ON THE APPLICATION OF LS-OPT TO IDENTIFY NON-LINEAR MATERIAL MODELS IN LS-DYNA

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Abstract

A response surface optimization algorithm for structural material or parameter identification is evaluated. The algorithm used is the Successive Response Surface Method (SRSM) available in LS-OPT. To illustrate the robustness of SRSM as a material identification tool, two test cases are presented. The first concerns the identification of the power-law material parameters of a simple tensile test specimen. The second test case involves the identification of a model to characterize the brittle damage in a composite laminated structure. It is shown that SRSM is an effective tool for material parameter identification involving strongly nonlinear materials.

Keywords: Material identification, parameter estimation, response surface methodology, adaptive trust region, LS-OPT, LS-DYNA.

Introduction

The material identification process is a non-linear optimization process that uses experimentally measured data to determine the parameters describing a constitutive simulation model of a material. A non-linear simulation is performed with the model parameters as input, and the deviation of the simulated performance from that measured, also called a distance function, is used as a criterion for minimization (Eschenauer *et al,* 1990).

Parameter and system identification or estimation have been applied in a variety of fields. More specifically, material identification has been used by various researchers to characterized materials used in structural analysis. Different optimization methods have been applied to minimize the resulting non-linear distance function. E.g., Reese (2001) uses a genetic algorithm to minimize the residual in the 'Parameter Estimation via Genetic Algorithm' (PEGA) approach, while Seibert *et al.* (2000) use a modified random search algorithm in the identification of viscoplastic constitutive models. Extended Kalman filters also find application, as in e.g. Li and Roberts (1999a, 1999b). Rikards *et al.* (1998, 1999, 2001) employ experimental design techniques as in the current study to identify the plastic properties of polymers (using microhardness test data) and the elastic properties of laminated composites (using vibration test data). Kok *et al.* (2000a) have applied the BFGS method with design sensitivity analysis (DSA) gradients to identify the parameters of a temperature and rate-dependent viscoplastic polycrystal model. In a more recent paper, Müllerschön *et al* (2001) applied the response surface method to the optimization of material parameters for rate dependent foam materials. LS-OPT was also used in that study.

This process is shown schematically in Figure 1. The material constitutive relationship on the left typically involves different quantities than the experimental results curve on the right. The material constitutive law is a point-wise relationship valid at all points in the structural continuum whereas experimental results are discrete values of response quantities, typically as a function of time or deformation. The arrow represents both a simulation (forward) and optimization (backward) process to be performed to match the two curve sets. Multiple load configurations or geometries, involving the same material, can be introduced resulting in multiple cases being defined for the same optimization run.

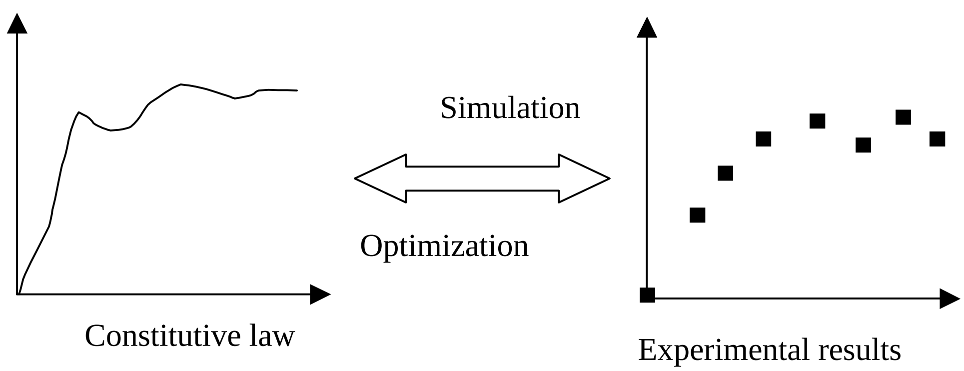


Figure 1 – Material identification process

Depending on the application for which the material identification is required, the formulation to be used in the optimization is adjusted accordingly. The two best known approaches are (i) the minimization of the maximum residual and (ii) the minimization of a residual norm constructed from the least squares residual (LSR) or RMS error:

$$\text{Minimize (LSR)} = \text{minimize } \sqrt{\sum_{j=1}^{R}\left(\frac{f_j(\boldsymbol{x})-F_j}{\Gamma_j}\right)^2}\,, \qquad (1)$$

where R represents the number of responses constituting the residual, and Γ_j is the scaling factor required for normalization or weighting of each respective response. In the current study, the residual is constructed as a composite, using a response surface for each $f_j(\boldsymbol{x})$. For the least-squares residual approach an unconstrained minimization problem (Equation 1) is solved unless other constraints related to e.g. monotonicity (see, e.g. Stander *et al*, 2000 where optimization was used for airbag system identification) in the curves to be matched, etc. are prescribed. The standard targeted composite in LS-OPT is used (LSTC, 1999).

