Strain Rates in Crashworthiness

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Abstract

Strain rates related tests, strain rates measurements, strain rates states during vehicle collisions, crashworthiness tests and simulations are discussed in the paper. Several papers (on which some of LS-DYNA strain rates options in constitutive models for metals are based) are considered in this paper from an automotive vehicle crashworthiness point of view. Strain rates effect on structure's metals during explosions are mathematically described in the papers. The objective of crashworthiness is not (like under explosion) to save a structure, but to sacrifice (making failing in a control manner) the structure to save its occupants during vehicle collisions. Sufficiently taking (during crashworthiness design) into account strain rates in vehicle structures during collisions will increase the structures energy absorption capability and increase their occupants' survival probability.

Introduction

Significant amount of publications have been dedicated to strain rates effect on material properties. Accordingly, new material theories and tests methods have been developed for both metal and non-metal (polymer, rubber) materials. Many constitutive models have been developed to mathematically describe the strain rates effect on metals during explosions and impact loads, and implemented in LS-DYNA.

Though there are many similarities of material performance in structures under explosion and vehicle structures during collisions, the objectives of the structures design under explosion and vehicle collisions may be fundamentally different.

If the objective of a structural design under explosion is to prevent structure's failure, the followings are considered. During explosions material strength reduces due to rising temperatures and accumulations of damage, while strain rates make metals harder/stronger. The underestimation of strain rates levels in structural design under explosions (by accepting lower strain rates levels than the levels would be during explosions, or not accounting for strain rates at all) would result in a higher probability of preventing the complete structures collapses, while increasing the structures weight.

The objective of crashworthiness related design is to keep during vehicles collisions occupants' body accelerations and impacts within the prescribed safe levels. Making a vehicle structure collapsing in a controlled manner during collision to absorb the collision impact energy does that. The underestimation or overestimation of strain rates in crashworthiness design may reduce the structure energy absorption capability. That would increase occupant's body accelerations, resulting in more injuries, which would reduce occupant's survival probability.

From the vehicle crashworthiness point of view, methods of strain rates related tests, strain rates measurements, strain rates states during vehicle collisions, crashworthiness tests and simulations are discussed below.

Strain Rates Effect in Constitutive Models

Some of constitutive models based on publications [1,2,3,4,5] and implemented in LS-DYNA, will be considered below. Selected information from these publications is presented here. The mathematical expressions from these publications are presented without details just as illustrations. The objective here is to understand these publications from a crashworthiness point of view.

The common assumption in these publications is that strain rates affect only the stresses after the yield point. These stresses are referred to as (dynamic, plastic) flow stresses. The common assumption is also that material tests were conducted under impacts at a series of constant strain rates.

Cowper G. R. and Symonds P. S [1], (CS), have developed their strain rates theory based on bending impact tests of cantilever beams with a mass on one of their ends. The impact velocity was up to 109 km/h and strain rates within 100 1/sec. The basic expression used is

$\sigma = \sigma_0 + \left(\frac{\dot{\varepsilon}}{D}\right)^{\frac{1}{p}}$, where σ is bending stress in a beam, σ_0 is static yield stress, $\dot{\varepsilon}$ is strain rate, p and D are experimentally defined material constants.

The theory's assumptions are that flow stress curve is constant, and the plastic zone in the beam is very small. The theory is based also on some considerations on strain hardening and damage effects. During the tests the plastic zone was [1] "actually so large as to invalidate the theory". The conclusion was that "the further development of the theory…is desirable".

Johnson G. R. and Cook W. H. [2], (JC) constitutive model uses von Misses flow stress, which is based on the expression

$\sigma = \left(A + B\varepsilon^n\right)\left(1 + C\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_O}\right)\left(1 - T^m\right)$, where *A, B, C, m, n* are experimentally defined material constants/parameters, *T* is a function of temperature.

The five parameters for 15 metal (copper, steel, etc.) materials were defined from torsion and tension tests of specimens under different levels of temperatures and strain rates. The model confirmation was done using "Taylor's cylinder tests": on cylinder-specimen compression tests under strain rates in excess of $10^5\ \text{sec}^{-1}$.

