Investigating the Vibration Behavior and Sound of Church Bells Considering Ornaments and Reliefs Using LS-DYNA

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Abstract

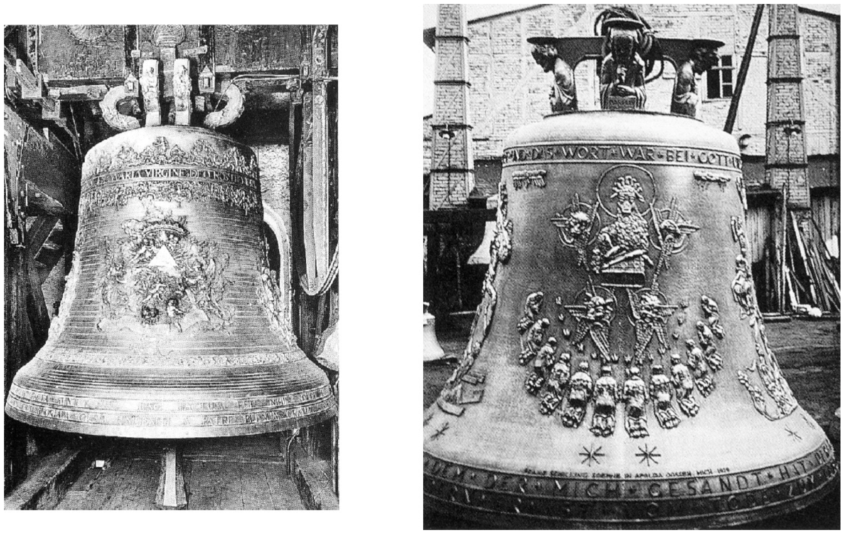
A numerical investigation of the vibration behavior and the sound of a specific bell is performed and validated by experimental modal analysis. In the numerical simulations a number of modifications of the geometry mimicking ornaments and reliefs is investigated as such ornaments have lead to mistunes in a very popular case in Germany. It is also shown, how the influence of ornaments on the modification of eigenfrequencies can be reduced.
The numerical results obtained by eigenvalue analyses as well as transient analyses with LS-DYNA compare very well with the experimental results. It is shown that LS-DYNA- Finite Element analysis can be well used for bell design [14].

Introduction

The casting of bells was and is still an art and the craftsmen or artists are asking for some luck to achieve the final goal of a well sounding bell. Recently in Germany the casting flaws in the making of the bells for the rebuild Frauenkirche in Dresden have been in the focus of attention. From seven newly cast bells six had to be redone as reported widely in newspapers [13] and television [5] under the title “Ornaments may be a burden”.

Bells appear to be axisymmetric at a first look, however, as a consequence of the casting process and due to the application of ornaments and reliefs – see figure 1 - there are certainly some deviations from axisymmetry. Thus the geometry is not fully symmetric and therefore also the mass and stiffness distributions are not symmetric which leads to the so called beat. This is the result of the interference of two or more natural frequencies, the so-called beat frequencies, which are very close to each other. The observed sound is often interesting and charming, even desired by the bell founders; however, if the difference in the frequency is beyond a limit, some dissonance may be the result. The measure which beat is acceptable and which not, is not an objective quantity.

Bells are three-dimensional shell like structures which can nowadays be investigated by computational methods such as finite elements. In our investigation the bells are first discretized with axisymmetric geometry then we modify the geometry in a similar fashion as found in the making process or by adding ornaments resulting in a non-axisymmetric structure. This leads directly from the multiple eigenfrequencies of the originally symmetric system to eigenfrequencies which are very closely spaced. This will be investigated by eigenfrequency analyses and by striking the bell and running a time history analysis.



a) b)

Figure 1: Typical Ornaments for bells from KRAMER [12], a) “Herrgottsglocke” of old Salem peal of bells, Herisau, Switzerland; b) Meißen, Dom, large bell

Further we perform an experimental modal analysis of a small size carillon bell given to us by the bell founder Perner, Passau, Germany and compare the results with our numerical analyses. In particular we obtain damping values for our computations. The strike of the bell is also measured and used in the time history analysis to allow a better comparison of the frequency response spectra. Finally the sound of the bell is recorded by a condenser microphone.

Bell Material and Support Structure

The sound requirements for bell material concern mainly a low material damping and a low material sound speed. The standard material for church bells is bell bronze. Bell bronze is a material with clear regulations, e.g. tin bronze bell must be cast from original bell bronze – 78 % copper and 22 % tin – with less than 2 % other material. The minimum concerning tin is 20 % leading to easy casting, high corrosion resistance, high microstructure stability, very good sound capabilities, small damping, low elasticity modulus, high density and low sound speed. Some average values are given in table 1.

hardness (Brinell)	180 HB 2.5/187.5
mass density	8,45 g/cm^3
speed of sound c	3400 m/s
Young’s modulus E	95000 N/mm^2
Poisson ratio	0,30
porosity P	1 \%
damping	3 x 10^{-4}
tensile strength	180 N/mm^2
strain at failure	0,01 %
area of melting temperatures	798 to 890° C

Table 1 Average bell bronze material data after SCHAD [9, 10]

In the manufacturing of carillons a material with slightly different – higher or lower – percentage of tin is often used. With more in the speed of sound of the bell is increased and higher tones are achieved.