The Successive Response Surface Method (SRSM) as implemented in LS-OPT (LSTC (1999)) is used to solve the minimization problem. This optimization algorithm uses a Response Surface Methodology (RSM) (Myers and Montgomery(1995)), i.e. a Design of Experiments approach, to construct linear surfaces to fit the computed responses. More detail of the algorithm can be obtained in e.g. Stander and Craig (2001). The algorithm has been proven to be robust for simulation-based optimization studies in the crashworthiness and other structural optimization fields (Roux *et al*. (1998), Stander and Craig(2001), Stander *et al.* (2000), Kok and Stander(1999), Akkerman *et al.* (2000)).

Two examples are introduced to demonstrate the procedure.

Formulation of optimization problems

The first example involves the identification of parameters in a power-law material model of a simple tensile test specimen and the second identification determines several parameters in a non-linear material law for brittle damage of a laminated composite.

Power law using tensile test (Müller (2000); Stander *et al* (2002))

In this example, the parameters of a power-law material model of a tensile test specimen are determined using the experimental reaction force, *F* and elongation, *u*. The stress-strain history of the specimen (Figure 2) is simulated using LS-DYNA (LSTC, 2001) and the objective is defined as the least-squares difference between the simulated and measured force-elongation history.

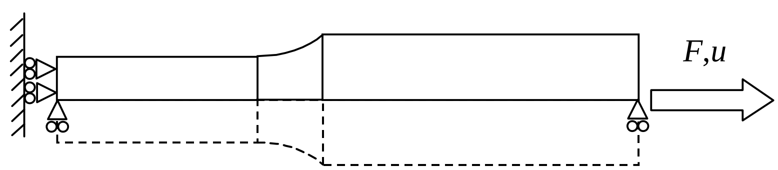


Figure 2 – Quarter symmetric model of tensile test specimen

The stress-strain relationship using the power-law material model, is defined in Equation 2:

$$\sigma_y = K\varepsilon^n = K(\varepsilon_{yp} + \varepsilon^p)^n \qquad (2)$$

where ε_{yp} is the elastic strain to yield and ε^p is the effective plastic strain (logarithmic). The strength coefficient, *K*, and strain-hardening exponent, *n*, are used as design variables.

Brittle damage law (LS-DYNA) using composite laminated beams

The composite beams (see e.g. Figure 3) is modeled after actual experimental samples (Harach, 2000). Each beam is a *metal-intermetallic laminate* (MIL) composite consisting of alternate layers of Titanium (Ti) and Titanium Tri-Aluminide (Al_3Ti). It is simply supported with a 25mm span and centrally loaded perpendicular to the laminae. The cross-sectional measurements are typically 7mm deep and 3mm wide. When subjected to a 1 mm central displacement, the beam exhibits brittle failure of the Al_3Ti layers as well as ductile failure of the Ti layers (Harach, 2000). The experimental result available is the midspan force as a function of the midspan displacement. The beam shown in Figure 3 and 4, denoted as 14Ti-MIL, has a 14% volume fraction of titanium (filled in black).