Zerilli F. J. and Armstrong R. W. [3], (ZA) analyzed and further developed "empirical JC constitutive model" using "dislocation-mechanics-based constitutive relations", which improved "Taylor's cylinder tests" simulation for metals of different microstructure types. For steel like metals, ZA's flow stress σ as a function of experimentally defined constants/parameters, strains ε and strain rates $\dot{\varepsilon}$ [3] is defined by the expression

$$\sigma = C_1 + C_2 e^Q + C_5 \varepsilon^n + C_6, \;\; Q = T(-C_3 + C_4 \ln \dot{\varepsilon}).$$

Moudlin P. J., Davidson R. F., Henninger R. J. [4] presented a constitutive model (referred to as MTS), which is based on a flow-stress σ,

$$\sigma = \sigma(\hat{\sigma}, \dot{\varepsilon}, T, P), \;\; \hat{\sigma} = \hat{\sigma}(\varepsilon, \dot{\varepsilon}, T, P),$$

where σ is based on dislocation mechanics and includes pressure *P*, strain rates $\dot{\varepsilon}$, temperature *T*, and a new variable $\hat{\sigma}$, referred to as mechanical-threshold-stress (MTS).

The MTS constitutive model has been also evaluated by simulating "Taylor's cylinder tests"(diameter 7.62mm, height 25.4mm for copper). Fourteen MTS parameters for each of several materials (copper, uranium, titanium) were defined, and the results were compared with the results of JC model. The evaluation of compression material tests results shows that MTS is accurate [4] "in the problems containing mostly normal strain with shear strain less than 0.2 but perhaps not as accurate for problems that contain large amounts of shear strains." MTS requires extremely large numbers (fourteen) of constants/parameters, which must be defined from material tests.

Cady C. M. at al reported in [5] their extensive research and number of tests "to characterize the dynamic mechanical properties of four different structural sheet steels used in automobile manufacturing". The parameters for JC, ZA and MTS constitutive models were defined by fitting experimental data at different series of constant strain rates and temperatures. Biaxial and through-thickness compression tests were considered to be [5] "the best option for crash-worthiness scenario, where the punching type loads are critical". The tests for strain rates below 500 1/sec were conducted on "a specially built high rate" MTS-testing machines. The tests for strain rates between 1000 1/sec and 7000 1/sec were conducted on Hopkinson-Bar testing machines.

Conclusions in [5] were based on the results of through-thickness compression tests of cylindrical sheet steel samples with diameters 0.1"- 0.2" and heights 0.063"- 0.1". The compression true stress-true strain curves dependence of specimens' aspect ratios was found.

If the results of their impact tests would be questionable in "a crashworthiness scenario": [5] "alternately, the lowest stress/strain curves could be used in the analysis of the constitutive models. This will ad to the factor of (product) safety, although it will also add weight to the product."

Crashworthiness & Constitutive Models

As mentioned in the Introduction, the objectives of the structure design under explosion and in crashworthiness may be fundamentally different. The objective of crashworthiness is not (like under explosion) to save a structure, but to sacrifice (make failing in a control manner) the structure to save its occupants.

Constitutive model CS [1] for bending cantilever beams looks to be the closest to the need of vehicle crashworthiness design, where large bending (normal and shear) deformations look to be the more desirable for the energy absorptions needs. But the model's theory was questionable by the authors [1], and further developments are needed, but unknown.

Constitutive models JC, ZA and MTS [2,3,4,5] parameters have been defined by fitting the results of (cylindrical samples') compression tests. The results include information from the tests at high temperatures, under pressure, and justified for strain rates in excess of $10^3\,\sec^{-1}$. Only flow stress is taken into account.

The justification for using the compression tests was based on the fact that for the strain rates higher than $10^3\,\sec^{-1}$ only Hopkinson-Bar testing devise must be used. Due to the compression tests, even the most advanced and extremely complex constitutive model MTS, is [4] "not as accurate for problems that contain large amounts of shear strains." That is probably logically true for JC, ZA models.

Constitutive models MTS, JC and ZA look to need more developments to be used efficiently for crashworthiness structural design. The results of the compression tests look questionable to use in vehicle crashworthiness impacts simulations. To save occupants, vehicle parts (like rails, doors, roof, etc.) must be designed to absorb impacts energy by having large (normal and shear) deformations and flow stresses.

The recommendation in [5] (which is reasonable for explosion) to use "the lowest stress/strain (strain rates) curves" does not look justifiable for crashworthiness design as safety of occupants is the objective of crashworthiness, not the safety of vehicle structure. Not taking into account correctly strain rate hardening of vehicle structure material during collision may reduce the structure energy absorption capability and reduce the occupants survival probability.