The support structure consists of crown, yoke and clapper. The clapper, see figure 2, is manufactured from low carbon steel, density 7.8 g/cm^3. The weight of the clapper is about 3.5% - 4.5% of the complete weight of the bell. In figure 3 a clapper is depicted which is hanging centrally in the bell and is striking against the rim of the bell to create sound. The crown consists of several hangers radially put together into one centrally located plate and is needed to fix the bell to the yoke. The yoke is a horizontally supported girder in the belfry made from wood or steel. Clapper and yoke form a double pendulum.

Figure 2 Different clappers, bell with crown and different cross sections of bells (bell ribs); from bell founder Perner, Passau, Germany

Bell Geometry

Bells are usually designed by defining so-called rib geometries which lead to different bell sounds. The real geometry of a rib is one of the secrets of each bell founder besides the composition of the alloy, the casting temperature and the cooling process. A typical rib for a church bell – the Gert de Wou rib - is given in figure 3 patterned after information given in [2]. It is clear that the form strongly defines the bell character [1], here a typical minor octave bell. This is the classical church bell. The name of the bell is stemming from the strike tone, which is the dominant sound heard and which is used to name the bell. It is a virtual tone which cannot be measured objectively and which depends on the major octave, the twelfth and the double octave dominantly. For minor octave bells it is about one octave below the major octave (Rayleigh's rule) thus about half the frequency of the major octave (1/2 $f_{\text{major octave}}$). To later tune up the bell often the inner radius is enlarged by trimming.

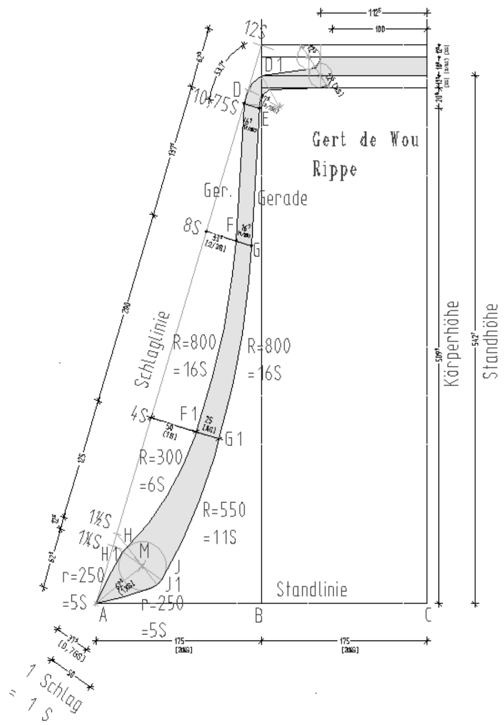


Figure 3 Classical Gert de Wou rib taken from [2]

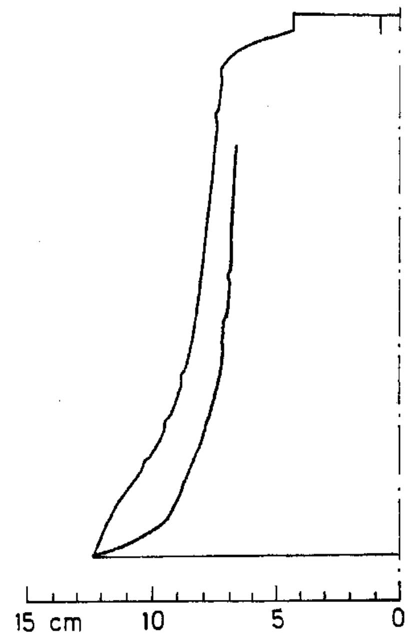


Figure 4 Typical carillon bell rib from FLEISCHER [4]

While Gert de Wou rib in figure 3 has a maximum diameter of about 700 mm and a total height of about 570 mm the rib of the carillon, see figure 4, investigated later in more detail has corresponding measures of 250 mm and 220 mm. In general the ratio of wall thickness to diameter of carillon bells is larger than for church bells.

Such a carillon bell is later investigated in detail concerning experiments and comparing simulations. Unfortunately the bell founder has asked for not publishing information about the specific geometry.

Eigenfrequency Investigations and Influence of Ornamentation

All investigations concerning eigenfrequencies and their modifications due to ornamentation, out-of-roundness and similar are investigated for the bell with the Gert de Wou rib as a typical example. The numerical analysis is performed with LS-DYNA® vs. 970; only eigenfrequency analyses are performed for shell and different solid element meshes, see figure 5. The shell mesh consists of a total of 757 shell elements (type 16 in LS-DYNA® [7]), 17 in axial, 44 in circumferential direction plus some for the closure on top. The solid element mesh consists of 12500 elements for the coarse mesh (63 in axis and 192 in circumferential direction plus elements for the hanger construction) and 41600 elements for the fine mesh. For the FE analysis solid element type 18 – an enhanced strain formulation - in LS-DYNA® was chosen.