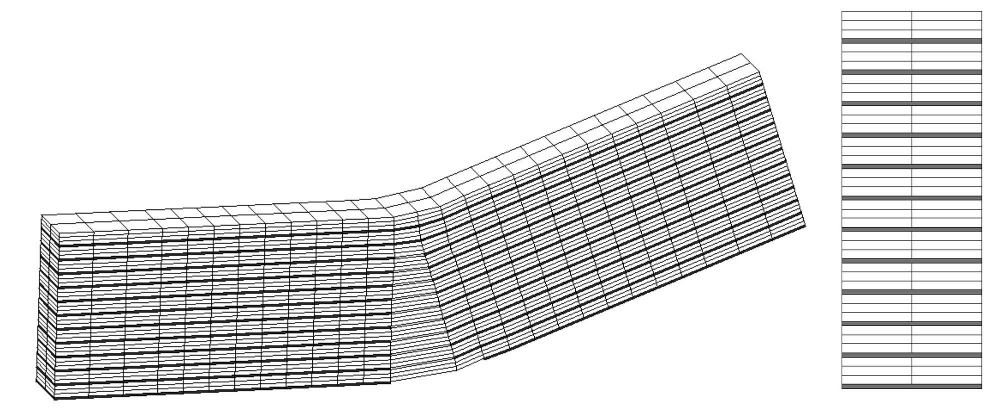


Figure 3 – 14Ti MIL simply supported composite beam: deformed state (2mm maximum displacement) and cross-section.

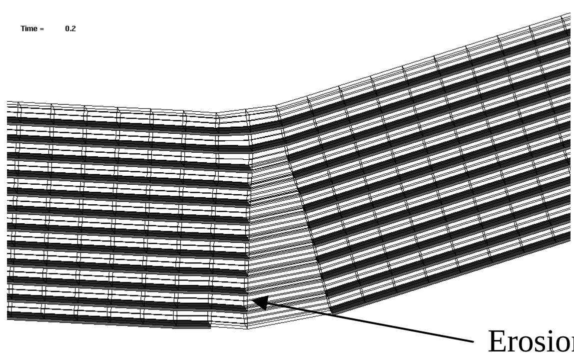


Figure 4 – 14Ti MIL simply supported composite beam: Deformed state (2mm displacement) showing failure and erosion of Ti layers.

The results of two other composite beams were also incorporated into the optimization namely 20Ti-MIL (20% Ti volume fraction) and 35Ti-MIL (35% Ti volume fraction). The material of interest is the Tri-Aluminide, hence the material parameters for the Ti are kept constant for the purpose of design optimization.

The titanium is modeled with a kinematic hardening plasticity model with Young's modulus, E=116GPa, Poisson's ratio, ν=0.3, Yield stress Y = 450MPa, Tangent modulus = 6GPa and Failure strain = 8%. The titanium layers are alternated with Al_3Ti layers. The Al_3Ti is a brittle material, not unlike concrete, with a relatively low tensile strength. Two material models were investigated:

1. A user-defined model (Benson (2002))
2. A brittle damage model available in LS-DYNA (Material 96) (Govindjee *et al* (1995))

The user-defined model consists of a piece-wise linear curve relating v. Mises stress to Effective strain (see Figure 5). The curve parameters are

1. Young's modulus.

2. The plastic strain increment between consecutive points on the curve (see Figure 5: a single parameter represents a uniform strain increment for all points on the curve).
3. The flow stress curve data (solid points in Figure 5).
4. A scale factor on the flow stress curve data when in compression.

The points defining the flow stress curve can be specified individually, but for the purpose of the optimization, a fixed shape was chosen and scaled using a single variable for all the stress ordinates. The Young's modulus is set at a fixed value of 215GPa.

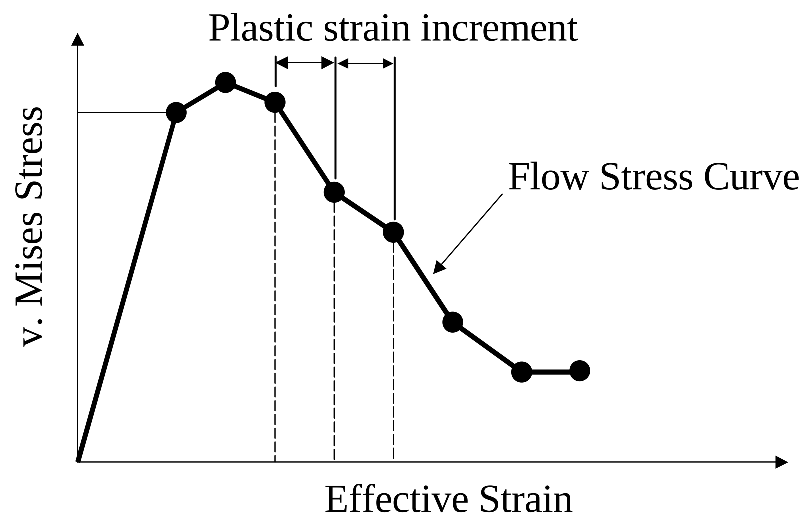


Figure 5 – Laminate: User-defined material model

The model was analyzed using an implicit quasi-static analysis. The optimization method used the RMS residual force quantity as the objective function with 33 target points on the experimental force-displacement curve. Three design variables were chosen in the optimization, namely (1) strain increment, (2) scale factor on the curve and (3) scale factor on the curve in compression.

