

# Mixed Mode Constitutive Driver

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## Abstract

*The mixed-mode constitutive driver is a software package that is dedicated to the efficient development, evaluation, and parameter identification (fitting) of material models used in finite element codes. The core driver calculates the stress-strain behavior of material models driven by combinations of strain increments and stress boundary conditions. Graphical user interfaces facilitate selection of the material model constants and desired load histories, and plot model output in two (stress-strain curves compared with test data) and three (yield surfaces) dimensions. Optimization routines fit the material models to test data. Optimization is accomplished by interfacing the driver with the LS-OPT code. The driver complements the performance of finite element codes. Its intended use is to help analysts efficiently fit and evaluate material models, with consistent results, prior to performing large-scale finite element analyses.*

## Introduction

The roadside safety community supplements crash test data with LS-DYNA simulations [LSTC, 2005]. The accuracy of such computer impact simulations depends upon a number of factors, such as the formulation (theory and coding) of the material models used to define the roadside structures and vehicles materials, the ability of each analyst to fit the appropriate material models to basic material property data, and the availability of material property data. Fitting material models is a time consuming and sometimes iterative process. The accuracy of each fit depends largely on the analyst's experience and judgment.

The Mixed Mode Constitutive Drive (MMCD) is a graphical software package that helps the user efficiently evaluate and fit material models to test data. The MMCD calculates the stress-strain behavior of material models for a variety of loading conditions, such as pure shear stress, and combinations of uniaxial, biaxial, and triaxial compression and tension. Material response may include elastic, plastic, damage, and high strain rate behaviors.

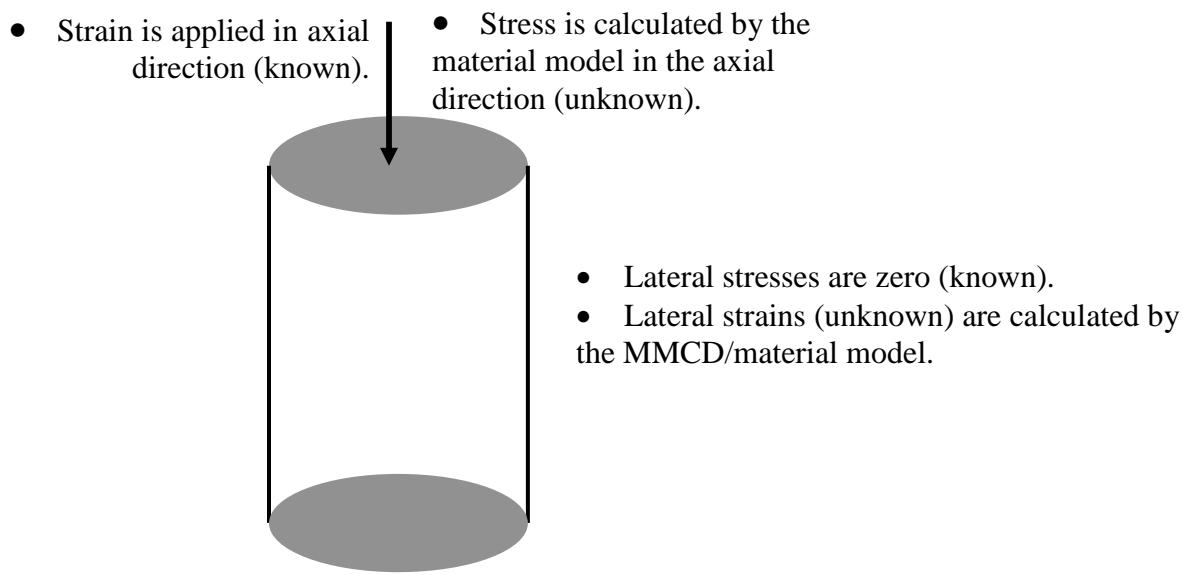
The main attributes and functions of the MMCD include:

- Web downloadable software package (pre-processor, driver, post-processor) that operates seamlessly as a single, easy to use code via a graphical user interface (GUI).
- A library of select LS-DYNA material models.
- An interface for user-defined material models.
- A library of predefined load histories that simulate common laboratory tests.
- An optional method for inputting user-specified load histories.

- A database of experimental results for a variety of materials, such as concrete, wood, and soil.
- A read curve capability for incorporating user-specific experimental data. This feature is useful for plotting new data or for data that is proprietary.
- An automated procedure for fitting each material model to experimental data by interfacing the MMCD with the LS-OPT code [Stander *et al.*, 2004].
- The capability to plot 2D curves (stress, strain, principal stresses and strains, and invariants) with and without experimental data.
- The capability to plot 3D yield surfaces, and to rotate and translate those surfaces about various axes.

### Driver Theory

The basic objective of the driver is to update all stress and strain components for a loaded material given a mixture of known strain increments and known stress boundary conditions. The mixture depends upon the loading condition selected. As an example, consider simulation of a material in unconfined compression (uniaxial stress), as shown in Figure 1. The applied axial strains and zero lateral stresses are known, while the axial stress and lateral strains are unknown. The lateral strains depend upon Poisson's ratio in the elastic regime, and the flow rule in the plastic regime. In addition, all shear strains are known to be zero, which results in zero shear stress in an isotropic material.



**Figure 1. Schematic demonstration of known and unknown strain and stress components for an unconfined compression simulation.**

The driver uses an iterative procedure to update the stress-strain response of the material model following the procedure and notation of Hansen [2000]. The driver partitions the stress and strain components into known and unknown components. The unknown (free) strains ( $\varepsilon_f$ ) and unknown (constrained) stresses ( $\sigma_c$ ) are determined from the known (constrained) strains ( $\varepsilon_c$ ) and the known (free) stresses ( $\sigma_f$ ) through the partitioning of the incremental elastic constitutive matrix, as follows:

$$\begin{matrix} \text{Known} \\ \text{Unknown} \end{matrix} \begin{bmatrix} \Delta\sigma_f \\ \Delta\sigma_c \end{bmatrix} = \begin{bmatrix} E_{ff} & E_{fc} \\ E_{cf} & E_{cc} \end{bmatrix} \begin{matrix} \Delta\varepsilon_f \\ \Delta\varepsilon_c \end{matrix} \begin{matrix} \text{Unknown} \\ \text{Known (applied)} \end{matrix} \quad (1)$$

Here,  $E_{ij}$  are the partitioned pieces of the elastic constitutive matrix  $E$ .