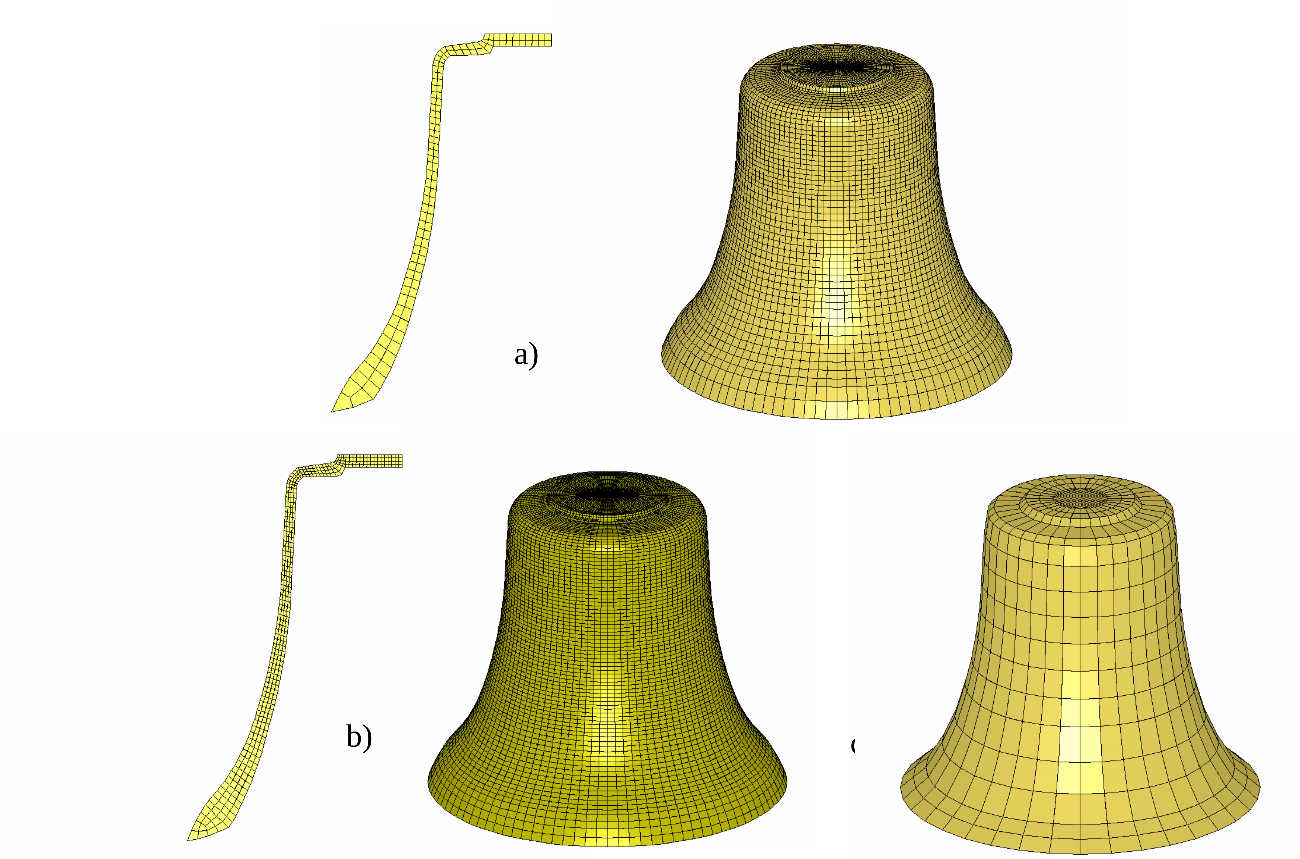
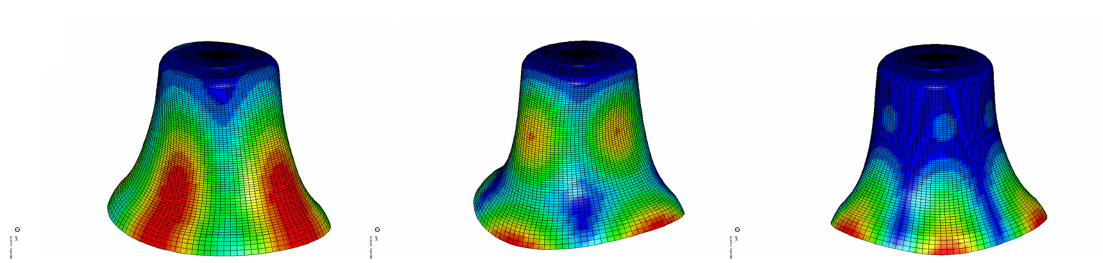