The second model investigated, a brittle damage model, admits progressive degradation of tensile and shear strengths across smeared cracks that are initiated under tensile loadings. Compressive failure is governed by a simplistic J_2 flow correction. This model is explained in more detail in Govindjee (1995). Some detail is available in LSTC (2001).

Results and discussion

Tensile test material identification problem

The starting design and optimum design values of the tensile test specimen material identification problem are shown in Table 1 together with the bounds on the design variables.

	Minimum	Initial	Maximum	Optimum
K [GPa]	0.7	1	2	1.23865
n [-]	0.01	0.1	0.2	0.106726

Table 1 - Design variable upper and lower bounds; initial and optimum values of design variables – Tensile test specimen material identification

The optimization history for the design variables is given in Figures 6 and 7 as a function of the initial range on *K*. It can be seen that although the algorithm is sensitive to this parameter, the optimum is obtained in about 6 design iterations. The stable convergence rate of SRSM can also be viewed in the objective function (least-squares error) history plot in Figure 8.

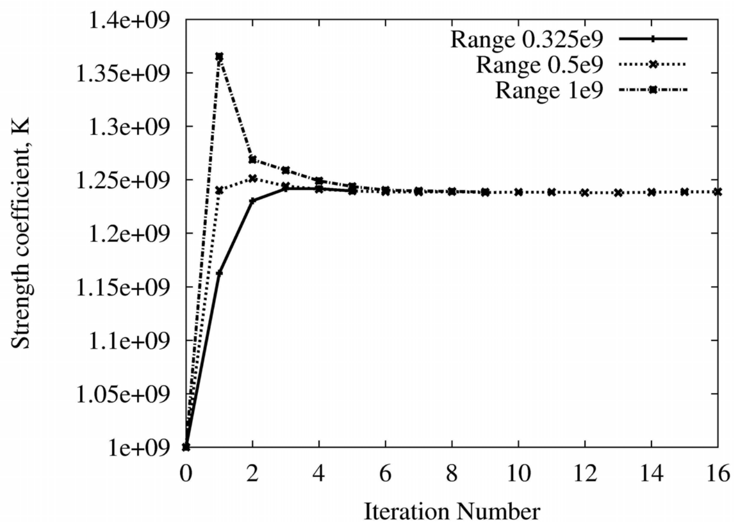


Figure 6 - Tensile test specimen material identification - Optimization history of *K* as a function of initial range on *K* (Range on *n* = 0.05)

