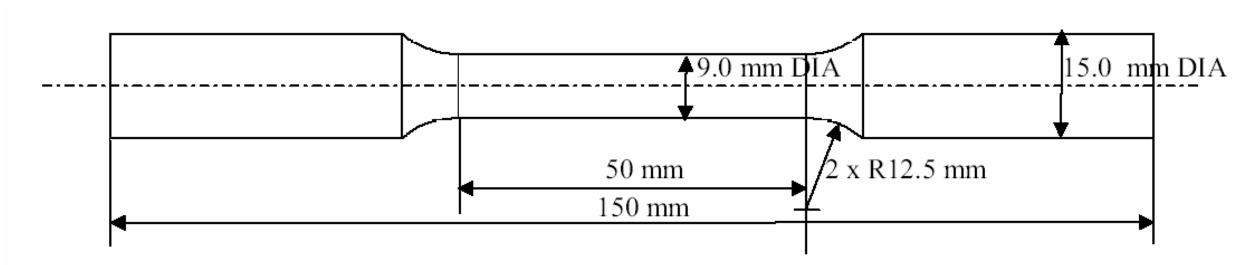


Figure 3. The geometry of a tensile round bar.

The simulation is performed and shows the realistic cup-cone fracture mode. Figure 4 shows the comparison with the experimental observations at various stages of deformation.

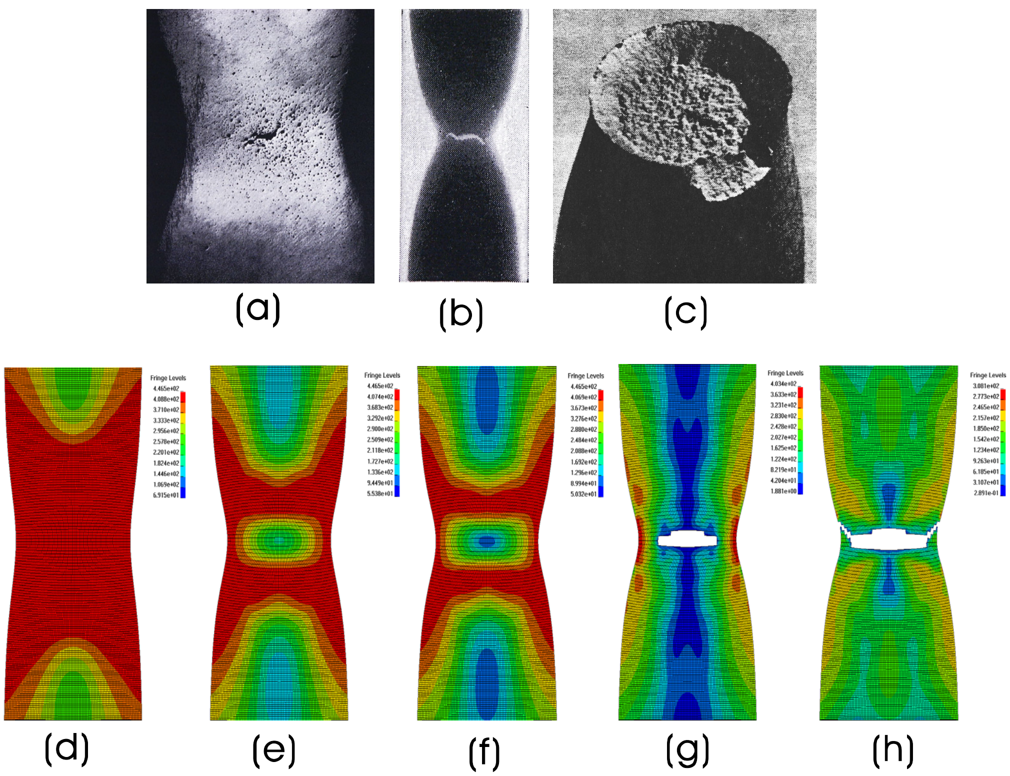


Figure 4. The microstructure and the corresponding equivalent stress contours at the neck of a tensile round bar at different stages of deformation showing the micro voids cluster (associated softening region), the center crack and the cup-cone fracture mode. (a) [16] ; (b) [17] ; (c) [18].