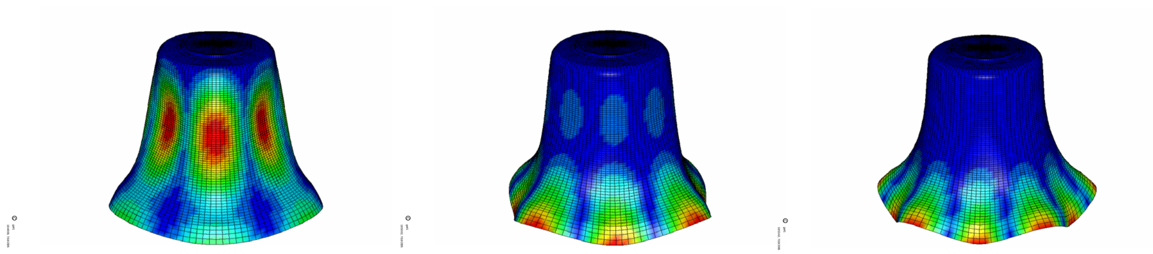
Figure 5 Bell with de Wou rib; a) and b) solid element meshes with the meshed rib geometry and the total mesh a) 12500 el. B) 41600 el., c) total shell mesh 757 4-node shell elements – all without hanger construction

The eigenfrequency analysis for the fine sold element mesh confirms that we have a typical minor octave bell, as the ratios of frequencies to the strike tone are: 1.17 for the third, 0.96 for the prime, 0.45 for the minor octave. These values are close to the ratios found in the literature [11]. The corresponding eigenmodes are given in figure 6.1 and 6.2 for the first six eigenfrequencies.



a) minor octave b) prime c) third

Figure 6.1 Eigenfrequencies for the first three partials of the de Wou rib given sequentially in rising order



d) quint e) major octave f) twelfth

Figure 6.2 Eigenfrequencies for the partials four to six of the de Wou rib given sequentially in rising order

In table 2 the computed values are given for the three different meshes shown. The difference between shell and coarse solid element mesh is 3.2 % for the highest given frequency; all other values are about 1% or less. The finer solid element mesh shows no further improved frequencies, as the differences in the computed values are mostly below 1 %. Thus the coarse solid element mesh can be considered as reasonably converged.

Partial	757 4-node-shell elements (element type 16)	41600 solid elements (element type 18)	Difference	12500 solid elements (element type 18)
	frequency [Hz]	frequency [Hz]	[\%]	frequency [Hz]
Minor octave	267,3	265,5	-0,7	263,9
Prime	555,7	563,1	1,3	564,1
Third	679,2	688,3	1,3	682,5
Quint	774,8	782,7	1,0	782,6
Major octave	1137	1173,6	3,2	1162,1

Table 2 Comparison of frequencies obtained for a shell discretization with different solid element discretizations for the de Wou rib

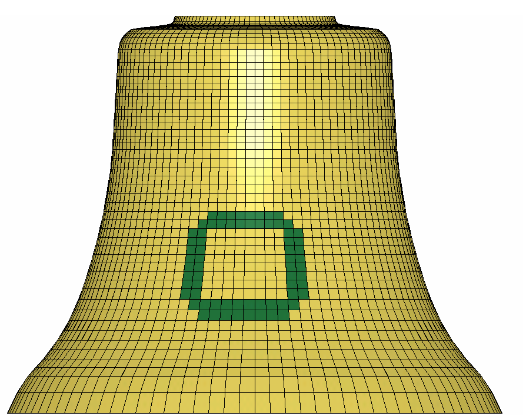
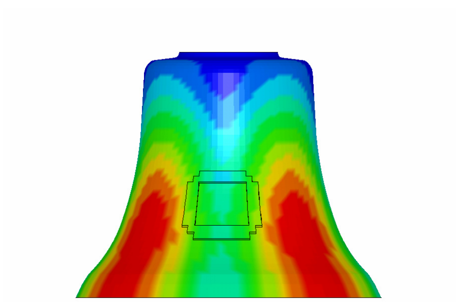


Figure 7 FE model for the ornamentation; 68 solid elements modified or added

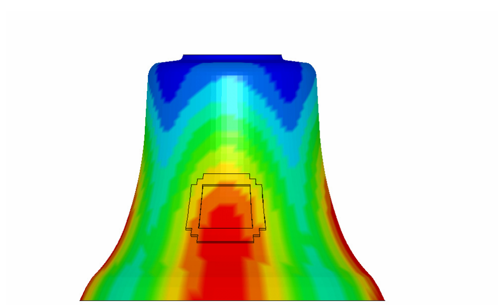
The **investigation of ornamentation** on the eigenfrequencies – the original reason of the study – is performed in three ways. The pattern of the ornament covering about 68 elements is shown in figure 7 assuming symmetry of the imperfections. In variant A the mass density is increased by a factor of 1.8 simulating the increase of thickness by about 1 cm. In variant B Young's modulus is increased by a factor of 5.8 again simulating the increase of thickness by about 1 cm. In the third variant C solid elements with the thickness of 1cm are added on the pattern.