These considerations suggest that while it is useful to learn from the many years of material research under explosions, and use what is applicable for crashworthiness design, some additional research work may be necessary for crashworthiness design.

Specimen Tests Under "Constant" Strain rates

To illustrate what kind of strain rates are generated during impact tests, specimen displacement and velocity curves versus time are presented in Fig.1. The curves were recorded from a testing machine programmed for "loading" and "unloading" stages with velocity up to6 km/h.

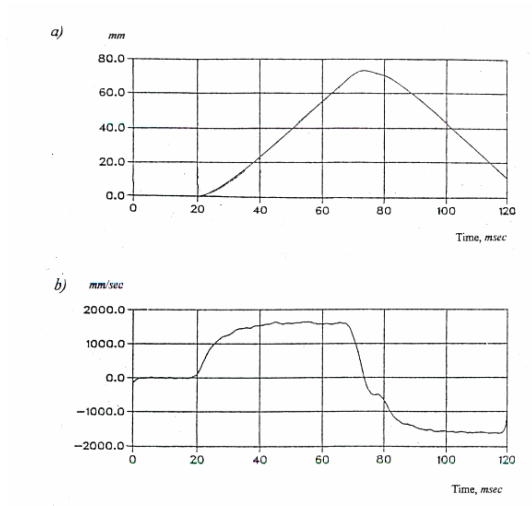

Fig. 1. Records from low velocity impact tests: a) displacements, b) velocity.

The curves of Fig.1 were filtered at SAE 180Hz. The filtering is a tradition (from the time when structure design was under static loads only) to consider all oscillations during all tests as noise (i.e. as errors). Under dynamic loads and particularly under impacts, structure's oscillations are a reality, not noise.

The filtered displacement curve in Fig.1a looks almost linear during the loading stage. Such displacement curve sometime is replaced with a strait line, which slope is used to define "constant" velocity during material tests. The velocity curve in Fig.1b is not constant. It takes time (to accelerate) to reach the required velocity and (to decelerate) to reach zero velocity. Even during the time (40 to 70msec), when the velocity is closer to constant, some oscillations on the filtered curve are seen. Testing machine has to fluctuate around the required velocity level to reach that level; therefore the test velocity can never be constant.

The displacement and velocity curves from Fig.1 can be used to assess possible strain rates during the tests. In this case we have one-dimensional displacements and velocities along a vertical coordinate axis *z*. For infinitesimal strains, strain rates [6] are approximately equal to the rate-of-deformation

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} \approx \frac{\partial v}{\partial z}, \tag{1}$$

where ε is strain, *v* is velocity along axis *z*.

The following approximations are used here

$$\left.\begin{aligned} \partial v &\approx v(t_i) - v(t_{i-1}) \\ \partial x &\approx u(t_i) - u(t_{i-1}) \end{aligned}\right\}, \tag{2}$$

where *u* is displacement along axis *z, t* is time at curves' points *i* and *i-1* .

The velocity and displacements curves from Fig.1 were digitized. Then using Eq.1 and 2, the approximation of a strain rates versus time curve was developed and shown in Fig.2. An approximation of strain rates versus engineering strain at the loading stage is shown in Fig.3.

As could be expected, the strain rates are not constant. The maximum strain rates are at the beginning and the end of loading, and they are significantly larger than during loading. After the beginning of loading and before the end of loading strain rates are smaller (at the level of plus/minus 50 1/sec), but they are not constant, they are oscillating. The strain rates oscillations and amplitudes should be expected to be higher at higher impact velocities.

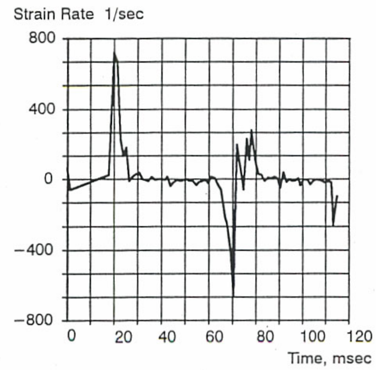

Fig. 2. Strain rates versus time from the Fig.1 records.

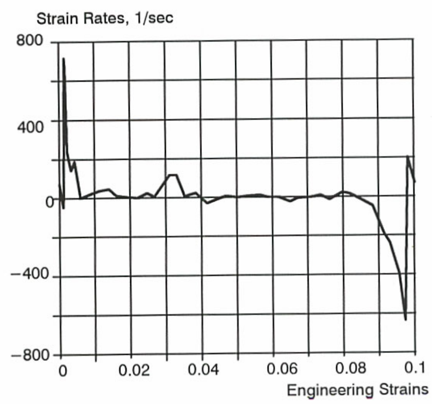

Fig. 3. Strain rates versus engineering strains from the Fig.1 loading stage records.

