Application of Dynamic Relaxation in Thermo-Elastic Structural Analysis of Highway Pavement Structures

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Abstract

This paper describes the application of the dynamic relaxation technique implemented in LS-DYNA in analyzing large transportation structures as dowel jointed concrete pavements under the effect of temperature variations. The main feature of the pavement model is the detailed modeling of dowel bars and their interfaces with the surrounding concrete using extremely fine mesh of solid elements, while in the bridge structure, it is the detailed modeling of the girder-deck interface as well as the bracing members between the girders. The 3DFE results were found to be in a good agreement with experimentally measured data obtained from instrumented pavements sections constructed in West Virginia. Thus, such a technique provides a good tool for analyzing the response of large structures to static loads in a fraction of the time required by traditional implicit finite element methods.

Introduction

Modeling concrete structures is very difficult because of the material and geometrical nonlinearity involved in such structures. For example, when modeling a concrete slab of a highway section, the interface between consecutive slabs must be modeled. If the adjacent slabs are joined using steel dowel bars embedded within the slab, this will create nonlinearity within the concrete slab. The same is true for a concrete bridge deck including steel rebar reinforcements or the interfaces with the steel girders. In general, when modeling concrete structures, their nonlinearity creates a situation that must be accounted for when choosing a modeling technique.

A type of modeling focused on in this study involves the analysis of the interface between two connected concrete slabs joined by steel dowel bars, which has been modeled previously by various other researchers [1, 2]. Previous studies have made simplifications or assumptions by idealizing dowel bars as spring or beam elements. This approach ignores the dowel-concrete contact that leads to triaxial stresses near the edges of the slab [3]. Also, the contact interface introduces the axial dowel forces that will resist the concretes expansion and contraction due to temperature change that cause stresses to develop at the mid-slab [4]. In order to accurately model the stresses caused by the change in temperature, the contact interface between the concrete and the dowel must be accurately modeled.

Accurately modeling the concrete-dowel interface requires a fine mesh around the dowel bar to accurately simulate the circular shape of the dowel. In the model demonstrated in this paper, a mesh of approximately 273,000 elements is used. Solving this problem using traditional implicit techniques is not practical because the calculation and recalculation of the stiffness matrix alone would be too expensive in terms of computing time. However, when looking at the literature and studying other possible techniques, dynamic relaxation was determined to be the best technique to use for this study [5].

Dynamic relaxation (DR) is an explicit iterative process that can be used to obtain static solutions of structural mechanics problems. DR can calculate static solutions because it is based on the fact that a damped system being excited by a constant force undergoes vibrations, however; the system will ultimately come to a fixed displacement at the position of the systems static equilibrium [6]. Dynamic relaxation becomes an attractive technique because of its low storage requirements as well as its faster computing time compared with other implicit techniques. Also, another characteristic of the dynamic relaxation method is that it is very useful in solving problems that exhibit nonlinear geometric and material behavior. Actually, the main engineering use of dynamic relaxation is for problems with nonlinearity. In fact, Newton's methods with iterative techniques are more useful in obtaining solutions of linear structural mechanics problems.

This paper explores the use of the dynamic relaxation technique to investigate the effect of temperature variations on concrete structures. Two examples of 3DFE models built for newly constructed concrete pavement and Reinforced Concrete Bridge are presented. The results obtained from the 3DFE models are validated versus field measured data obtained from the instrumentation installed in such structures. The success of the dynamic relaxation technique is manifested by the good agreement between the 3DFE-calculated results and the field measurements.

Dynamic Relaxation Formulation in LS-DYNA

As stated before, DR is a technique that uses a dynamic analysis to produce a static solution. Since DR is an explicit dynamic analysis technique based on viewing the steady-state solution of the damped system as the static solution, the equation of motion governing dynamic response of a system can be the starting point in finding a solution:

$$\mathbf{M}\ddot{\mathbf{x}}^{n} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{P}(\mathbf{x}^{n}) = \mathbf{f}(\mathbf{t}^{n}) \tag{1}$$

where **M** is the mass matrix, **C** is the damping matrix, **f** is the vector of external forces, t is the time, n is the nth time increment, and the dots represent derivatives with respect to time. In this equation, $\mathbf{P}(\mathbf{x}^{n})$ is substituted for **Kx** where **K** is the stiffness matrix, **P** is the vector of internal forces, and **x** is the vector of dependent discrete variables (the solution vector) as stated by Underwood [18].