	axisymmetric	Variant A ρ_{mod}		Variant B E_{mod}		Variant C Additional layer of 68 solid elements	
Partial	frequency [Hz]	frequency [Hz]	Δ %	Frequency [Hz]	Δ %	Frequency [Hz]	Δ %
Minor octave	263,9	262,5 263,3	-0.5 -0.2	264,5 266,1	0.2 0.8	263,9 265,0	0.0 0.4
Prime	564,1	562,1 563,2	-0.4 -0.2	567,0 569,9	0.5 1.0	565,0 565,1	0.2 0.2
Third	682,5	681,2 681,8	-0.2 -0.1	685,5 686,2	0.4 0.5	683,9 684,3	0.2 0.3
Quint	782,6	778,6 779,4	-0.5 -0.4	788,7 790,4	0.8 1.0	785,5 787,4	0.4 0.6
Major octave	1162,1	1160,9 1161,1	-0.1 -0.1	1167,0 1167,7	0.4 0.5	1164,4 1165,5	0.2 0.3

Table 3 Comparison of the results for the frequencies for different model variants of the ornaments on the bell with the de Wou rib; basis coarse mesh with 12500 solid elements



Minor octave 263,9 Hz, ornament on line of vibration nodes



Minor octave 265,0 Hz, ornament on maximum vibration amplitude

Figure 8 Spinning Modes of minor octave; ornaments modeled by adding solid elements

In table 3 we recognize the modifications of the eigenfrequencies due to each modification. Clearly visible is the splitting of the original single eigenfrequency into two frequencies f_1, f_2 with the beat frequency $f_s = f_1 - f_2$ in each case, see also [3]. For the minor octave the corresponding eigenmodes are depicted in figure 8. The general pattern shows the same number of waves, however there is a shift in circumferential direction. The deviation of the new frequencies from the original ones is in the range of 1 %. For variant A the frequencies decrease – as expected, as the mass increases. For variant B and C the eigenfrequencies increase, which is more realistic.

A further variation by adding solid elements over the complete area of the shown pattern leads to a slightly larger splitting of the eigenfrequencies. The maximum beat frequency of the minor octave becomes now 3.8 Hz compared to previously 1.1 Hz. This may be rather dissonant.

The **investigation of out-of-roundness** on the eigenfrequencies as a result of the manufacturing process is also modeled in three ways. In variant D the geometry complete bell model is scaled in one direction by 1 % achieving an oval form. This leads to a total mass increase of about 1%. In variant E an oval form is achieved by a rotation of some areas around the axis through the crown of the bell. By this action advantage is taken from the morphing functionalities in HYPERMESH® [15]. The mass increase amounts then to about 4%. In the third variant F a similar modification is performed as in variant E, however, the surfaces are modified such that the mass is not increasing. The results for the eigenfrequencies are depicted in table 4. Again the original eigenfrequencies are split up into two closely spaced frequencies with the beat frequency f_s. For variant D and F the eigenfrequencies are slightly reduced while for variant E the eigenfrequencies are increased. The largest beat frequency in the minor octave is found for variant E with f_s = 3.3 Hz.

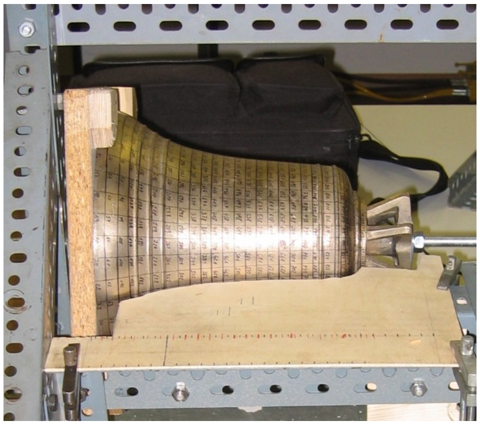
	axisymmetric	Variant D		Variant E		Variant F	
Partial	frequency [Hz]	frequency [Hz]	Δ %	Frequency [Hz]	Δ %	Frequency [Hz]	Δ %
Minor octave	263,9	262,7 262,7	-0.5 -0.2	269,4 272,7	2.1 3.3	263,1 264,5	-0.3 0.2
Prime	564,1	563,4 563,4	-0.1 -0.1	568,9 569,8	0.9 1.0	563,5 563,6	-0.1 -0.1
Third	682,5	679,3 679,3	-0.5 -0.5	697,8 698,6	2.2 2.4	680,9 681,9	-0.2 -0.1
Quint	782,6	779,9 779,9	-0.3 -0.3	807 810,9	3.1 3.6	780,1 780,3	-0.3 -0.3
Major octave	1162,1	1156 1156	-0.5 -0.5	1190 1195	2.4 2.8	1159 1160	-0.3 -0.2

Table 4 Comparison of the results for the frequencies for different model variants for the out-of-roundness of the bell with the de Wou rib; basis coarse mesh with 12500 solid elements