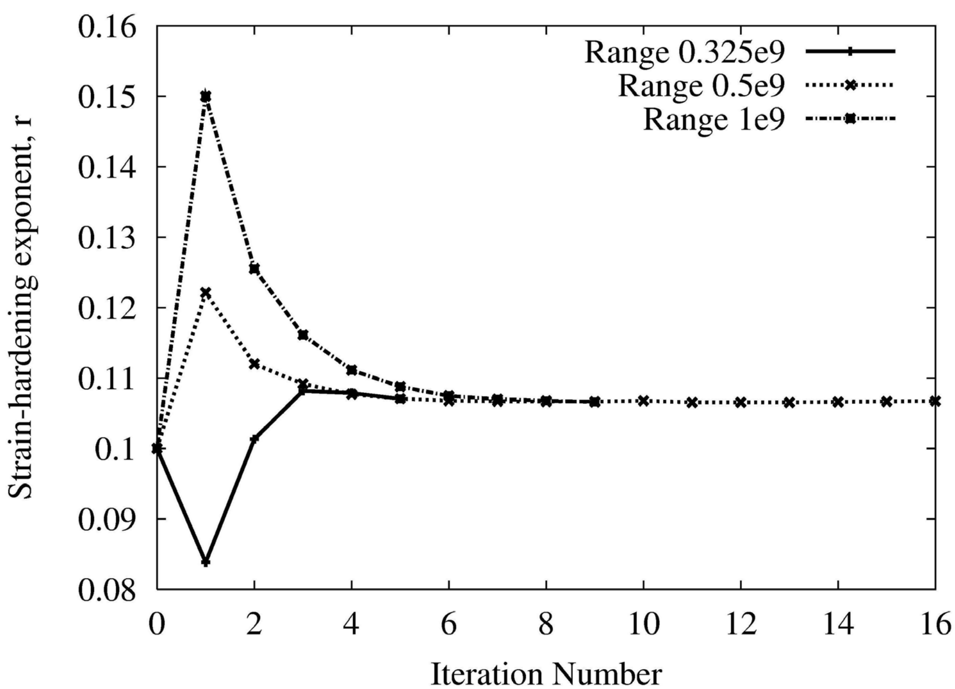


Figure 7 - Tensile test specimen material identification - Optimization history of *n* as a function of initial range on *K* (Range on *n* = 0.05)

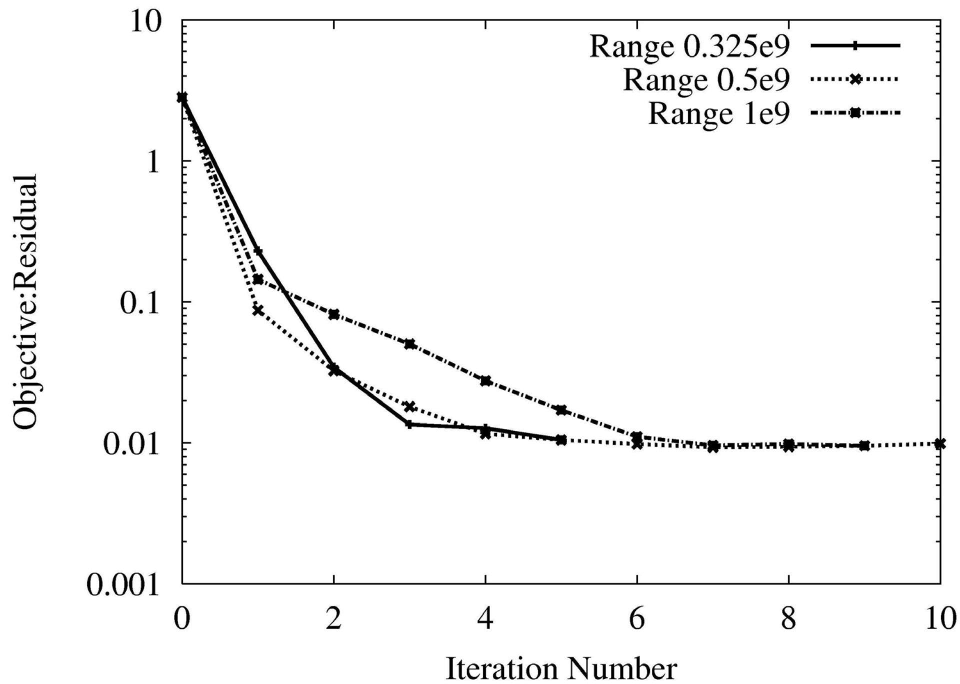
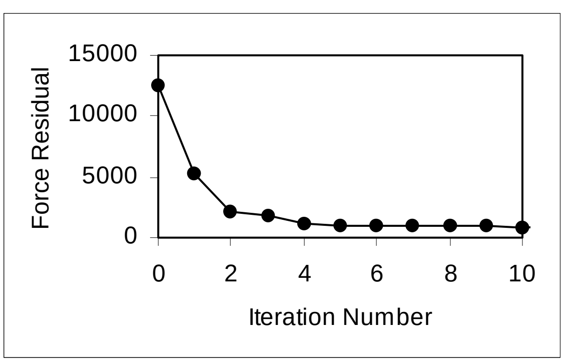


