Thin-Walled Beams Research and Development

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Abstract

An integrated warped FEM beam element has been implemented in LS-DYNA and is considered here as a very important beginning. Accounting for warping is a fundamental part of Thin-walled beam theory, having more than three quarters of century history of research and developments, which are still active. Information related to thin-walled beams looks to be very useful to LS-DYNA users, may define steps for further beam FEM elements implementations and wider usage, and therefore some of the information is presented in this paper. The principal idea of the Thin-walled beam theory to represent three-dimensional thin-walled cylindrical shell structures as one-dimensional thin-walled warping beams was very useful in the past and very important today taking advantage of that beams computational efficiency. So far, however, thin-walled beams are modeled mostly using shell FEM elements, which is computationally more expensive

1. Introduction

Implementation of integrated warped beam element based on Thin-walled beam theory, Vlasov [1], by Borrval [2], in LS-DYNA is considered here as a very important beginning of wider usage computationally efficient beam FEM elements in FEM structure models.

The basic idea of the Thin-walled beam theory is to represent mathematically three-dimensional cylindrical shell structures as one-dimensional thin-walled beams. That is similar to the computationally efficient Strength of Material theory mathematical representations three-dimensional structures as one-dimensional objects referred to as beams. Strength of Material theory is based on two hypotheses: hypothesis of rigid and hypothesis of plane (not warping) cross sections. The hypotheses are justified for beams with solid cross sections, which dimensions are significantly smaller then beams length.

Three-dimensional cylindrical thin-walled shell structures cross sections are designed to have insignificant deformations during normal applications. To make shells cross sections not deformable, sufficient shell thickness is selected for small cross sections and for large cross sections reinforcements are used. Vlasov [1] had confirmed theoretically and experimentally the applicability of rigid cross sections hypothesis in cylindrical open cross sections shells analyses. Therefore hypothesis of rigid cross sections is used in the Thin-walled beam theory.

However, during normal operations, even reinforced thin-walled shell structures cross sections not always remain plain, i.e. their cross sections might warp. Therefore Vlasov's Thin-walled beam theory does not accept the hypothesis of plane (not warping) cross sections. That makes the Thin-walled beam theory a generalization of the Strength of Material theory. Thin-walled beam theory uses additional generalized force referred to as bimoment B, Fig.1, which graphical representation comprises of two self-equilibrium moments. The distance between the two moments is considered to be infinitesimal. Bimoments create cross sections warping. Thin-walled beam theory, Fig.1, represents warping as longitudinal normal cross section deformation under torsion. The warping deformation is plane within each linear part of a cross section's profile.

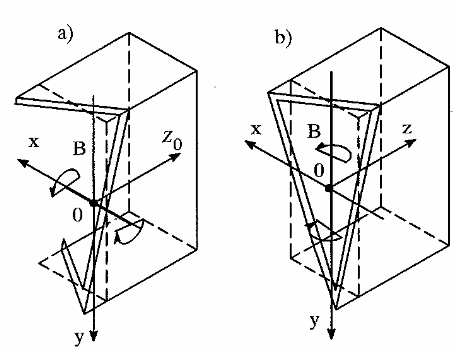

Fig.1. Bimoments B and warping of cross sections: a) open; b) closed

2. Thin-Walled Shells and Beams

Similar to FEM model, an open cross section cylindrical shell is geometrically represented in Thin-walled beam theory as a set of rectangular plates in axes xyz_0, Fig.2.