The procedure follows three steps. The first step is estimation of the trial strain increment vector  $\Delta\varepsilon_f$  from Equation 1.

$$\Delta\varepsilon_f = E_{ff}^{-1} : (\Delta\sigma_f - E_{fc} : \Delta\varepsilon_c) \quad (2)$$

The second step is the equilibrium check. The trial elastic strain increments are passed through to the material model, which updates all stress components. The *trial* free stress components are compared with their *known* free values to see if they are equal (within a small tolerance). Equilibrium is automatically achieved by the partitioning scheme with convergence in one iteration if the trial stress state remains elastic within the material model. However, the residual stress difference will not be zero on the first iteration once the trial stress state enters the plastic loading regime. Equilibrium must be enforced via multiple iterations.

The third step is the iterative loop. Residual strain increments  $\Delta r$  are estimated from the residual stress difference  $\Delta R$ :

$$\Delta r = E_{ff}^{-1} : (\Delta R - E_{fc} : \Delta\varepsilon_c) \quad (3)$$

The residual strain increments at equilibrium iteration  $i$  are added to the previously calculated free strain increments to update the free strain increments at equilibrium iteration  $i+1$ .

## Material Models and Load Definitions

Three material models are currently implemented with two more under development:

- Wood model 143 called MAT\_WOOD [Murray, 2005] [Murray& Reid, 2005].
- Soil model 147 called MAT\_FHWA\_SOIL [Lewis, 2004] [Lewis& Reid, 2004].
- Concrete model 159 called MAT\_CSCM for Continuous Surface Cap Model [Murray, 2005] [Murray, Abu-Odeh, & Bligh, 2005].
- Model 24 called MAT\_PIECEWISE\_LINEAR\_PLASTICITY, which is commonly used for steel.
- A user defined material model for proprietary models and models under development.

The driver calls the material model subroutines in the same form as implemented in LS-DYNA, without alteration.

The user must define the load to calculate the material response. Default load definitions currently implemented generate loading histories for:

- Uniaxial stress.
- Biaxial stress.
- Triaxial stress.
- Simple shear.
- Pure shear.
- Uniaxial strain.
- Triaxial compression or extension.

These loading histories are commonly used to evaluate material models and to test material specimens. The user specifies the appropriate strain increments, total number of increments, a non-zero strain rate (if desired), and an option to unload and reload (cyclic loading). User-specified load definitions may also be input in addition to the default load definitions. User-specified loads may take the form of load increments, strain and stress histories, or velocity histories.

An example material model GUI for concrete is shown in Figure 2. The material response calculated with the MMCD for these concrete properties is identical to that calculated with LS-DYNA using a single element, as shown in Figure 3. Example calculations are shown in Figures 4 through 7, which demonstrate the load/unload/reload and user-defined load increment capabilities.

## Two-Dimensional Plots

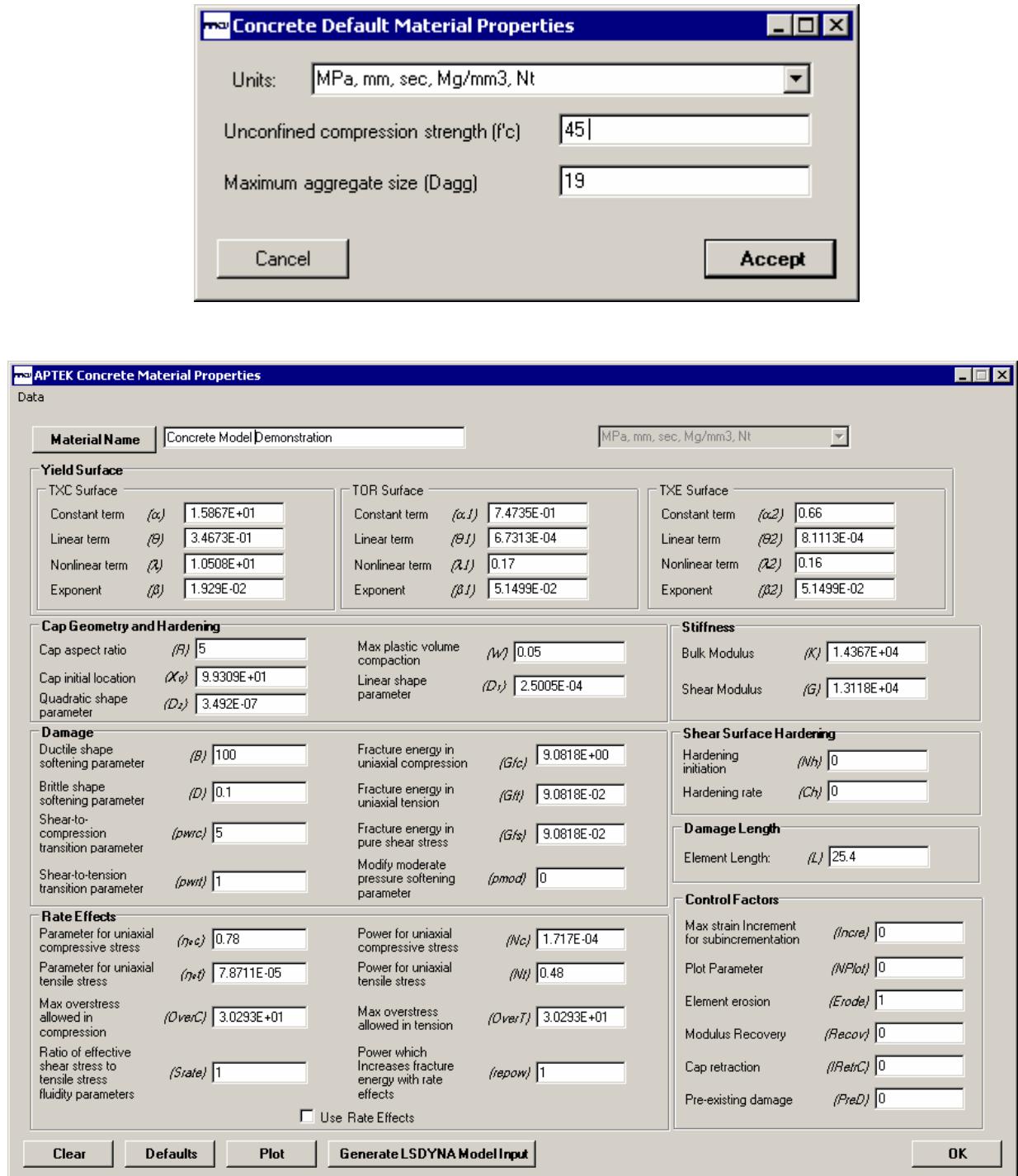
An example 2D plot database is shown in Figure 8. The direct output from driving each material model is a set of six stress histories and six strain histories stored as a function of load increment number. The user may plot any one of these components versus any other component, *e.g.* stress versus strain, stress versus stress, or strain versus strain. Numerous derived quantities are also available for plotting. These include:

- Principal stresses and strains.
- Stress and strain invariants.