(2) Uniaxial tension of a hollow round bar

It is interest to investigate the uniaxial tension of a hollow round bar. In this example, a 4.5mm diameter hole is drilled in the center of the round bar. With the removal of material, the inner surface no long subjected to a radial stress after necking. The fracture point at the inner surface is now in the combination of an axial tension and a hoop stress. Therefore the symmetry of stress is broken. The experimental results show that the entire cross-sectional area was fractured in a shear mode as shown in Fig. 5.

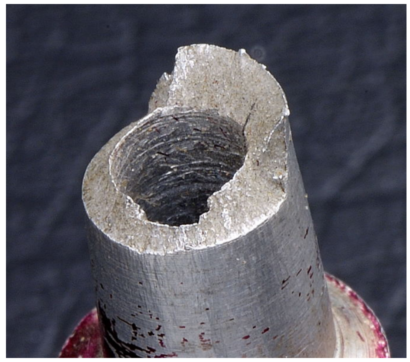


Figure 5. A hollow round bar fails in a shear mode. (Courtesy L. Anand [19].)

A shear fracture is observed in the numerical simulation as shown in Fig. 6.

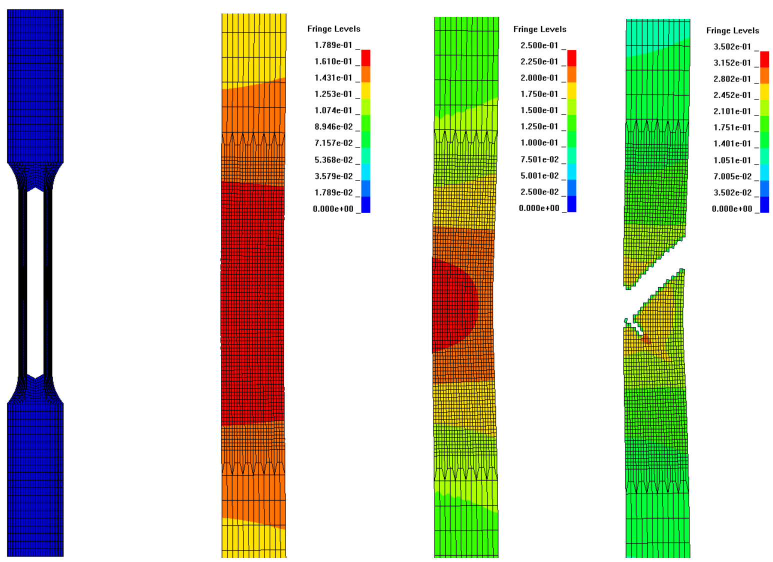


Figure 6. The original geometry and the damage accumulation process near the fracture site.

(3) Three-point bending of a narrow beam

The three-point bending of a narrow rectangular bar shows a complicated fracture mode. The material of the bar is 2024-T351 aluminum alloy. The fracture appears at the center of the opposite face of the central pin, which is subjected to tension. The fracture surface of central shear zone and the two shear lips that develops as the crack propagates toward the compressive side appears to be smooth, as shown in Fig. 7. The middle portion fracture surface is rough, which indicates a void nucleation and growth dominated fracture mode.

The numerical simulation results of the final fracture pattern of beam under three-point-bending are shown in Fig. 8. Shear lips are clearly visible, especially in the first one-third of the depth of the crack. Plotted are the plastic strain contour.