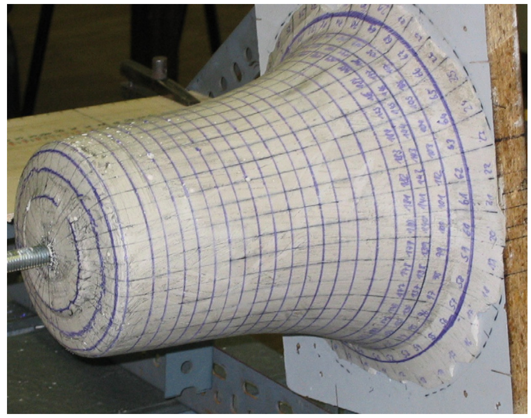
The conclusion from this small study is that both – ornaments and out-of-roundness – have some clear effect creating beat frequencies. Whether this is a discord or desired tone quality is certainly an open question.

Experimental Investigation of a Carillon Bell

The bell founder Rudolf Perner, Passau, Germany made a carillon bell available to us with the following general measures, mass 16.2 kg, height about 28 cm and lower diameter 26 cm. The geometry of this bell was first accurately measured from the outside and the inside by a laser tracker. The inside was captured by a cement model. For both sides we used a measuring grid of about 1000 nodes as depicted in figure 9.



External measurement grid



Cement bell with internal measurement grid

Figure 9 Measurement of carillon bell, Perner Passau; measurement grids

Then a modal analysis was performed using the software IDEAS® [16] measuring the acceleration at 240 nodes for each excitation. The excitation is achieved by an impulse hammer with almost identical intensity for each location. This leads to a broadband excitation of the bell. In figure 10 the result for a typical transfer function is displayed when the hammer is striking at the lower rim of the bell. All standard tones are clearly visible: the minor octave at 999 Hz, the prime at 1817 Hz, the major octave at 3907 Hz and some mixed tones. The response in the higher frequencies also shows increasing noise.

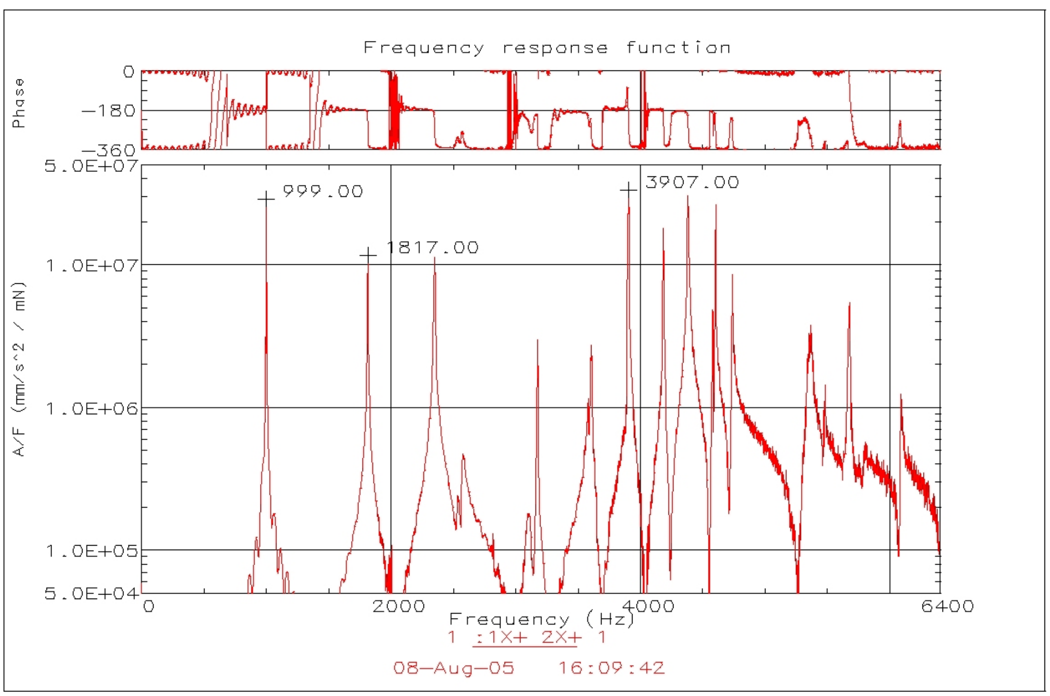


Figure 10 Transfer function for Perner bell; excitation by hammer strike at rim of bell

The eigenmodes obtained by the experimental modal analysis applying IDEAS® [16] are shown in figure 11. Beat frequencies are found for the prime and the twelfth. The frequency ratios expected for an ideal minor octave bell are fairly well matched by the real bell with some minor deviations for the prime and the quint. For the listing of the obtained frequencies we refer to the comparison with numerical results in table 5.