Using **z** as the fixed time increment and employing the central difference method, the following expressions can be stated:

$$\dot{\mathbf{x}}^{n-1/2} = (\mathbf{x}^{n} - \mathbf{x}^{n-1})/z \tag{2}$$

$$\ddot{\mathbf{x}}^{n} = (\dot{\mathbf{x}}^{n+1/2} - \dot{\mathbf{x}}^{n+1/2})/z \tag{3}$$

and the equality for $\dot{\mathbf{x}}^{n}$ can be obtained using the average value

$$\dot{\mathbf{x}}^{n} = \frac{1}{2}(\dot{\mathbf{x}}^{n+1/2} + \dot{\mathbf{x}}^{n-1/2}) \tag{4}$$

By substituting Eqs. (2)-(4) into Eq. (1), the future values for displacement and velocity can be found:

$$\dot{\mathbf{x}}^{n+1/2} = \left(\frac{\mathbf{M}/z - 1/2\,\mathbf{C}}{\mathbf{M}/z + 1/2\,\mathbf{C}}\right)\dot{\mathbf{x}}^{n-1/2} + \frac{\left(\mathbf{f}(t^n) - \mathbf{P}(\mathbf{x}^n)\right)}{\left(\mathbf{M}/z + 1/2\,\mathbf{C}\right)} \tag{5}$$

$$\mathbf{x}^{n+1} = \mathbf{x}^n + z\dot{\mathbf{x}}^{n+1/2} \tag{6}$$

In order to maintain the explicit form of the central difference integrator, **M** and **C** must be diagonal matrices. In the dynamic relaxation technique, **C** can be written as:

$$\mathbf{C} = c\mathbf{M} \tag{7}$$

where c is a scalar quantity. Substituting this value for **C** into Eqs. (5)-(6)

$$\dot{\mathbf{x}}^{n+1/2} = \left(\frac{2-cz}{2+cz}\right)\dot{\mathbf{x}}^{n-1/2} + 2z\mathbf{M}^{-1}\left(\frac{\mathbf{f}^n - \mathbf{P}^n}{2+cz}\right) \tag{8}$$

$$\mathbf{x}^{n+1} = \mathbf{x}^n + z\dot{\mathbf{x}}^{n+1/2} \tag{9}$$

where $\mathbf{f}^n = \mathbf{f}(t^n)$ and $\mathbf{P}^n = \mathbf{P}(\mathbf{x}^n)$ for simplicity. $\mathbf{M}^{-1}$ is the inverse of **M**. Since matrix **M** is a diagonal matrix, we can write Eqs. (8)-(9) in terms of each of the individual components of the vectors or matrices.

$$\dot{x}_i^{n+1/2} = \left(\frac{2-cz}{2+cz}\right)\dot{x}_i^{n-1/2} + 2zm_{ii}\left(\frac{f_i^n - P_i^n}{2+cz}\right) \tag{10}$$

$$x_i^{n+1} = x^n + z\dot{x}^{n+1/2} \tag{11}$$

Where the subscript i indicates the i^{th} component in the vector and m_{ii} corresponds to the i^{th} diagonal element of $\mathbf{M}^{-1}$.

The integration cannot be started by using Eqs. (8) and (10)-(11) because they require the velocities to be known at $t^{-1/2}$, which they cannot be. However, the velocity is known at t^0. Since $\dot{\mathbf{x}}^0$ must be zero for a static solution, the starting condition of the DR procedure can be assumed to be:

$$\mathbf{x}^0 = 0;\ \ \dot{\mathbf{x}}^0 = 0 \tag{12}$$

and by using Equation (4) and the second equation in Eq. (12), another equality can be deduced:

$$\dot{\mathbf{x}}^{-1/2} = -\dot{\mathbf{x}}^{1/2} \tag{13}$$

By combining Eqs. (10)-(11) and (13), an expression can is obtained for the velocity after the first time increment:

$$\dot{\mathbf{x}}^{1/2} = z\mathbf{M}^{-1}(\mathbf{f}^0 - \mathbf{P}^0)/2 \tag{14}$$