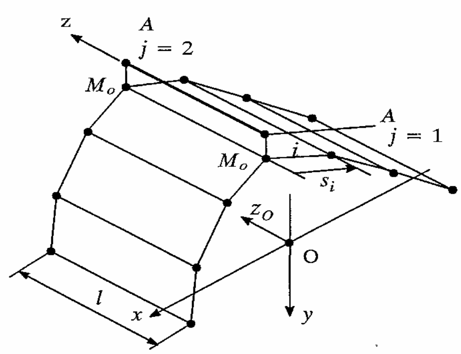

Fig.2. Three-dimensional thin-walled cylindrical shell with constant open cross sections and xyz_0 axes; and one-dimensional thin-walled beam with longitudinal axis z

By solving a similar to FEM system of equations for the set of rectangular plates, formulas for components of generalized deformation and forces δ_{jA}, F_{jA}, Eq.1, and F_{ji}, Eq.2, were derived by Vlasov [1]. Those formulas represent analytically the shell as one-dimensional thin-walled open cross section beam with longitudinal axis z and nodal points $j = 1,2$, Fig.2. Axis z also referred as centroidal axis, on which centers of bending/torsion points A, Fig2, are located. Axis z is parallel to the cross section principal axis z_O.

$$\left.\begin{aligned} &\delta_{jA} = \left[\zeta_{jA} \quad \xi_{jA} \quad \xi'_{jA} \quad \eta_{jA} \quad \eta'_{jA} \quad \theta_{jA} \quad \theta'_{jA}\right]^T, \\ &F_{jA} = \left[N_{jx} \quad M_{jxA} \quad N_{jy} \quad M_{jy} \quad N_{jz} \quad M_{jzA} \quad B_j\right]^T, \\ &j = 1,2 \end{aligned}\right\}, \tag{1}$$

where F_{jA} generalized forces, δ_{jA} generalized deformations from external generalized forces F_{ji}, Eq.2.

$$F_{ji} = [N_{jix} \quad M_{jix} \quad N_{jiy} \quad M_{jiy} \quad N_{jiz} \quad M_{jiz} \quad]^T \tag{2}$$

In Eq.2, F_{ji} are external generalized forces applied to profile nodal points with profile coordinates s_i, which origin is at profile points M_0, Fig.2. The first six components of generalized forces F_{jA} and deformations δ_{jA}, Eq.1, as well as all components of generalized force F_{ji}, Eq.2, are from the Strength of Materials theory. The sevenths Eq.1 components: bimoment $B = \int_{\hat{A}} \sigma \omega_A d\hat{A}$ and measure of warping $\theta' = \frac{\partial \theta}{\partial z}$ are the new generalized force and deformation introduced by the Thin-walled beam theory. Thin-walled beam theory introduces new type of coordinates ω_A referred to as sectorial coordinates. Beams warping are Fig.1 type of normal to cross section longitudinal deformation ζ under torsion. Warping is a function, $\zeta = \theta' \omega_A$, of sectorial coordinates ω_A, which will be described in some details below.

The Thin-walled beam theory normal stresses σ at profile nodal points with coordinates x, y, z, ω_A may be computed using Eq.3, in which the first three components are from the Strength of Materials theory.

$$\sigma = \frac{N}{\hat{A}} - \frac{M_y}{J_y} x + \frac{M_x}{J_x} y + \frac{B}{J_\omega} \omega_A . \tag{3}$$

The Eq.3 fourth component $\frac{B}{J_\omega}\omega_A$ defines normal stresses from torsion due to warping and includes sectorial moment of inertia J_ω.

In Fig.3, a thin-walled open variable cross sections shell structure is presented, Wekezer [3]. Unlike the shell of Fig.2, the shell of Fig.3 is modeled by triangular, not rectangular plates. One-dimensional beam, Fig.3, with axis z and bending/torsion centers A_1 and A_2 may or may not be parallel to longitudinal axis z_O.

In Fig.4, a three-dimensional thin-walled cylindrical shell with closed cross sections is presented. Geometrically that shell is presented by a set of rectangular plates. One-dimensional beam, Fig.4, for such a shell centers of bending/torsion A (points $j = 1,2$) are usually located on principal beam axis z, which coincide with the beam centers of gravity. Thin-walled beam with closed cross sections may have warping both from normal and shear stresses. Two bimoments are defined for beams with rectangular cross sections, Vlasov [1]: longitudinal bimoment B^L created by normal stresses and transverse bimoments B^T created by shear stresses τ, Eq.4. Warping of closed cross section beams is described in details in Lujin [4].