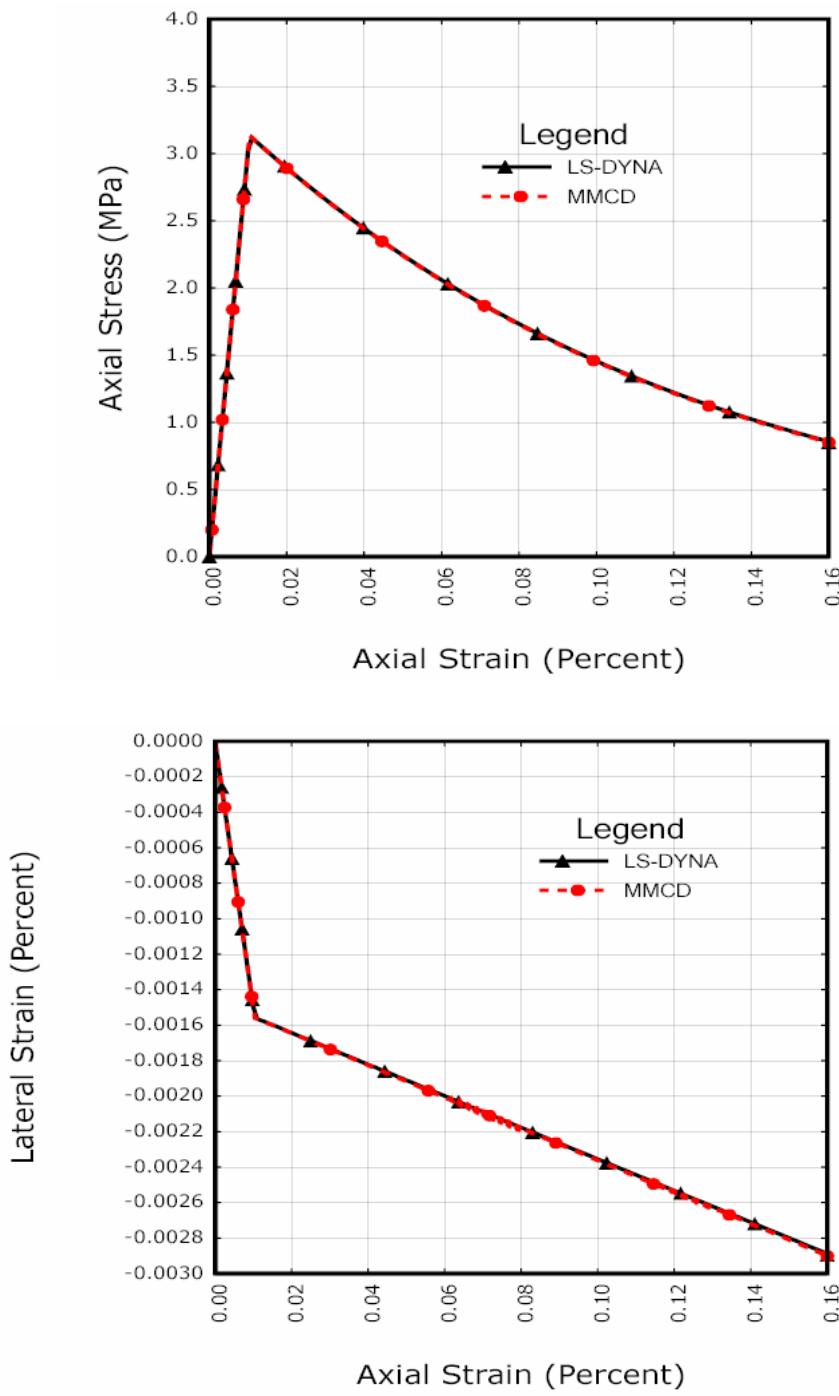
Once the user selects the *x* and *y*-components to be plotted, they are entered into the database of plots as a curve. Additional curves (data, calculations) may be read directly into the database for comparison with the MMCD computed results. Multiple curves can be combined and displayed as a single Graph. The user may generate a report quality plot by specifying the title, subtitle, axes labels, scale factors, curve colors, line and symbol types, and by interactive positioning of the legend. For each axis the user can choose between logarithmic or linear scale and specify whether tics or grids will be displayed. Notes can be attached to and displayed with a curve for further documentation. The user may also manipulate one or more curves via the database curve manipulations option. These include:

- *Cross Plots.* Plots the *x* or *y*-component of one curve versus the *x* or *y*-component of a second curve.
- *Integration.* Integrates the *y*-component as a function of the *x*-component over a user-specified range in *x*-values.
- *Shift.* Shifts the *x* and *y*-components by user-specified values.

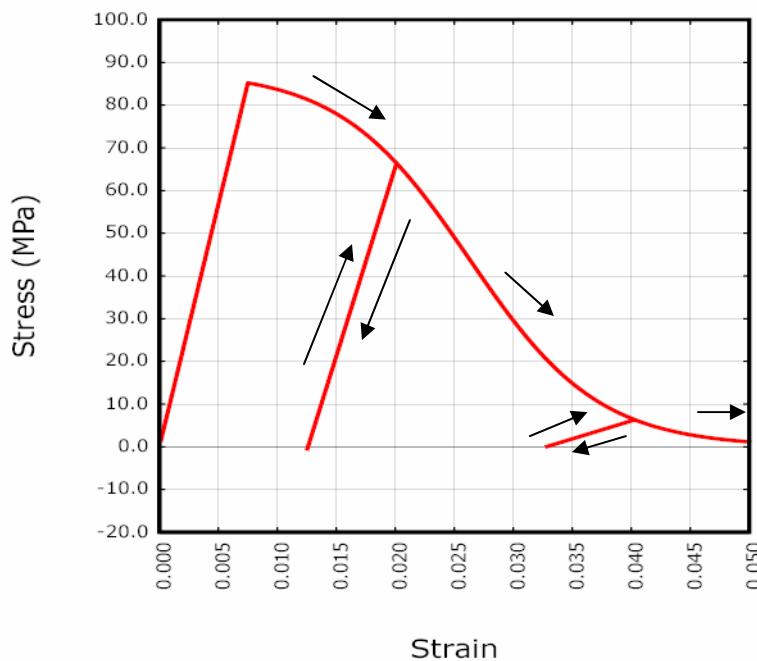
- *Subtract*. Takes the difference between two curves for either the  $x$ -values or  $y$ -values.
- *Swap*. Swaps the  $x$  and  $y$ -components with each other.



