Dynamic Pulsebuckling Analysis of FRP Composite Laminated Beams Using LS-DYNA

Zheng Zhang and Farid Taheri *

Department of Civil Engineering

Dalhousie University.

P.O. Box 1000, Halifax, NS, Canada B3J 2X4

Abbreviations

FE	Finite Element
FRP	Fiber Reinforced Plastic
FS	Fixed-end Support
PS	Pinned-end Support

Keywords: composites, impact, buckling, axial, momentum, finite element

* Corresponding author. Tel.: +1-902-494-3935, Fax: +1-902-484-6635.

E-mail address: Farid.Taheri@dal.ca

ABSTRACT

Buckling and post-buckling of composite structures have been important research topics since composite materials became widely used in engineering. As a result, significant volume of research has been done on their static stability, while relatively less has been devoted on characterizing their dynamic buckling and post-buckling response. The literature became particularly scares when considering the dynamic pulsebuckling and post buckling of axial components subjected to axial impact. This paper, therefore, presents the findings of our finite element analysis of dynamic pulsebuckling response of slender laminated fiber reinforced plastic (FRP) composite beams, with initial geometric imperfection, subject to axial impulse using LS-DYNA.

Dynamic pulsebuckling, as an instability form, or in the form of excessive growth of lateral or out of plane displacements, is resulted from a transient loading function of a single pulse with a magnitude greater than the static Euler buckling load. The FRP laminated composite beam with initial geometric imperfection, subject to axial impact of a moving object, is modeled by the Belytschko-Tsay shell element. The moving object is defined as a rigid wall with a mass and initial velocity. Dynamic pulsebuckling of an imperfect beam is characterized by the sudden and drastic increase in the lateral deflection while the axial load bearing capacity remains unchanged relatively when the impact momentum reaches a critical value. Numerical results show that momentum of the moving object may be considered as a viable parameter for predicting the dynamic pulsebuckling limit of the beam.

In this investigation, the effect of initial geometric imperfection used to promote instability was investigated and was shown to be a significant factor in promoting pulsebuckling. The effect of boundary conditions was also investigated and the significances of the axial and rotational restraints were demonstrated with numerical examples. A predictive criterion for the onset of pulse buckling was also presented.

INTRODUCTION

The buckling and post-buckling analysis of composite structures has been an important research topic since due to their high strength-to-weight ratio, composite materials have became widely used in engineering structures. As a result, a significant amount of research has been done on their static buckling, while relatively less has been devoted on characterizing their dynamic buckling and post-buckling. On the topic of dynamic buckling and post-buckling, many of the works concentrated on instability of plates/shells/beams under in-plane and transverse periodic loads (Lam and Ng, 1998, Liaw and Yang, 1990, and Zhou, 1991). To the authors' knowledge, much less work has been done on dynamic buckling and post-buckling of slender laminated composite beams subject to an axial impulse, although axial impulse is a very common loading type for slender engineering structures.

Dynamic pulsebuckling, as an instability form, or in the form of excessive growth of lateral or out of plane displacements, is resulted from a transient loading function of a single pulse with a magnitude greater than the static Euler buckling load. Considerable work (Housner and Tso, 1962, Abrahamson and Goodier, 1966, Ari-Gur, et. al., 1982, et. al.) has been done regarding the dynamic pulsebuckling analysis of metallic components (isotropic) since the earlier work of Koning and Taub(1934).

As stated, relatively much less volume of research has been performed on the dynamic pulsebuckling and post buckling response of FRP beams and columns in comparison to the works on their metallic counterpart. Ekstrom (1973) investigated the elastic buckling of a simply supported rectangular orthotropic plate, with initial imperfection, under a compressive pulse load. Ari-Gur and Simonetta (1997) constructed an analytical model of dynamic pulsebuckling of rectangular composite plates based on the Kirchhoff thin plate deformation theory with the assumption of small rotation of the cross-section. Wang et al. (1998) focused their investigation on the dynamic buckling of laminated composite bars based on the Timoshenko beam theory. Recently, Zhang and Taheri (2002[a], 2002[b], 2002[c]) also reported their investigations on dynamic pulsebuckling of laminated beams with initial geometric imperfections. The finite difference method was the main scheme to solve the dynamic differential equations.