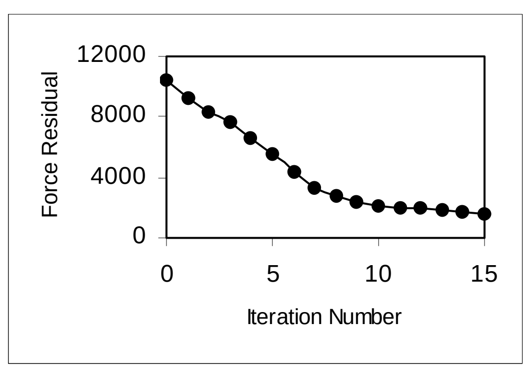
Figure 8 - Optimization history of residual – Tensile test specimen material identification

Composite laminated beam

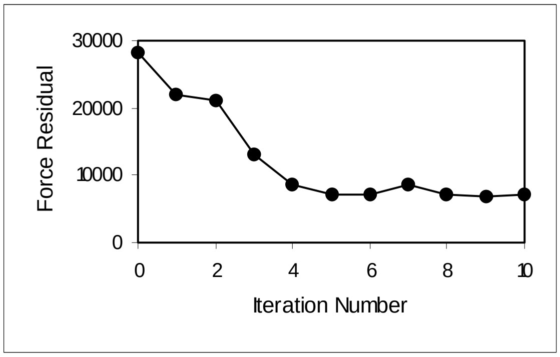
User-defined material model:
The optimization histories of the three individual beams 14Ti, 20Ti and 35Ti are shown in Figure 9. Most of the gain towards a calibrated material was made in the first few iterations and depends somewhat on the initial step size (e.g. note the small step used for Figure 9b). In Figure 9d all three beams with perpendicular loading were incorporated in the same run as a multi-case optimization. Note that, in this multi-case optimization, the residual did not decline very much beyond the first iteration. It is likely that, due to either experimental error or deficiencies in the material model, a more suitable material cannot be found.



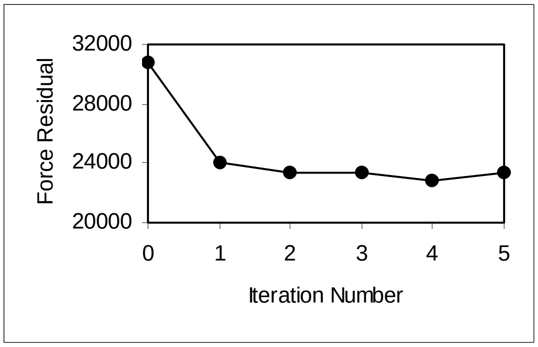
(a)



(b)



(c)



(d)

Figure 9 - Optimization history – MIL composite beams:
(a) 14Ti-MIL, (b) 20Ti-MIL, (c) 35Ti-MIL and (d) All beams (multi-case).
The force units are in Newton

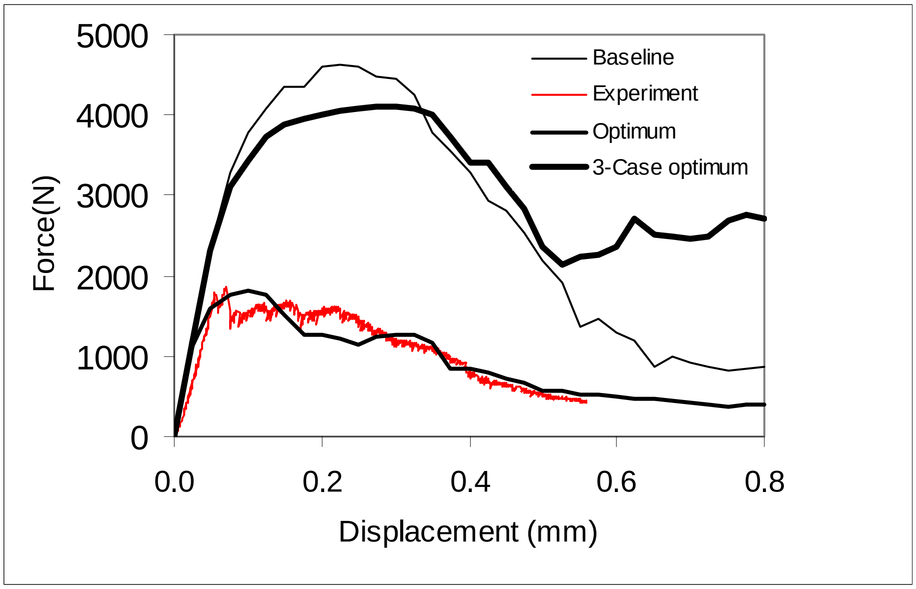


Figure 10 – Baseline and optimal force-displacement curves vs. experimental results:
14Ti-MIL composite beam