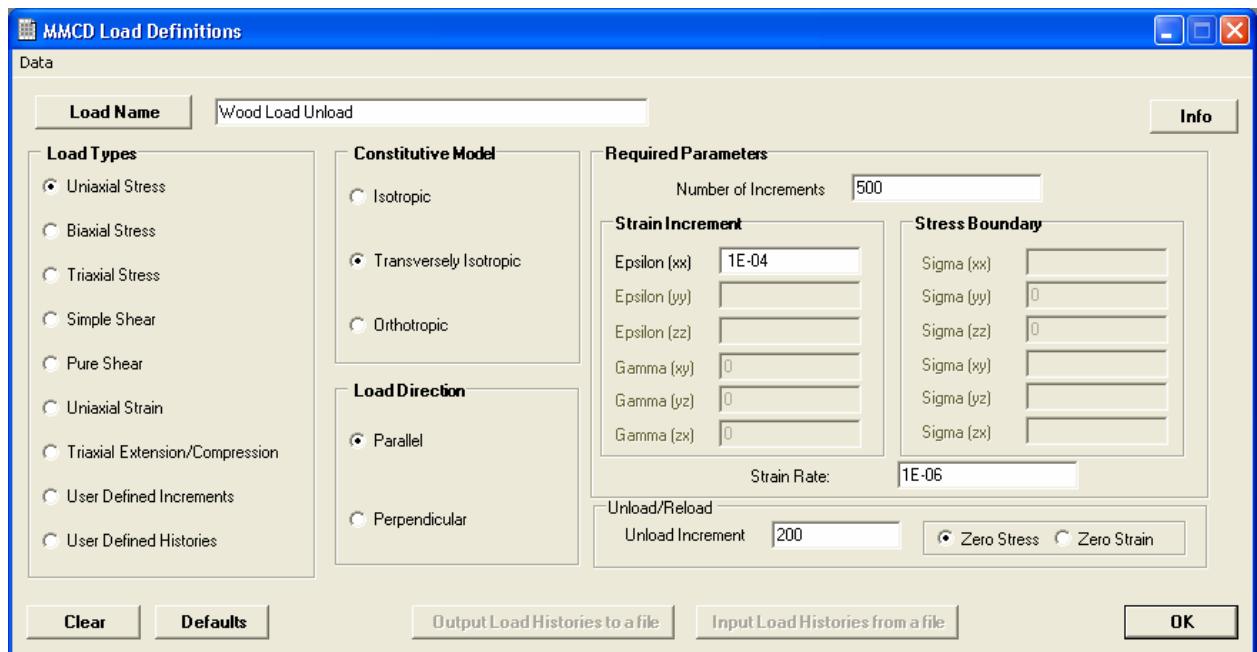
**Figure 2. Example material model GUI for concrete model 159 with default material properties and units.**



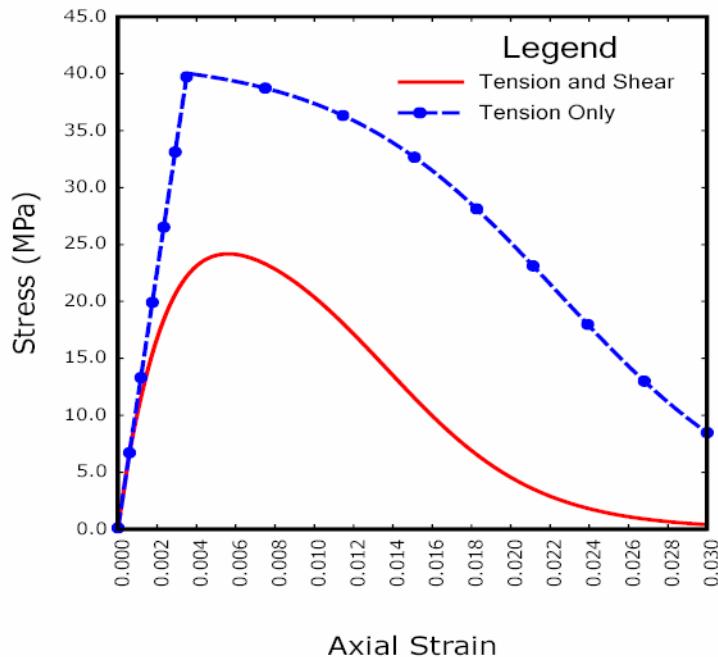
**Figure 3.** The driver accurately simulates the stress-strain response of concrete model 159 in direct pull (uniaxial stress in tension).