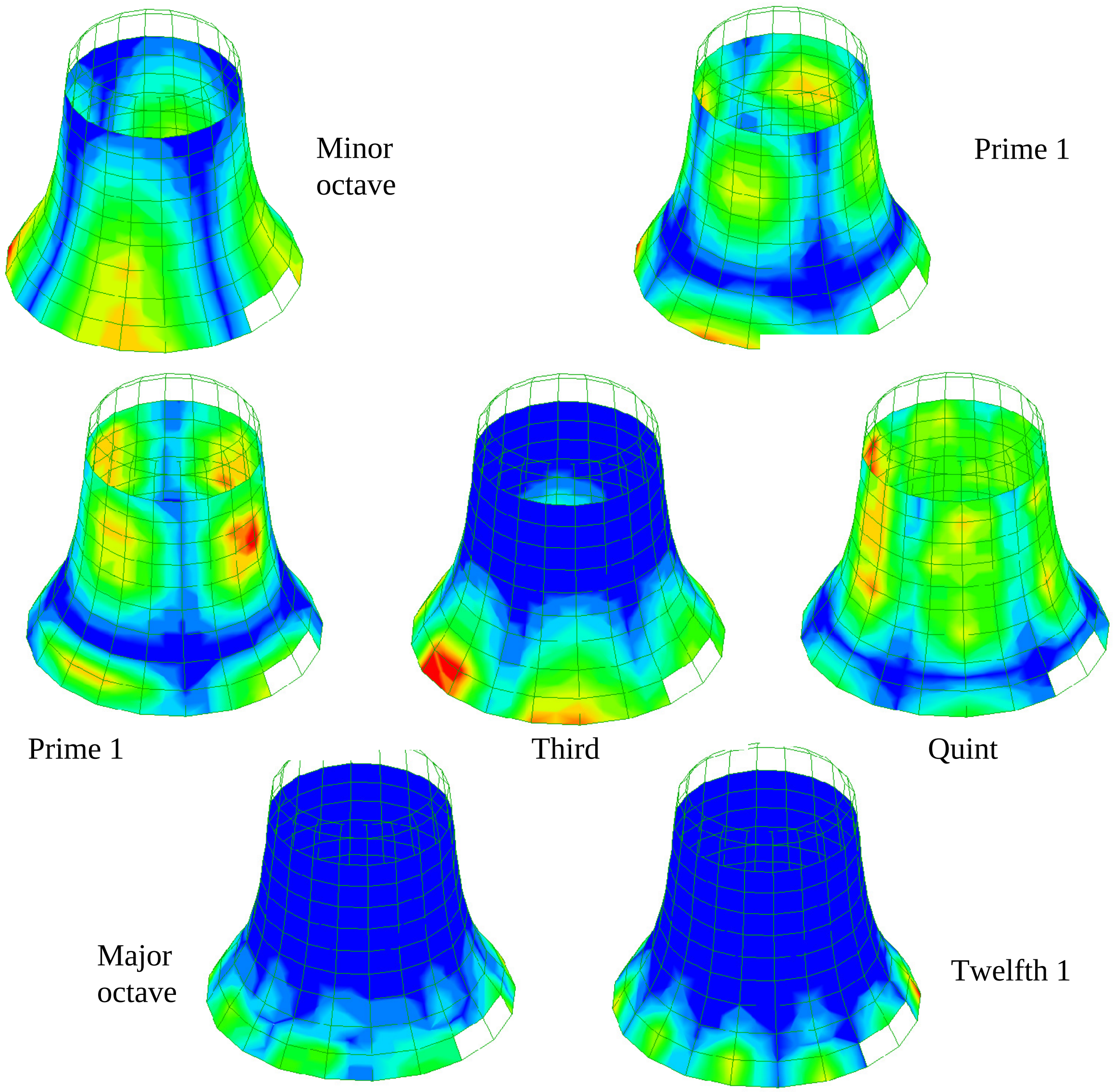


Figure 11 Eigenmodes for partials of the Perner bell obtained by modal analysis with I-DEAS®

Numerical Investigation of a Carillon Bell

Similar to the de Wou bells analysed in the previous sections the Perner bell is discretized with solid and shell elements. For both types of elements – type 16 for the shells and type 18 for the solids – also a convergence study is performed with meshes depicted in figure 12 and 13.