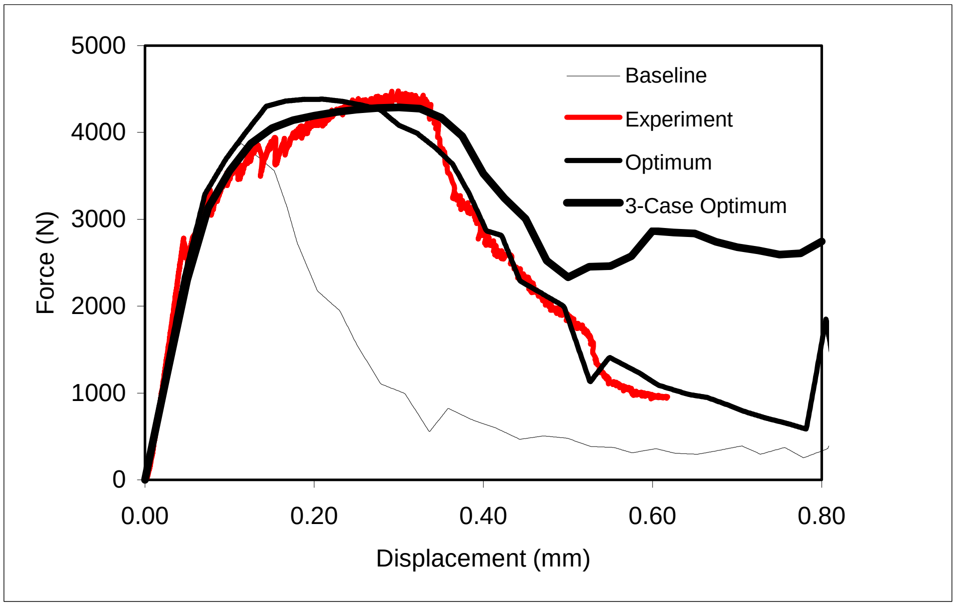


Figure 11 – Baseline and optimal force-displacement curves vs. experimental results: 20Ti-MIL composite beam

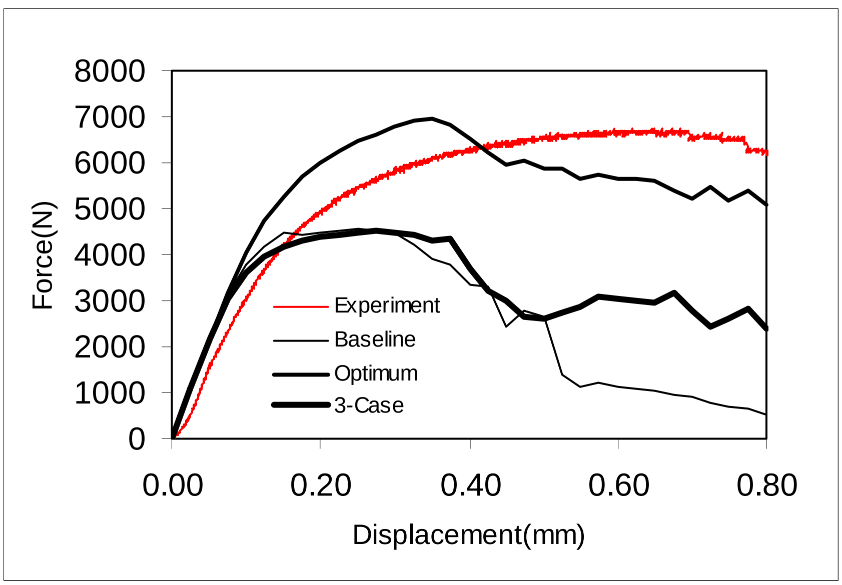


Figure 12 – Baseline and optimal force-displacement curves vs. experimental results: 35Ti-MIL composite beam

Figures 10, 11 and 12 show the force result comparisons for the 14Ti, 20Ti and 35Ti MIL beams respectively. The ‘Optimum’ shown in each case is for a single beam optimization whereas the ‘3-Case’ curve represents the optimum of the multi-case optimization involving the three beams. The figures, as well as Table 2, confirm that the 3-Case optimization failed to obtain the same material quantities (Table 2). Hence, although experimental error may be playing a role, a new model, the brittle damage model was also investigated.