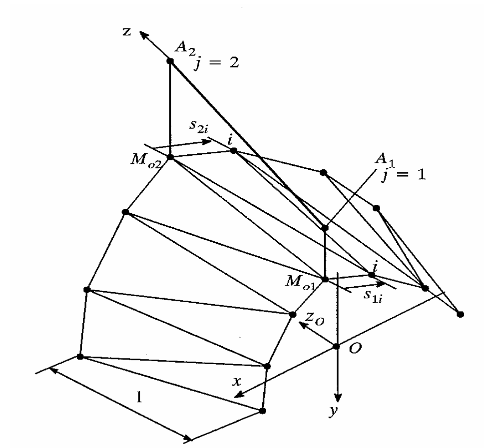

Fig.3. Three-dimensional thin-walled shell with variable open cross sections and axes xyz_0; and one-dimensional thin-walled beam with longitudinal axis z

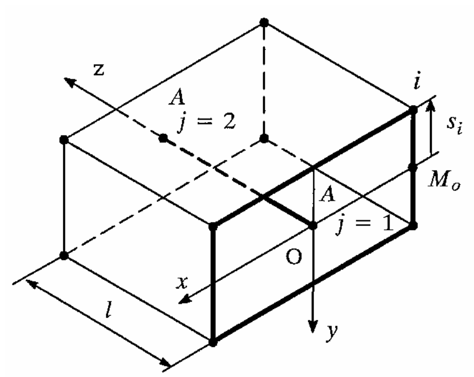

Fig.4. Three-dimensional thin-walled shell with closed cross sections and one-dimensional thin-walled beam with nodal points $j = 1,2$, both in axes xyz

$$B^L = \int_{\hat{A}} \sigma \phi_L d\hat{A}, \;\; B^T = \int_{\hat{A}} \tau \phi_T d\hat{A}, \tag{4}$$

where ϕ_L and ϕ_T are functions of profile coordinates s, Vlasov [1]

Due to the Thin-walled beam theory formulas derivation approach, should be the same results from modeling cylindrical shell, Fig.2, by using one FEM one-dimensional thin-walled beam element with 2 nodal points $j = 1, 2$ and by using seven FEM shell elements with 14 nodal points $i = 1,2,\cdots,14$. That is also true for shell structures of Fig.3 and Fig.4. There should be cases, in which beam elements usage is preferable with respect to shell elements. However regardless of beams types, if their FEM models' geometrical representations are defined just by one line, sufficient modeling structure geometry is not always possible. Also, one-dimensional FEM beam elements interactions with other types of FEM elements in a FEM model are not always possible (Section 5. Three-dimensional thin-walled beam elements).

3. Sectorial Coordinates

3.1 Open Cross Section Beams

Complete information on sectorial coordinates is presented in Vlasov [1]. Thin-walled open cross section beam sectorial coordinates for a simple channel-type and angle-type cross sections are presented here by Eq.5, and the coordinate's diagrams by Fig.5.

$$\left.\begin{aligned} \omega_A(s_i) &= \sum_{k=1}^{i} R_k \, \Delta s_k \,, \\ \Delta s_k &= s_k - s_{k-1} \end{aligned}\right\}. \tag{5}$$

In Eq.5, $\omega_A(s_i)$ is principal coordinate of cross section profile point coordinate s_i, which origin is at profile point M_0. Generalized radius R_k with origin at principal sectorial pole (bending/torsion center) A is the distance from pole A to the profile part Δs_k. The position of pole A is a function of cross sections profile geometry.

Sectorial coordinates $\omega_A(s)$ diagram in principal coordinates xy with principal pole A is presented in Fig.5a. The sectorial coordinate diagram, Fig.5a, reflects the non plane warping deformations shown on Fig.1a. Cross sections with such types of sectorial coordinate's diagrams warp, and normal stresses σ and strains from torsion (bimoment B) are proportional to the sectorial coordinate diagrams, Eq.3.