Combining Eqs. (8)-(9) and (14) the following set of iterative equations is developed that can be used to obtain converged values for the velocity and the displacement of a system:

$$\dot{\mathbf{x}}^{n+1/2} = \left(\frac{2-cz}{2+cz}\right)\dot{\mathbf{x}}^{n-1/2} + 2z\mathbf{M}^{-1}\left(\frac{\mathbf{f}^n - \mathbf{P}^n}{2+cz}\right) \tag{15}$$

$$\mathbf{x}^{n+1} = \mathbf{x}^n + z\dot{\mathbf{x}}^{n+1/2} \tag{16}$$

$$\dot{\mathbf{x}}^{1/2} = z\mathbf{M}^{-1}(\mathbf{f}^0 - \mathbf{P}^0)/2 \tag{17}$$

With iterative processes, there are parameters that must be chosen that allow the solution to either converge slowly or quickly. These parameters are either not known, and an estimated value is chosen and verified with convergence, or they can be varied for sake of speeding up convergence. For DR, the damping coefficient *c*, the time increment *z*, and the mass matrix **M** are all values that could be varied in the analysis. However, for a specific finite element mesh, the mass matrix is known and the time increment is specified, so the damping coefficient is the variable that must be optimized to obtain convergence in minimal time. This procedure can be followed in Hallquist [7]. Figure 1 illustrates the effect of the damping ratio on the convergence of the dynamic solution compared to the static solution for a single degree of freedom system. The plots in Figure 1 indicate that the DR technique acts as a critically damped system.

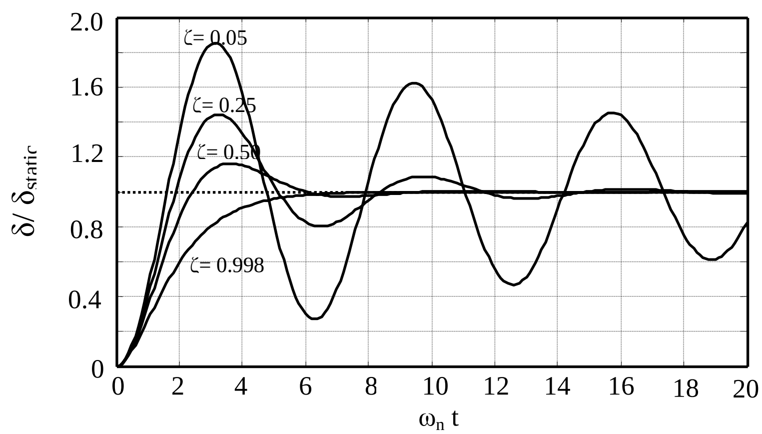

Figure 1. Displacement-Time Histories for Different Damping Ratios.

Response of Concrete Pavement to Temperature Variations

The 3D finite element model developed to simulate this pavement section consists of one full length slab having a half slab before and after it in the direction of traffic supported by both a base and a subgrade as shown in Figure 2. To allow for contraction and expansion of the slabs and to ensure that the dowel bars are the only means of load transfer between the slabs, a 10 mm gap was assumed between the two slabs. In order to avoid problems that may arise with boundary conditions, the full width of the concrete slab was modeled as 3.66 m. Also, the base and subgrade were widened 0.61 m on each side of the slabs. The contact between the base and the slab is maintained by the self-weight of the slabs and no other external constraints are applied. The subgrade depth is chosen as 1.92 m and non-reflective boundaries are applied to the bottom of the model along with the sides of the base and subgrade layers. The non-reflective boundaries simulate the semi-infinite extent of the subgrade by allowing the propagating stress waves to pass through the edges. A mesh of 8-node solid brick elements is used throughout the model. However, a finer mesh is used when modeling the dowel bars as well as the concrete around the dowel bars and the subgrade layers in close proximity to the dowel joint. This fine mesh allows modeling of the circular cross section of the dowel bar which is integral in order to account for dowel contact as well as the associated states of stress that develop surrounding the dowels due to traffic and/or thermal loading. The concrete elastic modulus was estimated based on the measured 28-day compressive strength f_c using the ACI relation ($E_c = 57{,}000\ f_c^{0.5}$). The elastic moduli of the base and subgrade layers were backcalculated from several Falling Weight

Deflectometer tests conducted months after construction. The model consists of 325,018 nodes and 272,992 elements with 1,637,952 degrees of freedom.
The loading placed upon this model is a temperature gradient profile that is applied through the