The purpose of this paper is to present the details of our finite element investigation on the dynamic pulsebuckling and post-buckling response of laminated composite beams with initial geometric imperfection subjected to axial impulse which was conducted by LS-DYNA(1999). Moreover, it was also to verify the results generated by the finite difference solutions presented elsewhere by the authors (2002[a], 2002[b], 2002[c]). The validity of a proposed dynamic pulsebuckling criterion, as well as the influence of geometric imperfection, slenderness ratio and boundary conditions of beams on their dynamic pulsebuckling response were all investigated and discussed.

FE MODEL OF LAMINATED COMPOSITE BEAM

We consider an *n*-layer FRP laminated beam with one end impacted by a moving mass (M_0) with velocity (V_0). The length of the beam is *L*. The cross section of the beam is retangular (as shown in Figure 1), with width *b* and thickness *h*. The initial geometric imperfection is $w_0(x)$. The laminated beam is modeled as orthotropic elastic laminated plate with the Belytschko-Tsay shell elements, as shown in Figure 1(c). The moving mass is defined as a rigid wall with a mass and initial velocity. The damping property is ignored during this analysis due to the short time duration of the impact.

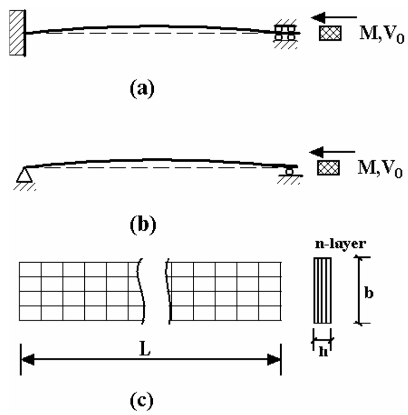

Figure 1: The Laminated FRP Beam impacted by a moving mass – (a) Fixed-end support (FS); (b) Pinned-end support (PS); (c) Finite element mesh.

DYNAMIC PULSEBUCKLING

Problem descriptions

The beam made of 8 layers of E-glass/epoxy $[(0)_8]$, with the boundary condition shown in Figure 1(a) is considered. The initial geometric imperfection of the beam is assumed to have a half sine wave form $w_0(x) = W_0 \sin(\frac{x\pi}{L})$. The right end of the beam is impacted by a moving mass. The geometrical and material properties are tabulated in Table 1.

Table 1: Geometric and material properties of the beam

Length (mm)	Width (mm)	Thickness (mm)	Imperfection W_0(mm) ($W_0 \sin(\pi x/L)$)	E11 (N/m^2)	E22 (N/m^2)	G12 (N/m^2)	γ_{12}	Density (Kg/m^3)
300	20	1.6	Factor * thickness	39E9	86E8	38E8	0.28	2100

Dynamic pulsebuckling of the beam

As mentioned, dynamic pulsebuckling, as an instability phenomenon, is characterized with excessive growth of the lateral, or out of plane displacements while the load bearing capacity of the component remains relatively unchanged. In our studies, the beam is impacted by a moving mass, in which the dynamic response of the beam varies with the different impact velocity of the moving mass. When the velocity of the moving mass is increased to the vicinity of some 'critical value', with a small increase of impact velocity, the deflection of the beam would increase dramatically, while the axial load capacity of the beam remains relatively constant, as shown in Figure 2, which shows the results of the beam with initial geometric imperfection amplitude $W_0 = 0.01h$, impacted by a moving object with mass $M_0 = 0.1Kg$.