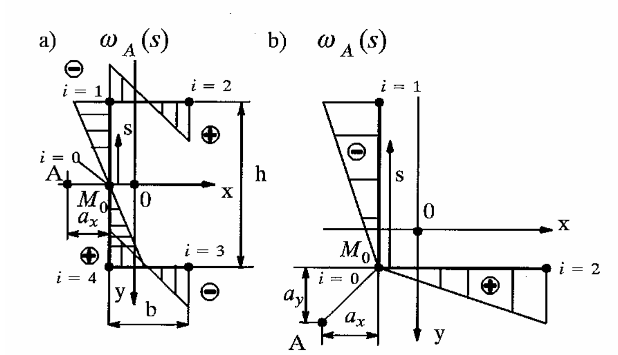

Fig.5. Open cross section thin-walled beam sectorial coordinates diagrams:
a) channel-type, b) angle-type

In Fig.5b presented are an angle-type thin-walled beam cross section, and its sectorial diagram $\omega_A(s)$ in arbitrary coordinates xy with arbitrary pole A. In such type of open thin-walled beam cross section, each sectorial coordinate diagram is proportional to a constant distance from pole A (radiuses $R_{i=1} = \alpha_x$ $R_{i=2} = \alpha_y$). Therefore, Fig.5b sectorial coordinates $\omega_A(s)$ diagram defines plane, not warping cross section deformations. The Fig.5b sectorial coordinate diagram is similar to x and y coordinates diagrams, i.e. similar to diagrams of bending deformations with respect to the neutral cross section line $A - M_0$. Thin-walled beams, which are formed by plates intersecting at one common longitudinal line and having cross sections similar to shown on Fig.5b and Fig.6, remain plane, do not warp, Vlasov [1], and do not have normal stresses under torsion.

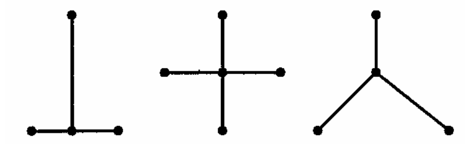

Fig.6. Not warping open thin-walled beam cross sections.

3.2 Single Cell Closed Cross Section Beams

Longitudinal deformations ζ under torsion in closed thin-walled cross section beams are generated both from normal and shear stresses. Therefore sectorial coordinates for closed cross section beams have additional component $\overline{\omega}_A$, and longitudinal deformations are defined as $\zeta = \theta^{'}(\omega_A - \overline{\omega}_A) = \theta^{'}\hat{\omega}_A$, where $\hat{\omega}_A$ is referred to as generalized sectorial coordinate, Lujin [4]. For a single cell cross section it is convenient to select center of bending/torsion A at cross section center of gravity, Fig.4 and Fig.7. Then the origin M_0 of profile coordinates s_i may be selected at any profile point, which particularly convenient when the cross section is not symmetric, Single cell closed thin-walled beam cross section generalized sectorial coordinates $\hat{\omega}_A(s_i)$ are defined by Eq.6, Lujin [4], Shkolnikov [5].

$$\left.\begin{aligned} &\hat{\omega}_A(s_i) = \omega_A(s_i) - \overline{\omega}_A(s_i),\ \overline{\omega}_A(s_i) = \hat{S}_i\hat{R},\ \omega_A(s_i) = \sum_{k=1}^{i} h_{k-1,k}\, s_{k-1,k}, \\ &\hat{S}_i = \sum_{k=1}^{i}\frac{s_{k-1,k}}{\delta_{k-1,k}},\ \hat{R} = \frac{2\widetilde{A}}{\hat{\Pi}},\ \hat{\Pi} = \sum_{i=1}^{n}\frac{s_{i-1,i}}{\delta_{i-1,i}}. \end{aligned}\right\} \tag{6}$$

In Eq.6, sectorial coordinate $\omega_A(s_i)$ is defined similar to the open cross section sectorial coordinate: $h_{k-1,k}$ is the distance (radius R_k) from the pole A to cross section profile between nodal pints $k-1$ and k; $\hat{S}_i$ is generalized profile coordinate, $\hat{R}$ is generalized radius, $\widetilde{A}$ is cross section area within cross section profile, $\delta_{i-1,i}$ cross section profile thicknesses, and $\hat{\Pi}$ is generalized profile perimeter.