While the Fig.2 and 3 strain rates values are approximate (due to the Fig. 1 curves filtering and digitizing), Fig.2 and 3 provide good illustrations to the fact that constant strain rates are not possible to generate even during low velocity impact tests. The higher impact test velocities the farther from constant should be expected strain rates values in the tested specimens.

If strain rates during specimens tests are not constant, then how accurate is the use of such tests results presented as “at constant strain rates” in FEM crashworthiness simulations requiring at each time step a material property at a particular level of strain rates? If strain rates always oscillate, what is really affects the material properties: strain rates or the rates of strain rates?

Strain Rates in Crashworthiness Tests and Simulations

The strain gages are considered here to be useful to define (for vehicle structure crashworthiness design analysis) the levels and states of strain rates generated during crashworthiness tests or vehicles collisions. Some general information of using strain gages to measure strain rates for crashworthiness is presented below.

a) Strain Rates Measurements

Traditionally strain gages are used to measure stresses under infinitesimal strains. The reason for the strains being infinitesimal is the need to use Young's modulus to define stresses from strains. Strain gages could be used to measure final strains, which magnitude is limited only by the gage strength. Strains in a vehicle structure under impact loads may be final.

Strain Gage Size and Locations

The selection of strain gages sizes are usually based on the assumption that strains vary along all structure surface directions, and each strain gage measurement represents an average of strains along the strain gage base (i.e. its length). Therefore the reasonably smaller strain gages bases are more reasonable to use to measure strains in a particular structure point. A five-mm-base strain gage looks practically reasonable to use in this particular application. Strains measured with such a gage could be small, but rather final, not infinitesimal.

Strain gages should be installed on the vehicle structure surfaces where the highest strains are expected. When strains are very low, the measurements may not be stable, and the strain rates defined from such strain measurements would be questionable. Therefore, a preliminary structure surfaces strain analysis may be required. It may be needed both analytical (FEM) and experimental analyses. It is important to predict both possible locations and directions of maximum strains.

In Fig.4 presented are some schematics [7] of strain gage locations on vehicle structure parts. In the vehicle body parts with rectangular cross sections, Fig.4a, the surfaces at cross sections' corners and panel's edges, Fig.4c, are the probable locations of maximum normal (longitudinal) strains. The shear strains at the corners and edges are close to zero. Single (longitudinal) strain gages, Fig.4a, could be used if their locations would be as close as possible to the corners or the edges. The measured strains in a part's rectangular cross section may be decomposed in three [7] components: tension/compression, bending about x and y cross section axes. To do the decomposition, three single strain gages located at across section as far from each other as possible are necessary. These three longitudinal strain gages in Fig.4 are referred to as required. If more than required gages are installed, statistical estimations of the measured strains precision may be done [7]. The additional gages are referred to in Fig.4 as redundant.

In the locations as in Fig.4c inside a panel, and as in Fig. 4b away from the corners, rosettes of strain gages are necessary to be able to define maximum strains. A rosette of two gages may be used if the direction of maximum normal strains is known. In general, a three gages rosette is

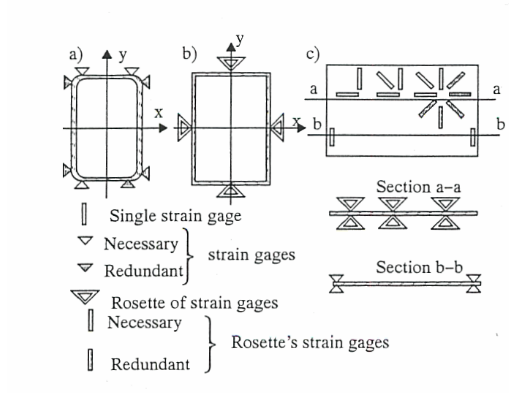

Fig. 4 Examples of strain gage locations on a vehicle structure: a) gages in cross sections b) rosettes in cross sections, c) rosettes and gages on panels

used to define the extreme or principal values of both normal and shear strains. These two rosettes are rosettes with required gages. Additional (redundant) gages in a rosette allow defining statistical estimations [7] of the measured strains precision.