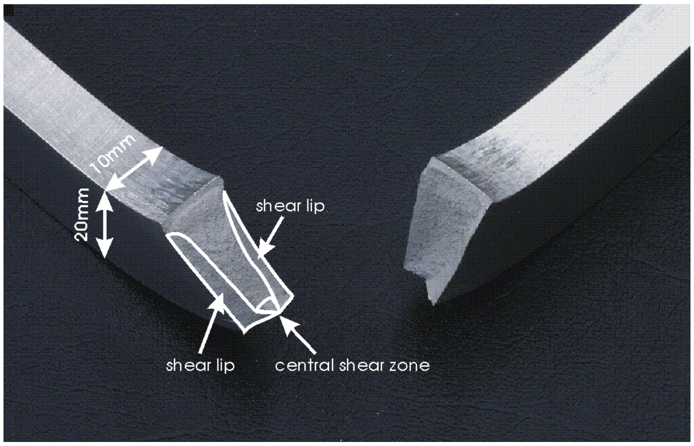
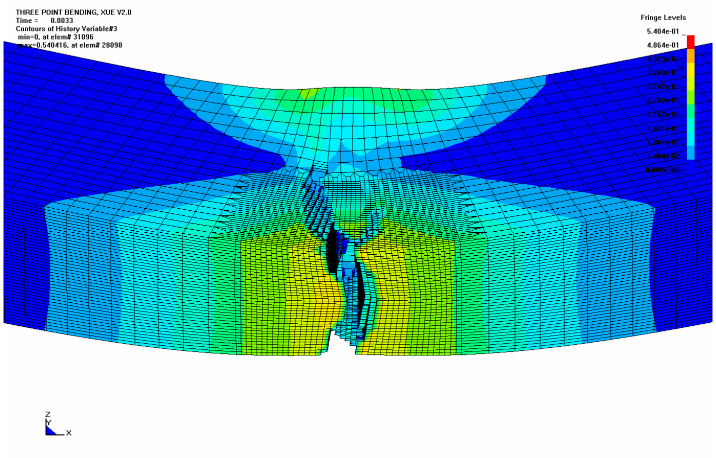
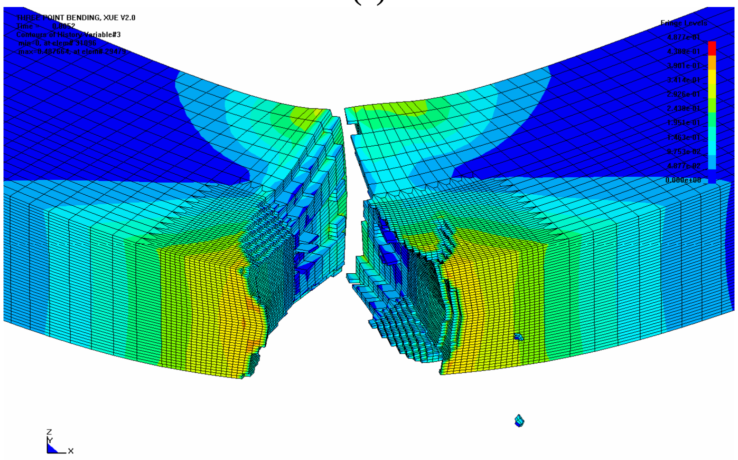


Figure 7. The fracture mode of three-point bending test of a rectangular bar.



(a)



(b)

Figure 8. The crack initiation and propagation of the rectangular bar subjected to three point bending.

Conclusions

A generic formulation an elasto-plastic constitutive model for ductile fracture was proposed in Ref. [2]. This model includes material deterioration, pressure sensitivity, Lode dependence and non-linear damage evolution law. The constitutive equations and the numerical integration scheme are presented. This model was implemented into LS-DYNA as a user subroutine. In the present paper, we focus on the crack initiation and propagation problems. These problems include a solid tensile round bar, a tensile hollow round bar and the three-point bending of a rectangular bar. The simulation results show good agreement with experiments. The cup-cone fracture mode is also predicted for the tensile round bar. The complex fracture model of the narrow rectangular bar in three point bending was predicted in a realistic way. A shear fracture is shown for the hollow round bar. These benchmark calculations demonstrate the predictive power of fracture of the newly developed model.

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