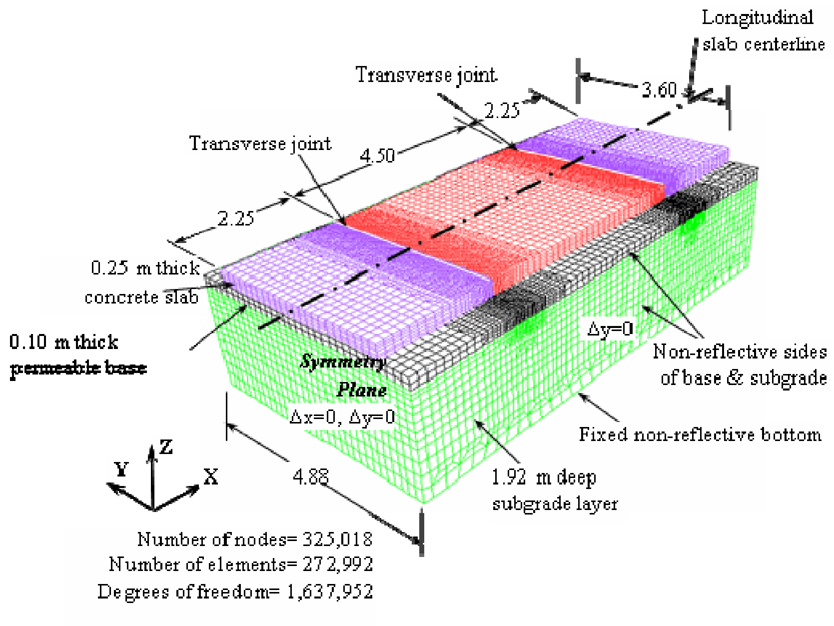

Figure 2 Finite element model of the concrete pavement

thickness of the concrete slab. The gravity was activated in the model by specifying it as base acceleration acting in the vertical direction. A data file containing the distorted model and the built up stresses becomes the starting file to which any magnitudes and configurations of traffic load and/or temperature gradient profile are applied prior to its reprocessing in a static or dynamic mode.

This model is an excellent example of when it is beneficial to apply the dynamic relaxation technique to obtain a static solution for the following reasons:

- The model consists of four different materials and a major area of concern for this study was the state of triaxial stress at the interface between the dowel bar and concrete. This clearly shows that the problem this study was concerned with is nonlinear in nature.
- The model consists of a high number of nodes and a high number of elements. Furthermore, because of the need for very accurate modeling at the dowel-concrete interface, there are a large number of very small elements present in the model. It can clearly be seen that the dynamic relaxation technique is more efficient to use than the iterative techniques to solve this problem because the storage requirements and computing time required for the iterative techniques would be much higher than that required for the dynamic relaxation technique.

From the instrumented highway section, temperature variation through the concrete slab thickness and the change in this variation can be measured. These values are used to calculate temperature gradients and temperature change profiles. The change in the measured temperature gradient profile through the slab thickness over a short period of time namely 6-10 hours was calculated. The 3DFE model was processed for different temperature gradients ranging from –8 °C to +8 °C; each is accompanied by a corresponding uniform temperature variation calculated using Equation 18. This temperature change is applied to the nodal points of the concrete slab, and the model is processed using LS-DYNA equation solver. The 3DFE-computed strains are found for all points in the model that correspond to the location of strain gages in the instrumented slab. The measured and 3DFE-computed longitudinal strains at slab top and bottom are shown in Figure 3. The comparison indicates an excellent agreement between the measured and 3DFE-calcuted strains. The differences observed for the bottom strains can be attributed to the built in construction curling induced in the slab at the early age and not accounted for in the 3DFE model, which influenced the contact area between the slab and the base layer. In the 3DFE model, the slab was assumed to be initially flat and in full contact with the underlying base layer.