Thin-walled vehicle body's panels usually locally buckle under impact loads. Therefore, strain gages on both panel sides, (sections a-a and b-b in Fig. 4c) are necessary.

Strain Measurement with a Single Strain Gage

Single strain gage is used to measure strain ε_φ on a structure surface along the strain gage longitudinal axis inclined with angle φ to the *x*-axis of an arbitrary *x-y* coordinate system. Strains measured during a period of time could be differentiated in time to compute strain rates.

From Fig.5 the expressions Eq.3, 4, 5 and 6 [7] can be defined for small but final longitudinal deformations of a strain gage with small length *L* (base) in a not deformed structure surface.

$$\left.\begin{aligned}\Delta x &= \varepsilon_x L_x + \frac{1}{2}\gamma_{xy} L_y \\ \Delta y &= \varepsilon_y L_y + \frac{1}{2}\gamma_{xy} L_x\end{aligned}\right\}, \tag{3}$$

where $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ the normal and shear strain components in the arbitrary *x-y* coordinates.

$$L^2 = L_x^2 + L_y^2 . \tag{4}$$

$$L_1^2 = L_{1,x}^2 + L_{1,y}^2 . \tag{5}$$

$$\left.\begin{aligned}L_{1,x} &= L_x + \Delta x \\ L_{1,y} &= L_y + \Delta y\end{aligned}\right\}. \tag{6}$$

The length of the strain gage on a deformed structure surface is

$$L_1 = L(1+\varepsilon_\varphi) . \tag{7}$$

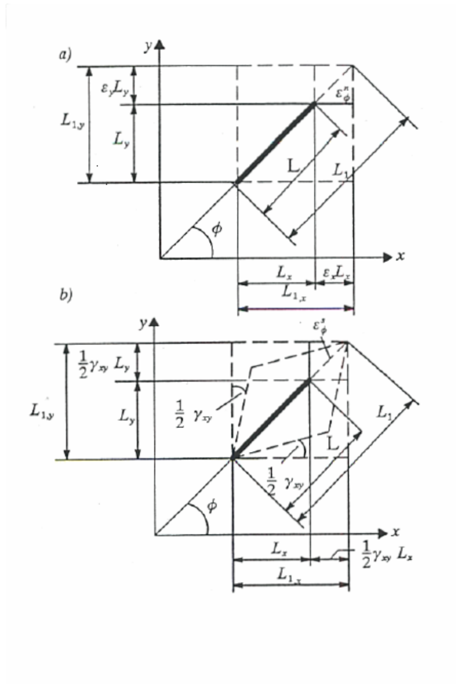

Fig. 5. Strain gage longitudinal deformations from a structure surface normal and shear deformations.

Using Eq.7 along with Eq.3, 4, 5, and 6 we can define the expressions for the measured by a strain gage on the structure surface final strain ε_φ from both normal and shear components as

$$\varepsilon_\varphi = l^2\varepsilon_x + m^2\varepsilon_y + lm\bar{\gamma}_{xy} + \frac{1}{8}\gamma_{xy}^2, \tag{8}$$

$$\bar{\gamma}_{xy} = \frac{1}{2}\gamma_{xy}\left[(1+\varepsilon_x)+(1+\varepsilon_y)\right], \tag{9}$$

where $l = Cos\,\varphi$ and $m = Sin\,\varphi$.

By minimization Eq.8 using $\frac{d\varepsilon_\varphi}{d\varphi} = 0$, angles φ_1 and $\varphi_2 = \varphi_1 + \frac{\pi}{2}$ (the slopes of principal coordinates 1 and 2 with respect to the arbitrary coordinate *x*) are defined as

$$2\varphi_{1,2} = arctg\,\frac{\bar{\gamma}_{xy}}{\varepsilon_x - \varepsilon_y}. \tag{10}$$

Note the independence of principal coordinates slopes φ_1 and φ_2, Eq.10, from γ_{xy}^2. By inserting φ_1 from Eq.10 in Eq.8 the principal normal strain ε_1 along the principal coordinate 1 can be defined. By inserting φ_2 from Eq.10 in Eq.8 the principal normal strain ε_2 along principal coordinate 2 can be defined. By traditional convention index 1 is assigned to the maximum value strain, while index 2 to the minimum value strain, both of which have been defined from Eq.8 and Eq.10. Eq.11 defines the principal shear strain (with angle $\varphi_3 = \varphi_1 + \frac{\pi}{4}$).