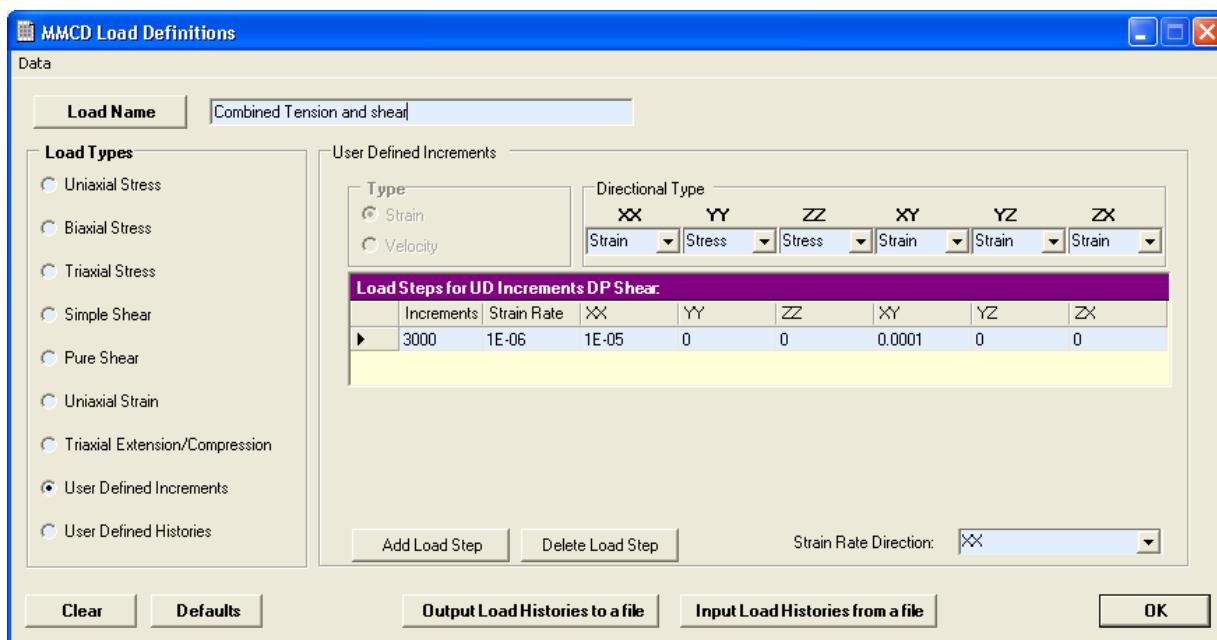
**Figure 4.** The MMCD uses internal feedback on the stress to load, unload to zero stress, and then reload (wood model 143 run parallel to the wood grain).



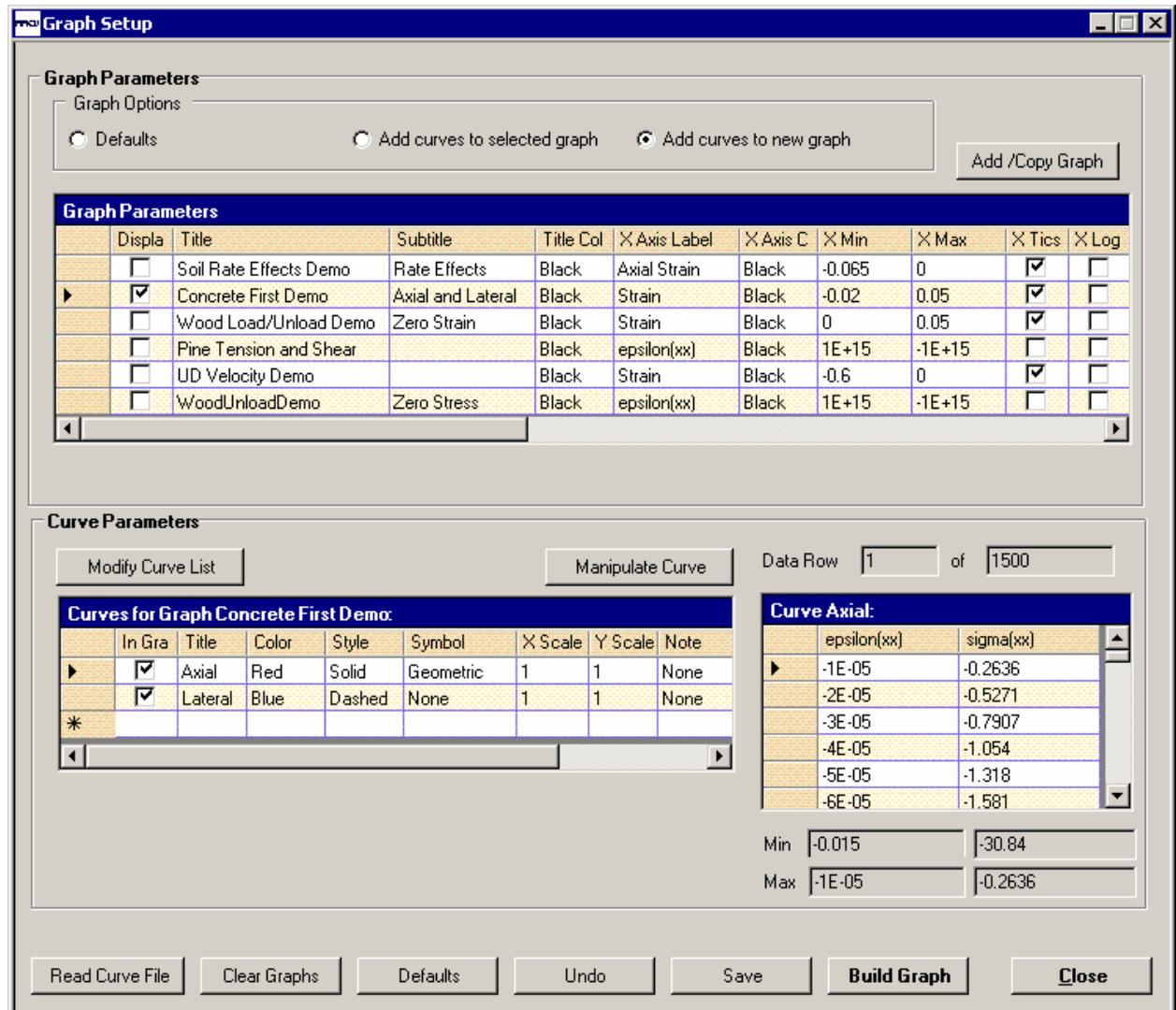
**Figure 5.** Example GUI demonstrating input for uniaxial stress (predefined load) with the unload/reload increment option selected.



**Figure 6. Demonstration of combined tension and shear loading of wood model 143 using the user-defined increments definition.**



**Figure 7. Example user defined increments definition for combined tension and shear loading.**



**Figure 8. Example database for making and storing two-dimensional plots of computed results versus test data.**

### Fitting a Material Model to Test Data

The process of fitting constitutive material model parameters to test data is typically called *parameter identification*. Two methods are available: manual and automated. In the manual method, the user selects the material properties and load definition and then runs the simulation, as shown in Figure 9. Then the user graphically compares the simulated results with test data. This procedure is repeated iteratively until the user decides that the model fits the test data via the visual comparison, as shown in Figure 10. The automated method uses optimization routines and is accomplished by interfacing the MMCD with the LS-OPT executable [Stander, 2005]. Iteration proceeds without intervention by the user until a specified number of iterations are attained or tolerances are achieved for goodness of fit. An example optimization GUI is shown in Figure 11.