	Plastic strain increment	Scale factor In compression	Scale factor On stress curve
14Ti	.00412	.629	.636
20Ti	.0091	1.046	1.488
35Ti	.0168	1.817	2.55

Table 2: Optimal parameters for 14Ti, 20Ti and 35Ti MIL composite beams

Brittle damage model (LS-DYNA):
An analysis was done with the 14Ti-MIL beam using the parameters in Table 3 to obtain the force-displacement result of Figure 13. These material parameters were based on a preliminary optimization that involved a calibration of the effective stress-strain curve obtained using the user-defined model. The force-displacement result shown in Figure 13 seems to indicate a promising, perhaps more accurate material model. However, the results of a multi-case optimization involving the 14Ti, 20Ti and 35Ti beam models are required to validate the model and will be presented at the conference.

Parameter Description	Parameter value
Young's modulus	215GPa
Tensile Limit	220MPa
Shear Limit	550MPa
Fracture Toughness	36.3N/mm
Shear retention	.0165
Viscosity	0.5MPa-sec
Compressive Yield Stress	607MPa

Table 3: Values of material parameters: brittle damage model

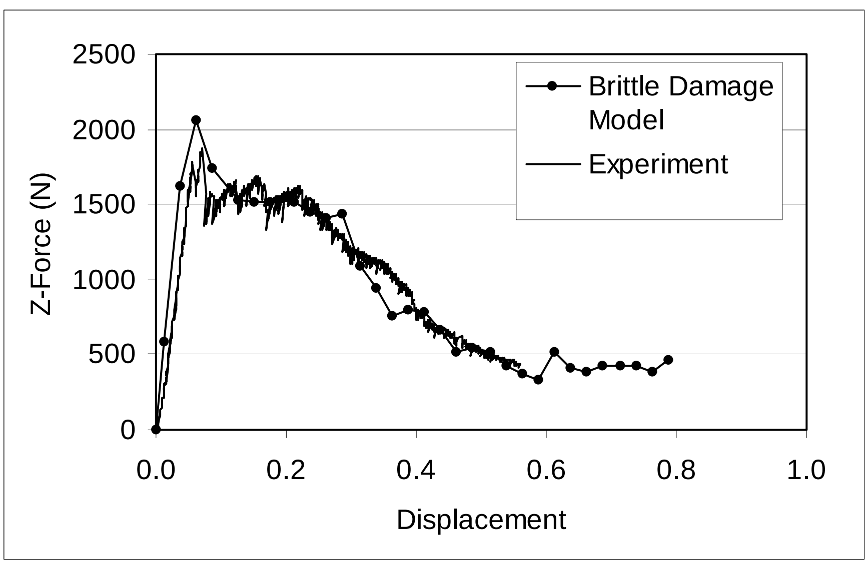


Figure 13 – Force Displacement Curve: 14Ti-MIL composite beam (brittle damage model in LS-DYNA)

Conclusions

The ability of the SRSM algorithm as employed in LS-OPT to perform material identification has been clearly illustrated in this paper. The following specific conclusions can be drawn:

1. The SRSM algorithm in LS-OPT is able to identify material properties of non-linear materials with a variety of material laws.
2. The identification process essentially converges in less than six optimization iterations in all cases considered. The method is not highly sensitive to the step-size applied in the optimization, except that a too small step may delay convergence to a limited extent as illustrated in the beam example.
3. The choice of material model and variables used in the optimization as well as experimental accuracy remain crucially important factors in material identification.

Acknowledgements

The authors would like to thank Dipl.-Ing. Tobias Müller for his assistance in setting up the first example problem. The support from DARPA for the modeling of MIL composites for the "Synthetic Multifunctional Materials Program" is gratefully acknowledged as is the support from the South African National Research Foundation, Grant No. GUN2046904, for the sabbatical of the fourth author at the Livermore Software Technology Corporation. The authors are also grateful to Dr. David Harach for providing digitized experimental data. The simulations for this example were performed at LSTC on a SGI Origin 3400.

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