$$\gamma_{12} = \frac{\varepsilon_1 - \varepsilon_2}{2}. \tag{11}$$

When the strains are infinitesimal (ε_x and ε_y are significantly smaller than 1, and the value of γ_{xy}^2 is negligible), Eq.8 transforms to a traditional expression for infinitesimal strains, [6]:

$$\varepsilon_\varphi = l^2\varepsilon_x + m^2\varepsilon_y + \gamma_{xy} lm.$$

In Eq.8, 9 and 10 strain components ε_x, ε_y and γ_{xy} are unknown. The normal and shear strain components ε_x, ε_y and γ_{xy} may be experimentally defined using strain gages as described next.

Experimental Strain Definition

To measure strain components ε_x, ε_y and γ_{xy} a three-gage rosette with two gages perpendicular to each other, and the third at 45-degree angle may be used .The two rosette's perpendicular to each other strain gages should be installed along *x* and *y* arbitrary *x-y* system coordinates. Then the normal strains ε_0, ε_{45} and ε_{90} will be measured by these three gages. By inserting each of the measured strains in the left side of Eq.8 as: $\varepsilon_\varphi = \varepsilon_0$, $\varepsilon_\varphi = \varepsilon_{45}$ and $\varepsilon_\varphi = \varepsilon_{90}$, a system of three equations, Eq.12, can be defined.

$$\left.\begin{aligned}
&\varepsilon_x = \varepsilon_0 - \frac{1}{8}\gamma_{xy}^2,\\
&\varepsilon_y = \varepsilon_{90} - \frac{1}{8}\gamma_{xy}^2,\\
&\frac{1}{8}\gamma_{xy}^2 + \gamma_{xy}\frac{1}{4}\left(2 + \varepsilon_x + \varepsilon_y\right) - \left[\varepsilon_{45} - \frac{1}{2}\left(\varepsilon_x + \varepsilon_y\right)\right] = 0.
\end{aligned}\right\} \tag{12}$$

The Eq.12 solutions ε_x, ε_y and γ_{xy} could be use along with Eq.9, 10 and 11 to define the values of principal coordinate exes slopes φ_1, φ_2 and φ_3 to *x*-axis, and principal strains $\varepsilon_1, \varepsilon_2, \gamma_{12}$. Note again that the definitions of φ_1 and φ_2 from Eq.10 do not depend on γ_{xy}^2. Therefore, for approximate estimations, it is possible to ignore γ_{xy}^2 in Eq.12. Eq.13 and 14 would define then the normal and shear strains components.

$$\varepsilon_x \approx \varepsilon_0,\ \varepsilon_y \approx \varepsilon_{90}. \tag{13}$$

$$\gamma_{xv} \approx \frac{2\left(2\varepsilon_{45} - \varepsilon_0 - \varepsilon_{90}\right)}{2 + \varepsilon_0 + \varepsilon_{90}}. \tag{14}$$

Eq.13 and 15 define normal and shear strains components if the strains are infinitesimal.

$$\gamma_{xy} \approx 2\varepsilon_{45} - \varepsilon_0 - \varepsilon_{90}. \tag{15}$$

b) Strain Rates from Measurements and Simulations

The presented above expressions for strains from strain gage measurements are not functions of time. But strain gages measurements conducted during vehicles crashworthiness tests or collision would provide strain versus time curves, which could be differentiated in time to define strain rates versus time curves.

The strain rates versus time curves defined using strain gages from vehicle crashworthiness tests show that the strain rates are not constant. The strain rates defined from LS-DYNA vehicle crashworthiness tests simulations are not constant as well.

In fact, the strain rates are oscillating. How these oscillating strain rates affect vehicle material properties and structure energy absorption? What should be taken into account: strain rates or the rates of strain rates?

Conclusions

1. It is beneficial to use what is applicable for crashworthiness design from the many years' experience of material research, constitutive model developments and structural design under explosion. More similar work on strain rates and their effect during vehicle collision may be necessary for crashworthiness design.

2. Physically it does not look to be possible to generate constant strain rates during impact tests. The use of averages of strain rates from the tests as constant strain rates may not be sufficient for crashworthiness design.

3. For crashworthiness design it is necessary to conduct more complex tests than the compression through thickness tests. The tests should account for both normal and shear strains.

4. For crashworthiness design, constitutive models should be capable to take into account both normal and shear strains.

5. One of the ways to measure strain rates is the use of strain gages.

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