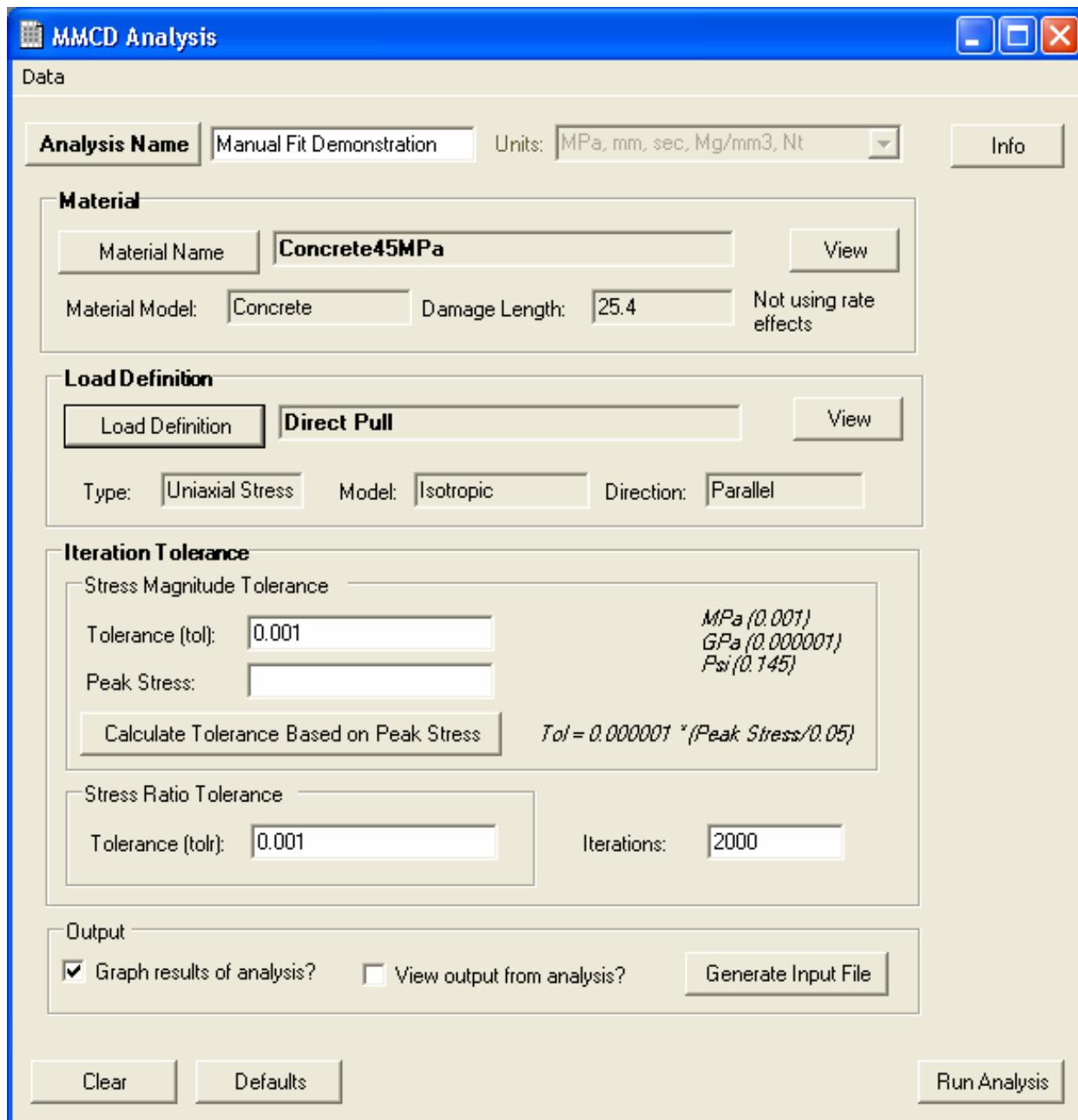
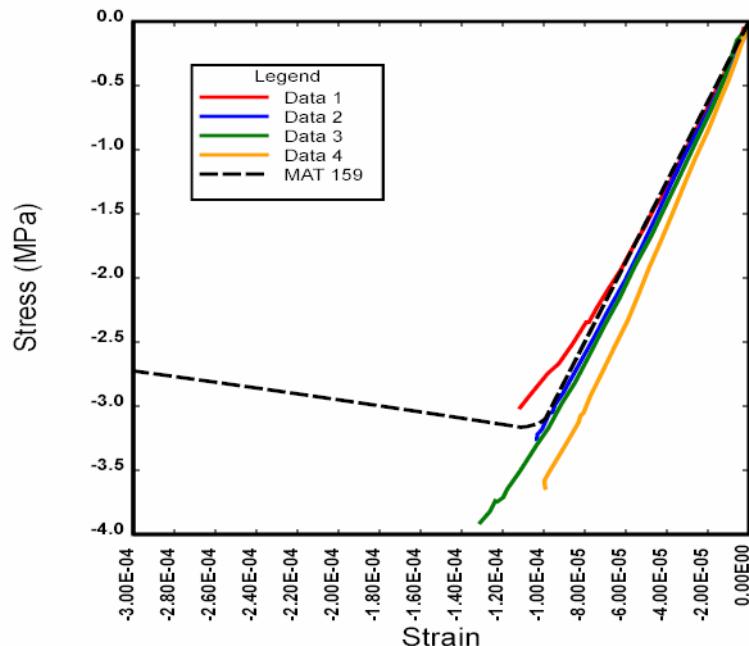
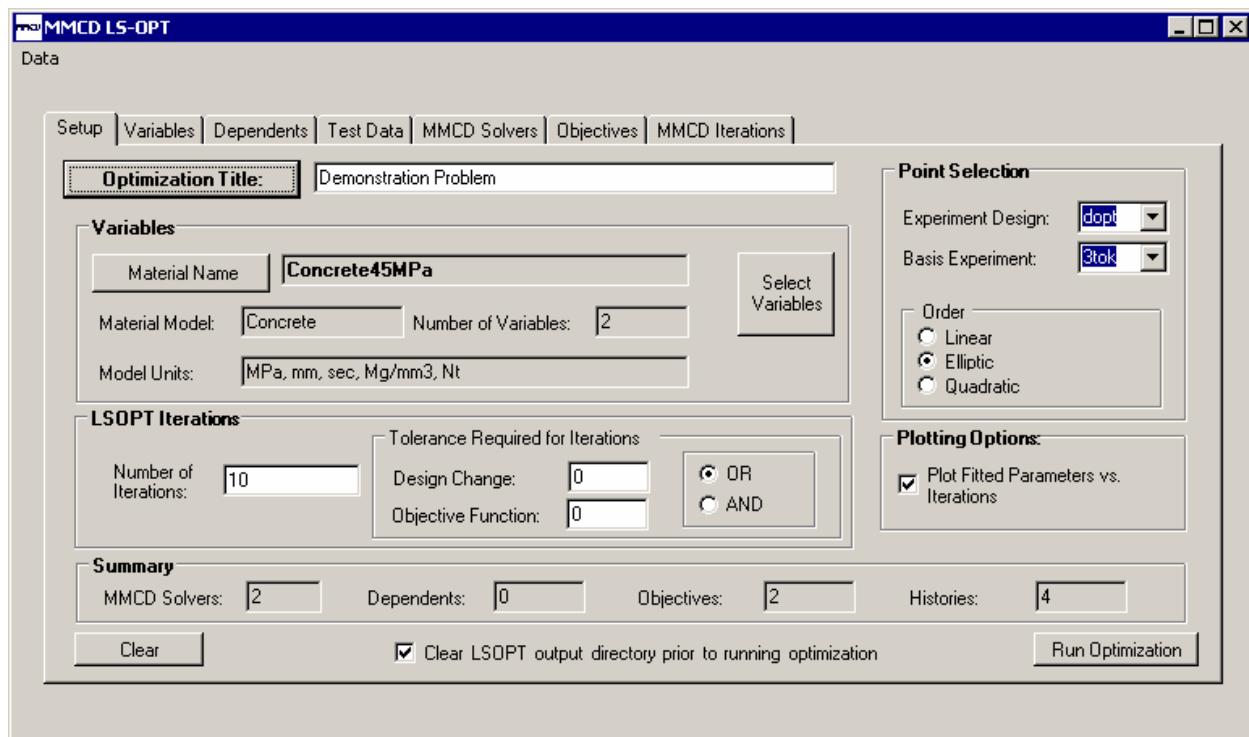