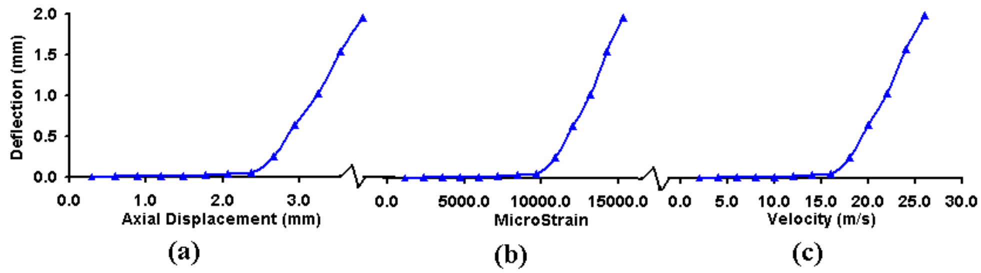

Figure 2: FE results of dynamic pulse buckling response of the beams - Deflection at mid-span of beam versus (a) axial displacement at the impacted end; (b) axial compressive strain at the neutral surface at mid-span of the beam; (c) initial velocity of moving mass.

INFLUENCE OF INITIAL GEOMETRIC IMPERFECTIONS

To investigate the influence of initial geometric imperfection of the beam on the critical velocity or momentum (in the next section we will demonstrate that the momentum can be viewed as a criterion of pulsebuckling for the beams), three beams with length of 450, 300 mm and 150 mm, each with width of 20 mm and different initial geometric imperfection of $W_0 = 0.001h$, $W_0 = 0.01h$, and $W_0 = 0.10h$, respectively, were analyzed subject to varying impact velocities. The resulting lateral deflection at the mid-span of the beam versus the impact momentum is shown in Figure 3. Figure 3 shows the resulting mid-span deflection versus impact momentum, for the varying length beams. From the figure it is obvious that the larger the initial geometric imperfection, the smaller the critical momentum. Figure 3 also indicates that regardless of the initial imperfection amplitude, once the momentum reaches a critical value, with a minor increase in the momentum, the beams undergo dynamic buckling (i.e., the lateral deflection increases rapidly and excessively). However, the axial load bearing capacity (i.e., the axial displacement and axial compressive strain), does not increase in the same rate as the lateral deflection (see details in Zhang Taheri, 2002[b]).

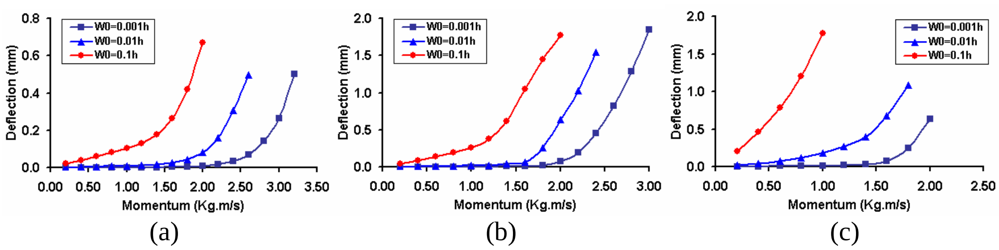

Figure 3: Deflection at mid-span of beam vs impact momentum for different beam lengths with initial geometric imperfection – (a) 450 mm; (b) 300 mm; (c) 150 mm.

PULSEBUCKLING CRITERION

Momentum as a dynamic pulsebuckling criterion

Momentum (M_t), defined as the product of mass (*M*) and impact velocity (V_0) of the moving mass, can be considered as a parameter that initiates dynamic instability. It is assumed that the beams undergo dynamic instability when their momentum approaches a critical value. This phenomenon can be seen from Figure 4. The figure shows the variation of mid-span deflection as a function of momentum for various imperfections and impact masses and velocities, for the 300mm long beam. This plot illustrates that the critical buckling momentum for a beam with a given initial imperfection is relatively fixed, irrespective of variation of the impact mass and velocity. Therefore, momentum may be used as a reasonable criterion for predicting the onset of dynamic pulsebuckling of FRP laminated beams subjected to axial impact of a moving mass.