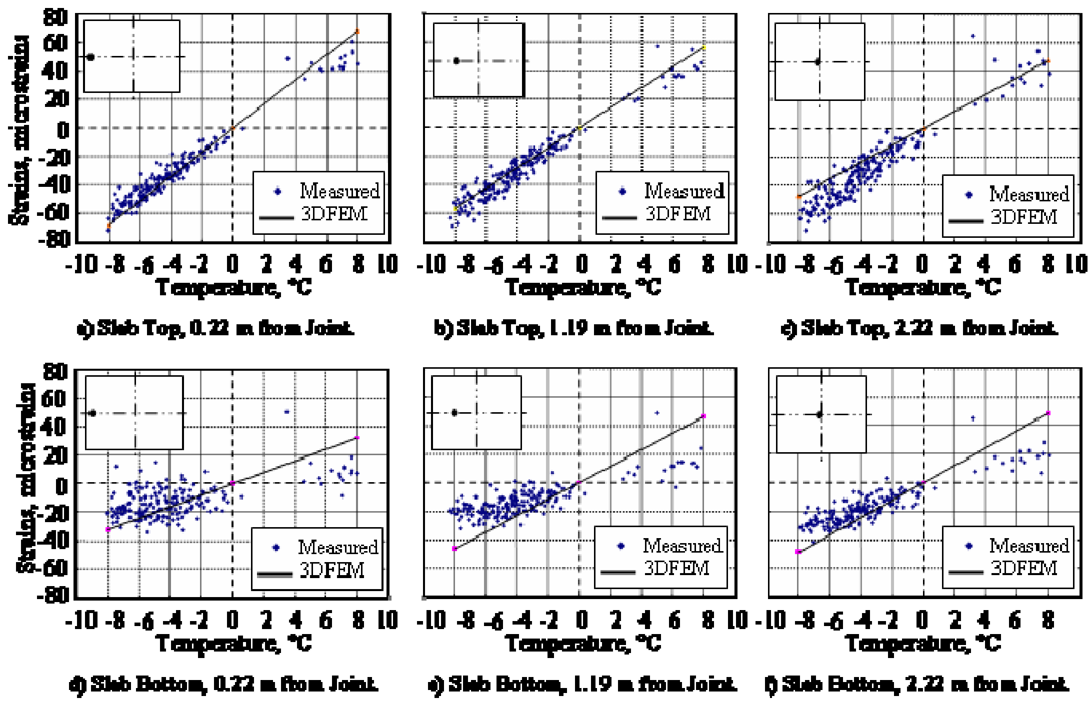

Figure 3. 3DFE-Calculated versus Measured Longitudinal Strains.

Conclusions

The study presented in this paper presents the LS-DYNA dynamic relaxation as an effective method for computing static solutions for large structures. Not only is this technique acceptable, but it is also efficient in solving highly nonlinear problems. To exhibit the superiority of DR in solving highly nonlinear structural problems, the longitudinal and transverse strains throughout a concrete slab due to temperature loading were compared from DR results obtained through FE analysis and experimentally obtained values from an instrumented highway section. The results show the effectiveness and versatility of the dynamic relaxation technique in modeling problems with intricate FE meshes along with a high degree of nonlinearity.

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References

[1] C. Channakeshava, F. Barzegar, and G.Z. Voyiajis, Nonlinear FE Analysis of Plain Concrete Pavements with Doweled Joints, Journal of Transportation Engineering, ASCE, vol. 119, no. 5, pp.763-781, 1993.

[2] W.G. Davids, 3D Finite Element Study on Load Transfer at Doweled Joints in Flat and Curled Rigid Pavements, The International Journal of Geomechanics, vol. 1, no. 2, pp. 309-323, 2001.

[3] S.N. Shoukry, G.W. William, and M. Riad, Characteristics of Concrete Contact Stresses in Doweled Transverse Joints, The International Journal of Pavement Engineering, vol. 3, no. 2, pp. 117-129, 2002.

[4] G.W. William and S.N. Shoukry, 3D Finite Element Analysis of Temperature Induced Stresses in Dowel Jointed Concrete Pavements, The International Journal of Geomechanics, ASCE, vol. 1, no. 3, pp. 291-307, 2001.

[5] T. Belytschko, W.K. Liu, and B. Moran, Nonlinear Finite Element for Continua and structures, John Wiley and Sons, LTD, West Sussex, England, 2000.

[6] M. Papadrakakis, A Method for the Automatic Evaluation of the Dynamic Relaxation Parameters, Computer Methods in Applied Mechanics and Engineering, vol. 25, pp. 35-48, 1981.

[7] J. Hallquist, LS-DYNA Theoretical Manual, Livermore Software Technology Corporation, Livermore, California, 1998.