The diagram of rectangular cross section generalized sectorial coordinate $\hat{\omega}_A$ is presented on Fig.7b; warping deformations diagram, Fig.1b, is proportional to $\hat{\omega}_A$ diagram. Generalized sectorial coordinate $\hat{\omega}_A$ diagram components $\omega_A(s)$ and $\overline{\omega}_A = \hat{S}_i\hat{R}$ are presented on Fig.8a and Fig.8b.

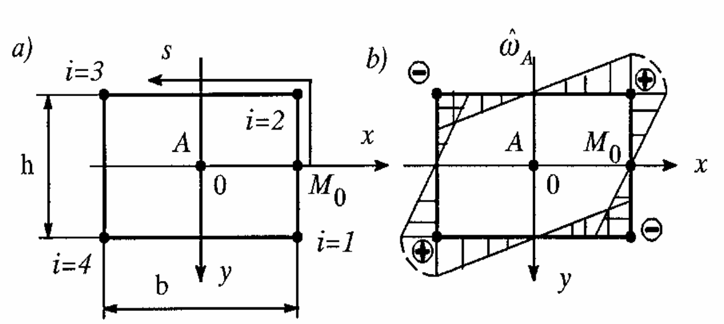

Fig.7. Closed cross section thin-walled beam: a) cross section definition, b) cross section sectorial coordinates diagram.

For a rectangular cross section with constant wall thicknesses δ sectorial coordinates $\hat{\omega}_A(s_i)$ may be defined by Eq.7. According to Eq.7, in a square cross section ($b = h$) all four sectorial coordinates are equal to zero, and such cross sections do not warp. Rectangular cross sections, Eq.6, with horizontal walls thickness δ_b, vertical walls thickness δ_h and dimensions $\frac{b}{\delta_h} = \frac{h}{\delta_b}$ also do not warp.

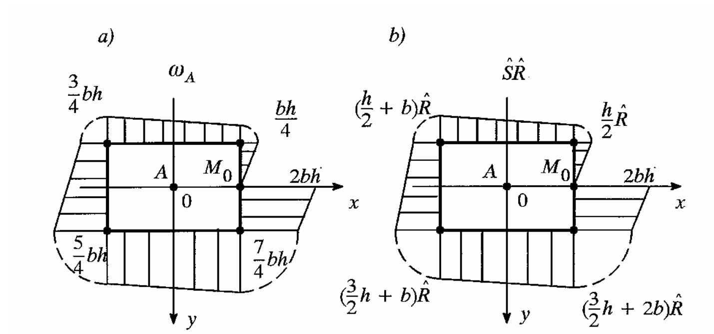

Fig.8. Generalized sectorial coordinate $\hat{\omega}_A$ diagram components: a) diagram $\omega_A(s)$ and b) diagram of $\overline{\omega}_A = \hat{S}_i\hat{R}$

$$-\hat{\omega}_A(s_1) = -\hat{\omega}_A(s_3) = \hat{\omega}_A(s_2) = \hat{\omega}_A(s_4) = \frac{bh(h-b)}{4(h+b)} \tag{7}$$

According to Eq.6, in all types of single cell thin-walled beam cross sections having $\omega_A(s) = \overline{\omega}_A(s)$ sectorial coordinates $\hat{\omega}_A(s) = 0$ and such cross sections do not warp under torsion.

Chiskis, P. and Parnes, R. [6] have presented by (similar to Lujin [4]) Eq.8 the derivative of constant thicknesses δ closed cross section longitudinal deformation defining cross section warping.

$$\frac{d\zeta}{ds} = \theta\left(\frac{2\tilde{A}}{\Pi} - R\right). \tag{8}$$

In Eq.8 θ is torsional angle, Π is cross section's profile perimeter, R is the radius of a circle, which touches all cross section profile parts or profile parts extensions, Fig.9. If the circle radius is $R = \frac{2\tilde{A}}{\Pi}$, Eq.8, $\frac{d\zeta}{ds} = 0$ and such cross section's warping deformations are equal to zero. As an example, in Fig.9 presented some thin-walled beams closed cross sections, which do not warp under torsion.