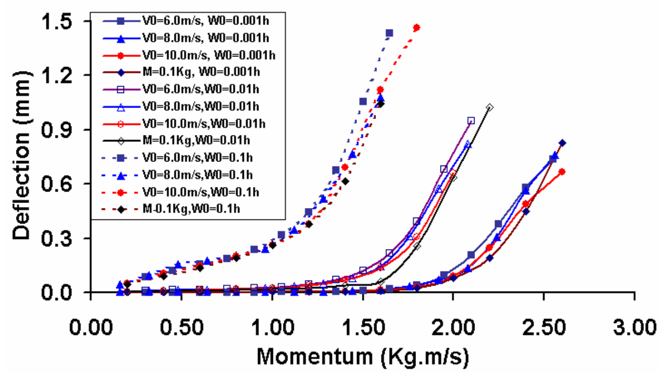

Figure 4: Variation of mid-span deflection as a function of momentum for various imperfections and impact masses and velocities.

Critical momentum versus slenderness and Curvature of beam

Under static and quasi-static loading conditions, the critical Euler buckling load increases with the decrease of slenderness ratio. This, however, does not hold necessarily in dynamic pulsebuckling. Comparing the graphs in Figure 3, one would observe that the beams' critical buckling momentum decreases as the length get shorter (i.e., the slenderness ratio decreases), for all initial imperfection values tested. This is opposite to what common sense would lead one to expect, as one would expect that the critical buckling capacity should increase as the slenderness ratio decreases. Such a response is due to the fact that for a specific imperfection amplitude, the shorter the beam, the larger the curvature of the beam. With the realization that the bending moment is proportional to the curvature, therefore, the larger the curvature, the higher the bending moment. This can be further seen in Figure 5, which shows the variation of critical buckling momentum as a function of slenderness ratio and maximum curvature of beam for different initial geometric imperfections. One can therefore predict the critical momentum (the momentum value causing pulsebuckling), of a beam with a specific imperfection from such a diagram. This phenomenon also conforms to the conclusion of the finite difference solution of Zhang and Taheri (2002[b]).

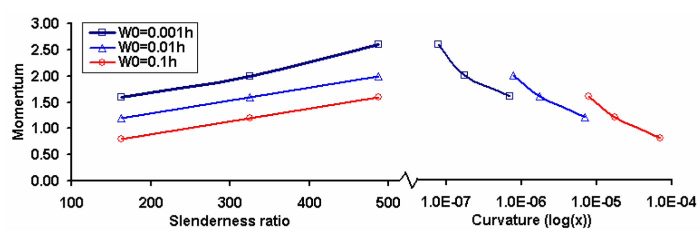

Figure 5: Variation of buckling momentum as a function of slenderness ratio and curvature of beam.

EFFECT OF BOUNDARY CONDITIONS

To investigate the effects of boundary conditions on the pulsebuckling and post buckling response, as shown in Figure 1, two types of boundary conditions were considered, fixed-end support (FS) and pinned-end support (PS),.

Effect of boundary conditions on axial displacement, axial compressive strain and deflection

Figure 6 shows the results of axial displacement, axial compressive strain and deflection for different boundary conditions when the beam is impacted by a moving rigid body with mass M=0.10 kg, initial velocity V_0 =10.0 m/s and the amplitude $W_0 = 0.1h$. From the figure, we can see that during the impact period (approximately 0.0~0.5ms), the two types of boundary conditions do not influence the response. In general, the axial displacement was much larger than the lateral deformation of the beam. From the one dimensional wave propagation equation of a perfect straight component,

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

one can forecast that the axial restraint affects the axial wave propagation. For the imperfect beam, the axial boundary condition is still a governing factor that affects the axial behavior of the beam. During the post-impact period, the deflection for the pinned-end support (PS) is larger than that of the fixed-end support (FS), as shown in Figure 6. This is mainly due to the fact that the PS provides no rotational constraint to the beam's ends. This phenomenon was also presented by the authors elsewhere (Zhang and Taheri, 2002[c]), in which the response was characterized by a finite difference based strategy.

Buckling criteria

Figure 7 shows the plots of deflection of the mid-span of beam as a function of momentum for different initial geometric imperfections under different types of boundary conditions. One can see from the figure that for both types of boundary conditions, the larger the magnitude of initial geometric imperfection, the smaller the critical buckling momentum. This also conforms to the conclusions drawn by Zhang and Taheri (2002[b]), based on their sensitivity analysis. For the effects of boundary conditions on the critical buckling momentum of pulsebuckling, one can see that the boundary conditions (fixed-end support and pinned-end support) do not significantly affect the buckling response. This phenomenon can also be seen from the beam profiles, as shown in Figure 8. Although the lateral response of beam is different for different boundary conditions, the critical impact momentum is very close.