Figure 9. Example GUI for running an MMCD simulation.



**Figure 10.** Example manual fit of model to concrete data in uniaxial tensile stress (plotted negative in tension).



**Figure 11.** Example GUI for interfacing LS-OPT with the MMCD.

The basic automated fitting procedure is to use the MMCD GUI to select the material model parameters that the user wants to fit to test data, define a range in values for these parameters, and then perform numerous MMCD calculations over the range in values. LS-OPT creates a multi-dimensional surface of the computed response versus parameter values. Then LS-OPT finds the parameter values that minimize the difference between the computed response surface and test data point using an iterative optimization routine. Multiple response surfaces are created, and the differences are summed up, if multiple data points are analyzed.

LS-OPT has many features that are not specifically needed for parameter identification. Therefore, the MMCD GUI prompts the user for pertinent information only, as follows:

- Analysis Name.
- Material Model.
- Variables (to be fit and their ranges).
- Dependents.
- Test Data.
- MMCD Solvers (load histories to be run).
- Point Selection (The *D-optimal* method determines the number of calculations performed).
- Objectives (comparison of computed results with test data using the mean squared error).
- Iterations and Tolerances (for both the MMCD and LS-OPT).

## Three-Dimensional Surfaces

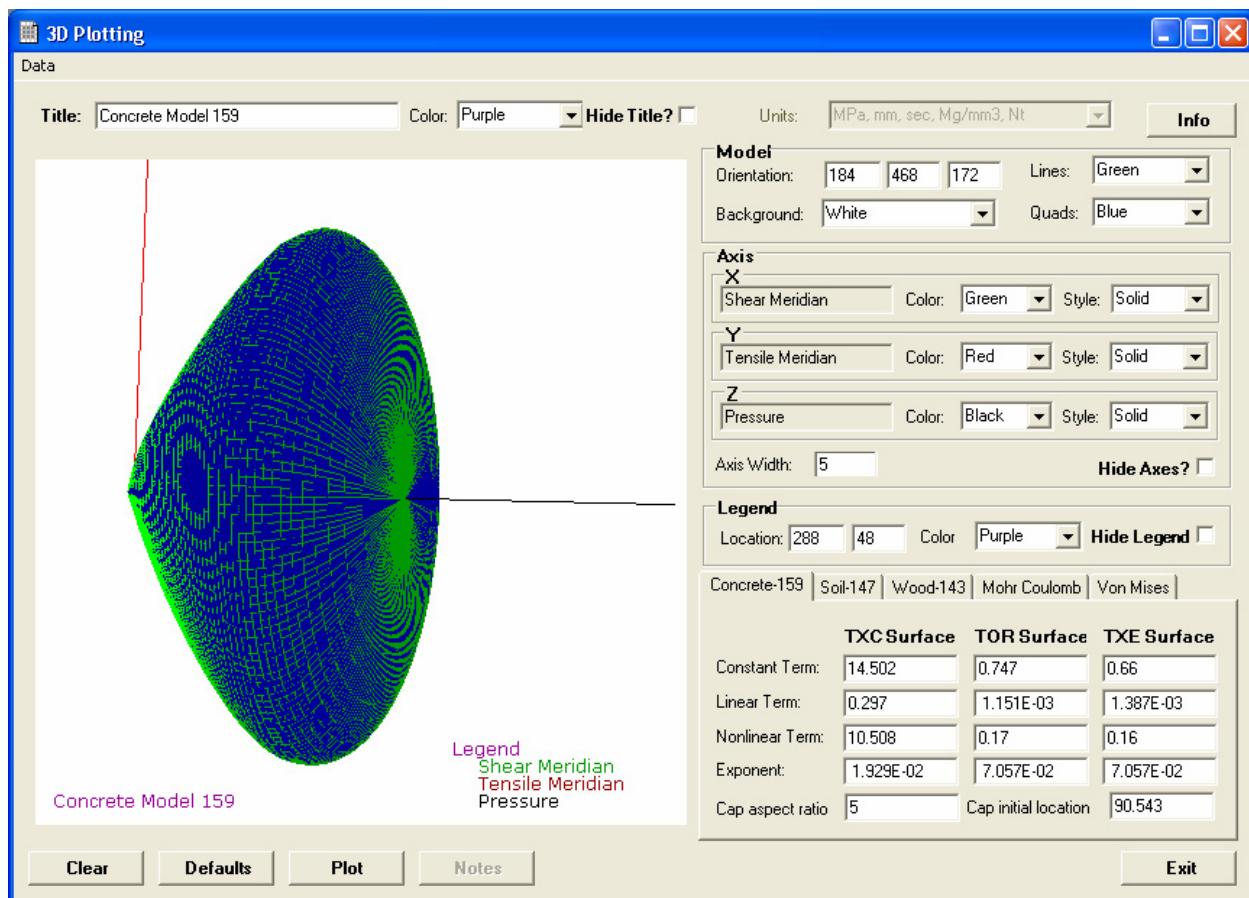
The MMCD plots three-dimensional yield or failure surfaces. Eight sets of surfaces are currently implemented. These are:

- Von Mises (not associated with a particular material model).
- Tresca (not associated with a particular material model).
- Linear Drucker-Prager (not associated with a particular material model).
- Parabolic Drucker-Prager (not associated with a particular material model).
- Mohr-Coulomb (simplification of Soil Model 147 Surface).
- Wood model 143 parallel and perpendicular to the grain.
- Soil model 147 modified Mohr-Coulomb.
- Concrete model 159 three-stress invariant shear surface with cap.

The implementation of the yield surfaces is separate from, and does not use, the FORTRAN coding of the LS-DYNA material models. Rather, the surfaces are plotted from knowledge of the yield surface equations. The only input parameters required are those used to define the particular yield surface. An example surface and GUI is given in Figure 12.

## MMCD Architecture

All functionality of the MMCD resides under an easy-to-use Graphical User Interface (GUI) written in C# (C-Sharp), which is a new programming language designed for building a wide range of enterprise applications that run on the .NET Framework. An evolution of C and C++, C# is simple, modern, type safe, and object oriented. The MMCD executable resides on user's network and runs under Windows 2000 or Windows XP.



**Figure 12. Example three-dimensional surface showing that concrete model 159 has a smooth intersection between the failure surface and hardening cap.**

All simulation input and output results are stored in a Microsoft JET database. This database is freely distributed with the MMCD. The database may be opened directly using Microsoft Access or modified via the MMCD.

The material model subroutines are written in FORTRAN without modification to their LS-DYNA format. The Livermore Software Technology Corporation supplied them to APTEK as a static library of files. The main number-crunching driver routines are also written in FORTRAN.

Report quality XY plots are generated with Scalable Vector Graphics (SVG), a [vector graphics](#) file format that enables two-dimensional images to be displayed in [XML](#) pages on the Web. Vector images are created through text-based commands formatted to comply with XML specifications. In contrast to [JPEG](#) and [GIF](#) images on the Web, which are bitmapped and always remain a specified size, SVG images are scalable to the size of the viewing window and will adjust in size and resolution according to the window in which it is displayed. Benefits include smaller file sizes than regular [bitmapped graphics](#) such as GIF and JPEG files, and [resolution](#) independence. This means that the image can [scale](#) down or up to fit proportionally into any size display on any type of Web device. The SVG plug-in is freely distributed with the MMCD, and is downloadable over the Internet.

Report quality 3-D plots of the yield surfaces are generated with OpenGL. The equations which describe the yield surfaces are written in C++ with a C# wrapper.

## Availability

MMCD preliminary development will be complete by the fall of 2006, with the expectation that the MMCD will subsequently be available for lease to the LS-DYNA community. The initial release will contain approximately five to ten material models, with additional models being added periodically and upon user request. The MMCD executable, updates, and User's Manual will be downloadable via the Internet from a distribution site. Please contact the authors at APTEK for updates on availability.

## Acknowledgements

APTEK developed the MMCD with Ms. Yvonne Murray as the program manager and Ms. Carolyn Yeager as the software developer. The Federal Highway Administration sponsored the development as a Phase I and II Small Business Innovative Research Project with Mr. Martin Hargrave as the technical monitor. The Livermore Software Technology Corporation supplied the material models and optimization methodology with Dr. Nielsen Stander as the optimization expert. The Colorado Advanced Software Institute sponsored industry-university collaboration between APTEK and the University of Colorado with Ph.D candidate Eric Hansen as the initial developer of the core driver technology.

## References

1. Hansen, Eric J., *Development of a Constitutive Driver Package for Material Modeling and Damage Detection*, Thesis to University of Colorado, 2000.
2. LSTC, *LS-DYNA Keyword User's Manual*, Version 971 Beta, Livermore Software Technology Corporation, Livermore California, March 2005.
3. Lewis, B.A., *Manual for LS-DYNA Soil Material Model 147*, Volpe National Transportation Systems Center, Federal Highway Administration, Report No. FHWA-RD-04-095, November, 2004.
4. Lewis, B.A. and J. Reid, *Evaluation of LS-DYNA Soil Material Model 147*, Volpe National Transportation Systems Center, Federal Highway Administration, Report No. FHWA-RD-04-094, November 2004.
5. Murray, Y.D., *Manual for LS-DYNA Wood Material Model 143*, Volpe National Transportation Systems Center, Federal Highway Administration, Report No. FHWA HRT-04-097, November 2005.
6. Murray, Y.D. and J. Reid, *Evaluation of LS-DYNA Wood Material Model 143*, Volpe National Transportation Systems Center, Federal Highway Administration, FHWA HRT-04-096, May 2005.
7. Murray, Y.D., *Manual for LS-DYNA Concrete Material Model 159*, Volpe National Transportation Systems Center, Federal Highway Administration, Report No. FHWA HRT-05-062, to be published.
8. Murray, Y.D., A. Abu-Odeh, and R. Bligh, *Evaluation of LS-DYNA Concrete Material Model 159*, Volpe National Transportation Systems Center, Federal Highway Administration, FHWA HRT-05-063, to be published.
9. Stander, N., W. Roux, T. Eggleston, and K. Craig, *LS-OPT User's Manual, Design Optimization for the Engineering Analyst*, Version 2.2, Livermore Software Technology Corporation, Livermore California, Version 2.2, 2004.