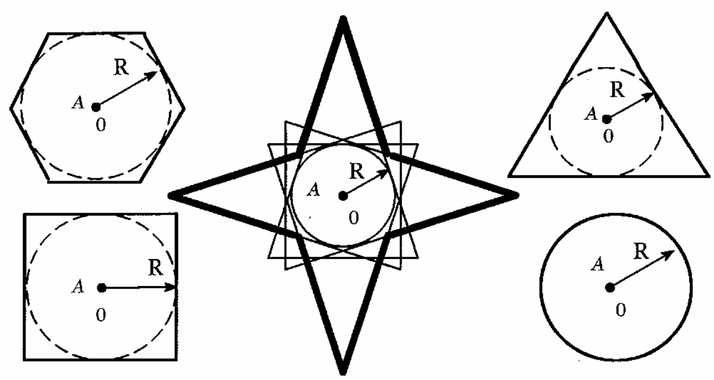

Fig.9. Not warping thin-walled constant wall thickness δ closed cross sections.

4. Thin-Walled Frames

Experiences of Thin-walled beam theory applications in analyses and tests have confirmed that modeling cylindrical shells as one-dimensional beams is practically sufficient and cost-effective. The problems, however, have accrued when using beams to model three-dimensional structures, in which beams intersect with each other via joints. In last century 60th and 70th several researches and publications, Belokurov at al [7], were dedicated to developing methods of three-dimensional frame-like strictures' joints design analyses. Fig.10 illustrates simplified frame joints. Frames with joints of Fig.10a and joints type "b" on Fig.10c, were considered in design analyses as completely constraining cross section warping. Therefore under frame torsion bimoments were generated, Fig.11, in frame rails and cross members. Maximums of bimoments, as self-equilibrium generalized forces were localized at joints, where warping was constrained. The level of localization however depends of cross sections walls' thickness, Fig.11a and b.

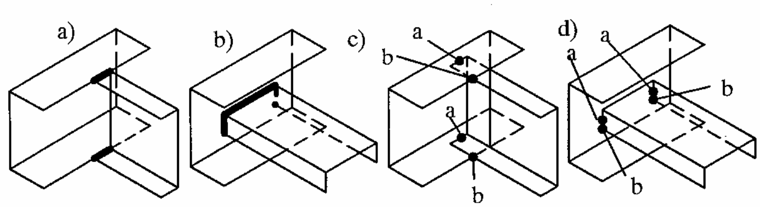

Fig.10. Thin-walled frame joints: a) and b) welded; c) and d) spot welded or riveted

Frames joints of type "a", Fig.10c and d, were considered as not constraining cross section warping. Such joints did not create bimoments or normal stresses under frame's torsion. Frames joints of Fig.10b type and joints of type "b" in Fig.10d were considered as insignificantly constraining cross section warping and creating practically negligible bimoments values.