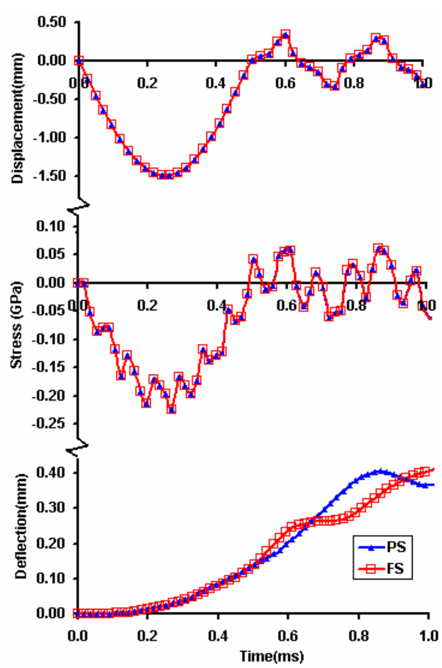

Figure 6: Comparison of the effect of fixed-end (FS) and pinned-end (PS) supports on the pulsebuckling response (time history results) - (a) Axial displacement at the impacted end; (b) Axial compressive stress at the neutral surface of mid-span of beam; (c) Mid-span deflection.

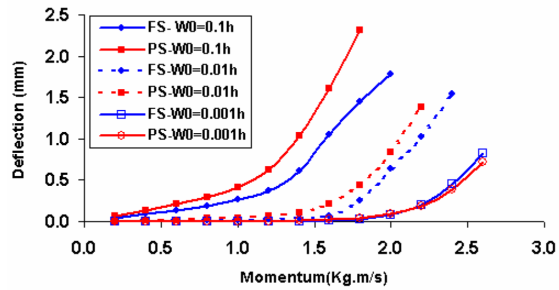

Figure 7: Comparison of the mid deflection of the beams as a function of momentum for different initial geometric imperfections under different types of boundary conditions.

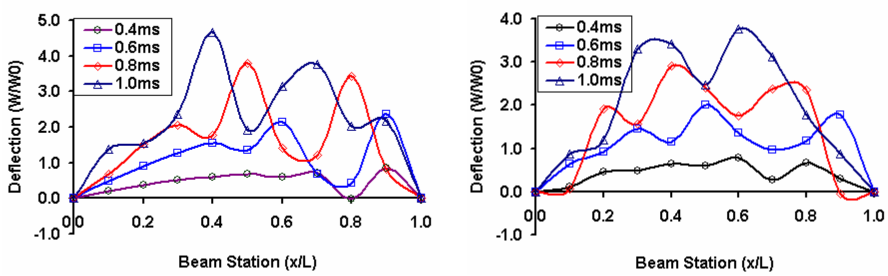

Figure 8: Dynamic profiles of the beams and distribution of deflection along the beam length for different boundary conditions – (a) PS; (b) FS.

CONCLUSION

Dynamic pulsebuckling of FRP laminated slender beams, having initial geometric imperfections, subject to axial impulse of a moving object load was investigated by the finite element method using LS-DYNA code. It was shown that momentum might be considered as a viable parameter for predicting the dynamic pulsebuckling limit of the beams. Initial geometric imperfection was shown to be an important factor that affected the dynamic pulsebuckling response. It is apparent that critical buckling momentum decreases with the increase in initial geometric imperfection amplitude. It was also observed that for a beam with a specific imperfection magnitude, the smaller its slenderness ratio (i.e., the shorter the beam), the smaller the critical buckling momentum. Our investigation also indicated that the amplitude of initial geometric imperfection played a more significant role in promoting dynamic pulsebuckling than the slenderness ratio of beam. It was shown that the onset of dynamic pulsebuckling could be predicted from the diagram of momentum versus curvature and slenderness ratio.