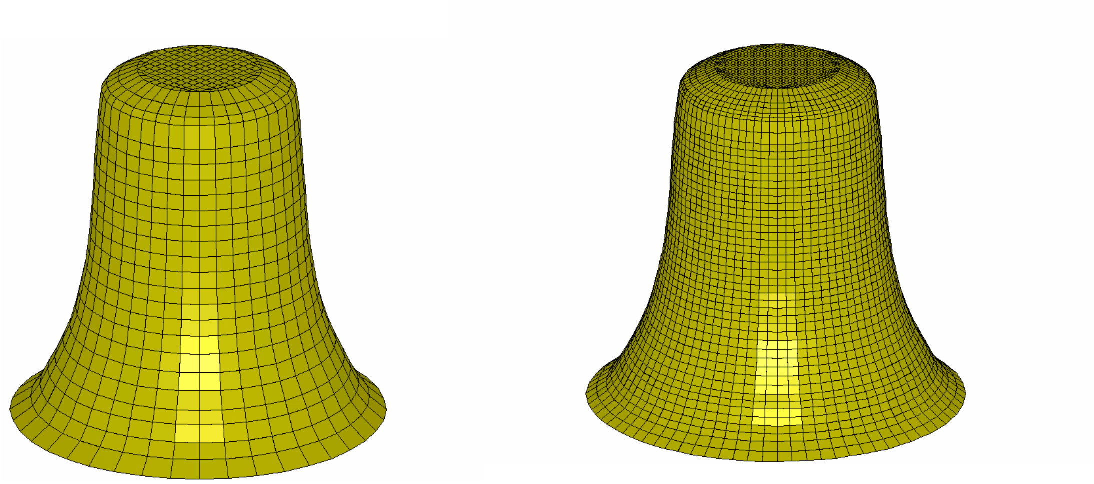


Figure 12 Perner bell, discretization with shell elements; coarse mesh 1132; fine mesh 4528 4-node shell elements

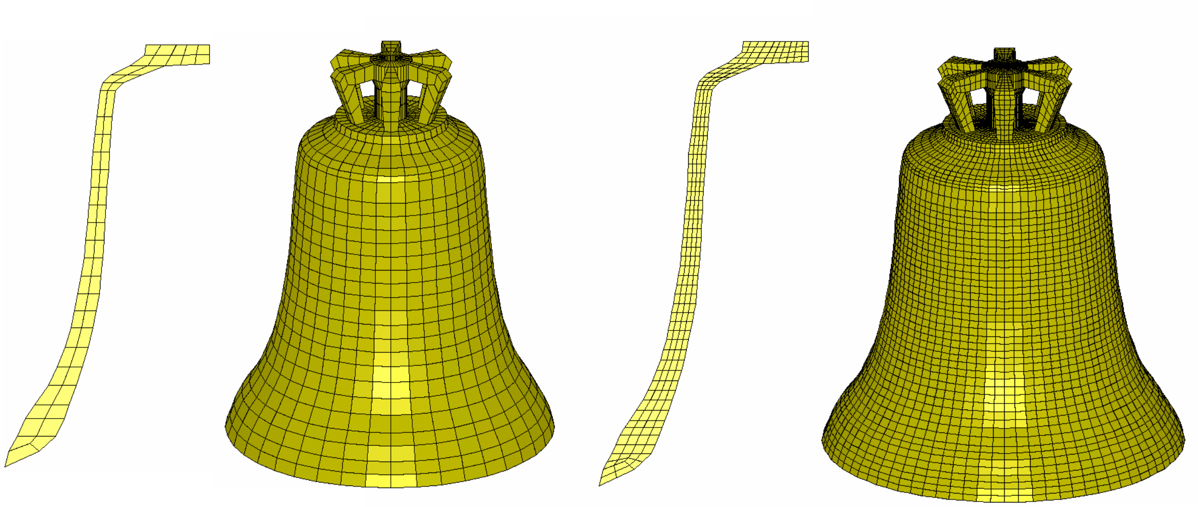


Figure 13 Perner bell, discretization with solid elements; coarse mesh 2531; fine mesh 20199 8-node solid elements; rib meshing left

In table 5 the computed frequencies for both coarse meshes are shown. The difference to the experimental results is a maximum of 4.3 % for the major octave using the shell model and 3.1 % for the minor octave using the solid element model. The refined meshes do not show a major improvement, as a maximum of 0.9% difference for the shell model and 0.5 % for the solid element model compared to the corresponding coarse mesh results is obtained. The major difference to the experimental results is the occurrence of spinning modes or beat frequencies for all partials in the computed results.

A final analysis was also performed simulating the experiment to some extend with a strike of a bell clapper. Then the acceleration frequency spectrum for selected nodes is computed showing the eigenfrequencies at the expected locations. Such an analysis shall serve in the future for the proper creation of the bell sound including the effect of beat frequencies.

		Experimental modal analysis	Shell analysis		Solid elements	
eigen-mode	partial	frequency [Hz]	frequency [Hz]	Δ %	frequency [Hz]	Δ %
1	Minor octave	999	1001	0.2	972	-3.1
2 2	Prime	1814 1817	1795 1801	-1.05 -0.88	1818 1825	0.22 0.44
3	Third	2351	2390	1.66	2306	-1.91
4	Quint	3176	3161	-0.47	3097	-2.49
5	Major Octave	3907	4043	3.48	3823	-2.15
6	Mode F	4187	4242	1.31	4135	-1.24
7 7	Twelfth	5668 5677	5912 5914	4.30 4.17	5566 5573	-1.80 -1.83

Table 5 Comparison of results for frequencies, Perner bell, experimental modal analysis, shell model with 1132 4-node shell elements, solid element model with 2531 solid elements, Young's modulus 95000 N/mm^2, mass density 8,38g/cm^3

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