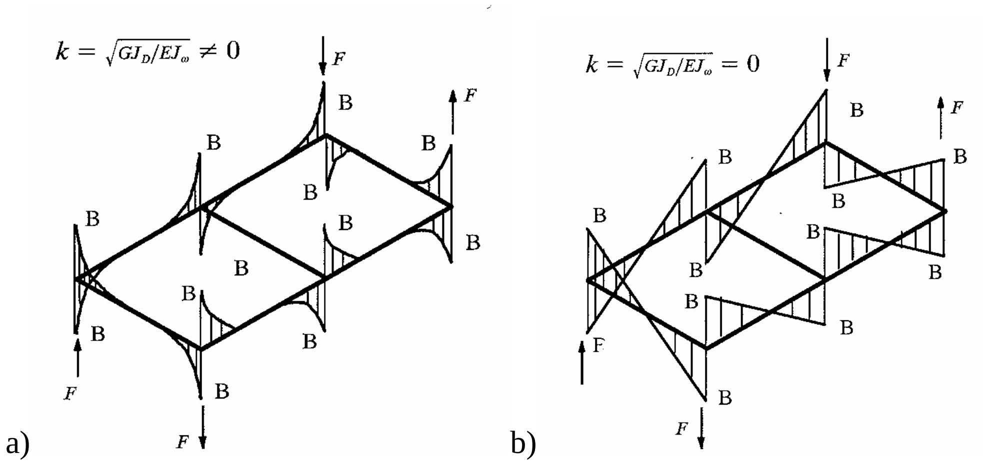

Fig.11. Bimoments B diagrams in frames with cross sections thickness: a) typical, b) extremely small ($k \Rightarrow 0$). J_D pure torsion moment of inertia, G shear modulus

Frames' rails, Fig12, are considered in design analyses having normal stresses σ from bending forces P and F . But under torsion by forces F , normal stresses σ in the rails are generated only with joints of type "b", while $\sigma = 0$ with joints type "a". Analytical modeling of frame joints directly connecting thin-walled beams to each other, (Fig. 10 and 12) is presented in publications: Belokurov at al [7] and Shkolnikov [8}], Fig.13a, Yang and McGuire [9], Fig.13b. Joints model in Fig.13a, comprises seven "rigid" or "flexible" pin-ended rods. The normal to cross section rods length dz is infinitesimal, therefore diagonal rods are considered to be parallel to cross sections' profiles. All rods are defined in beams principal xyz coordinates.

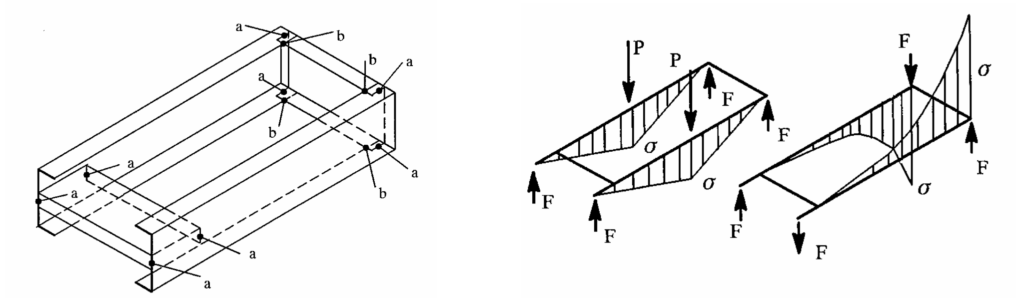

Fig.12. Thin-walled frame rails normal stresses σ under frame bending and torsion.

The rods location in Fig.13a is just an example of one of possible rods locations. Joints model in Fig.13b referred to as based on "warping spring concept", comprises springs with special linear constants, Yang and McGuire [9]. That model looks to be more sophisticated then in Fig.13a.

Fig.13 models look to be designed for modeling simple joints presented in Fig.10. In reality many of frame joints are made of complex geometry thin-walled shell-type parts, for which the applications of described methods may be questionable.