The effects of boundary conditions on the buckling and post-buckling responses were also examined. From the results of analysis, one may conclude that the axial restraint mainly controls the axial behavior of the beam, while the rotary restraint of the beam end do not affect the buckling criterion much.

ACKNOWLEDGEMENTS

The financial support of NSERC in the form of an operating grant to the second author in support of this work is gratefully acknowledged.

REFERENCE

ABRAHAMSON G.R., and GOODIER J.N. (1966). Dynamic flexural buckling of rods within an axial plastic compression wave, J of Appl Mech, Vol. 33, pp. 241-247.

ARI-GUR J. and ELISHAKOFF I. (1997). Dynamic instability of a transversely isotropic column subjected to a compression pulse. J Comput Struct, Vol. 62, pp. 811-815.

ARI-GUR J. and SIMONETTA S.R. (1997). Dynamic pulsebuckling of rectangular composite plates, J Composite B., Vol. 28B, pp. 301-308.

ARI-GUR J., WELLER T. and SINGER J. (1982). Experimental and theoretical studies of columns under axial impact. Int J Solids Struct, Vol. 18, pp. 619- 641.

EKSTROM R.E. (1973). Dynamic buckling of a rectangular orthotropic plate, AIAA J, Vol. 11, pp. 1655-1659.

HAYASHI T. and SANO Y. (1972). Dynamic buckling of elastic bars, 1st Report, The case of low velocity impact, Bulletin of the JSME, Vol. 15, pp. 1167-1175.

HAYASHI T. and SANO Y. (1972). Dynamic buckling of elastic bars, 2nd Report, The case of high velocity impact, Bulletin of the JSME, Vol. 15, pp. 1176-1184.

HOUSNER G.W. and TSO W.K. (1962). Dynamic behavior of supercritically loaded struts, J Engrg Mech Div, ASCE, Vol. 88(EM5), pp. 41-65.

KENNY S. PEGG N. and TAHERI F. (2000). Dynamic elastic buckling of a slender beam with geometric imperfection subject to an axial impulse, Finite Elements in Analysis and Design, Vol. 35, pp.227-246.

KONING C. and TAUB J. (1934). Impact buckling of thin bars in the elastic range hinged at both ends, Technical Memorandums 748, National Advisory Committee For Aeronautics, Washington, D.C., June.

LAM K.Y. and NG T.Y. (1998). Dynamic stability analysis of laminated composite cylindrical shells subjected to conservative periodic axial loads, J Composite B Vol. 29B, pp. 769-785.

LIAW D.G. and YANG T.Y. (1990), Symmetric and asymmetric dynamic buckling of laminated thin shells with the effect of imperfection and damping, J Composite Mater, Vol. 24, pp. 188-207.

LS-DYNA (1999). Keyword User's Manual, LS-DYNA Version 950, Livermore Software Technology Corporation, (http://www.lstc.com)

SUGIURA K. MIZUNO E., and FUKUMOTO Y. (1985). Dynamic instability analysis of axially impacted columns, J Engrg Mech, Vol. 111, pp. 893-908.

WANG D.Y., CHEN T.Y., and XIN G.N. (1998). Dynamic buckling of laminated composite bars subjected to axial impact, China Ocean Engrg, Vol. 12, pp. 127-134.

ZHANG Z. and TAHERI F. (2002a), Numerical Investigation on Dynamic Pulsebuckling of Slender FRP Composite Laminated Beams Subject to an Axial Impulse, to appear in the proceedings of the SECTAM-XXI Conference, Orlando, Fl, 2002.

ZHANG Z. and TAHERI F. (2002b), Numerical Studies on Dynamic Pulsebuckling of a FRP Composite Laminated Beams Subject to an Axial Impact, J Comp Struc, in press.

ZHANG Z. and TAHERI F. (2002c), Dynamic Pulsebuckling and Postbuckling of Composite Laminated Beam Using Higher Order Shear Deformation Theory, submitted to J Composite B.

ZHOU C.T. (1991). Theory of nonlinear dynamic stability for composite laminated plates, J Appl Mathematics and Mechanics (English Edition), Vol. 12, pp. 113-120.