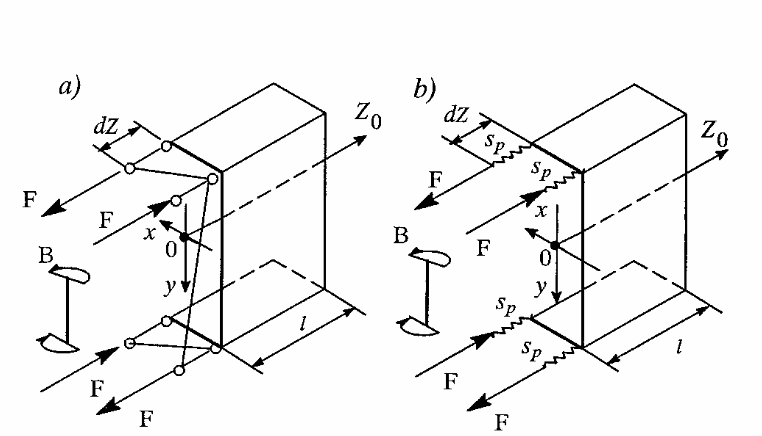

Fig.13. Thin-walled beam frame joints models

5. Three-Dimensional Thin-Walled Beam Elements

In the summary of previous Section 4, Thin-walled beam theory representing 3-D thin-walled cylindrical shell structures as 1-D thin-walled beams in application to 3-D structures has limitations. Example: based on the theory mathematical model, Fig.14, of a thin-walled frame represents the frame parts just by one line and the parts intersection just by one point. Modeling the frame, Fig. 14, using 1-D FEM beam elements, Shkolnikov [8], would have the same deficiency. Therefore, so far, thin-walled beams are modeled mostly using shell FEM elements, which is computationally more expensive.

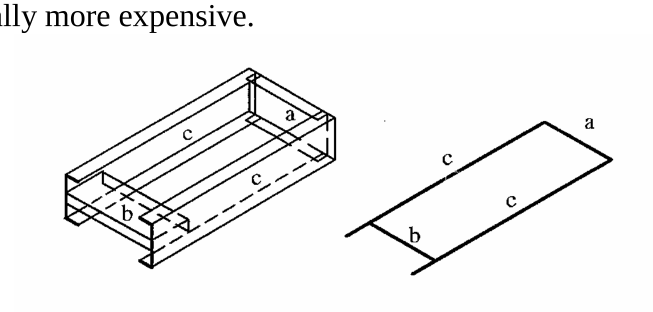

Fig.14. Thin-walled frame parts representation using Thin-walled beam theory

To eliminate the 1-D beam element deficiency a 3-D beam element, Shkolnikov [10], has been developed. The objective of the 3-D element development is to use computational advantages of Thin-walled beam theory, while making that element capable working with all other types of FEM elements in 3-D structures FEM models.

Fig.15 presents an example of modeling a Fig.14 thin-walled frame parts intersection using shell and thin-walled 3-D beam elements. Such models correctly represent structures geometry and details, while having lesser number of nodal points than with shell elements only and therefore having lesser systems of equations size to solve. The need of Section 4 assumptions and simplifications including simplified joints modeling, Fig.13, in most cases might be not necessary.

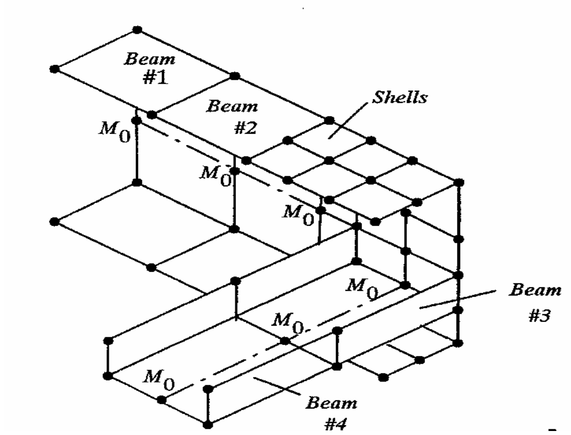

Fig.15. Thin-walled beam and shell elements FEM model of Fig.14 thin-walled frame parts intersections

Conclusion

1. Implementation of open cross section thin-walled FEM beam element in LS-DYNA is an important beginning.
2. Further implementations in that area would result in wider usage of computationally efficient beam FEM elements in FEM models.
3. Thin-walled closed cross section beam FEM element implementation in LS-DYNA might be the next step.
4. The advantage of implementing in LS-DYNA three-dimensional thin-walled open, closed and solid cross section FEM beam elements has been illustrated.
5. Presented information on thin-walled beam design analyses, research, and three-dimensional thin-walled structures joint modeling will be helpful to LS-DYNA users.

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