Effects of Initial Geometrical Imperfection on Square Tube Collapse

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ABSTRACT

Random geometric imperfections are natural in structures. The initial imperfections used to be ignored in structure strength analysis and thus geometric-perfect models were used in most case of numerical simulation. However, collapse of axially compressed square tubes is not such a case. LS-DYNA is used to simulate the effects initial geometrical imperfection has on square tube collapse. This study proves that dynamic progressive buckling of square box columns is sensitive to initial geometrical imperfections. The simulation results show that ideal square tubes tend to buckle in extensional mode, though is not likely to happen in experimental studies. Previous theoretical analysis suggests, that from the view of energy absorption extensional mode is a dynamic procedure of higher energy absorption characteristics than that of each of symmetric and asymmetric mode of square tube in case of high c/h. This phenomenon suggests that extensional mode is an unstable equilibrium that will easily change to another equilibrium – symmetric mode. In a real world, geometrical imperfection renders extensional mode almost unachievable for hollow square tubes. Three kinds of imperfections: deflection of wall, thickness deviation and length of section side unequal were discussed in this paper. The amplitude of imperfection was compared with the geometry tolerance. Numerical simulations are then performed using LS-DYNA. Compared with the experimental datum, deflection of wall is the main reason for the predominance of symmetric mode of axially impacted hollow square tubes. Several characteristic values with regard to the amplitude of wall deflection are discussed in particular. It is found that when the amplitude of deflection is less than a certain critical value λ_{cr}, the initial impact force peak value and the critical buckling load are almost the same and unchanged at a determined impact velocity. When deflection exceeds the critical value, buckling take place in elastic area and critical buckling force drops quickly. Energy absorbed before buckling also quickly drops to near zero when deflection is considerably large.

INTRODUCTION

Thin-walled columns are typical energy absorbing units in frontal car crash. During the impact event, such tubes crush progressively and dissipate energy. The dynamic buckling of axially loaded square tubes has been examined in numerous experimental, theoretical and numerical studies which have been undertaken in order to clarify various features of this complex phenomenon. A representative diagram of axial crush of square tubes using a drop hammer rig is shown in Figure 1. Progressive buckling may occur from top to bottom or reverse.

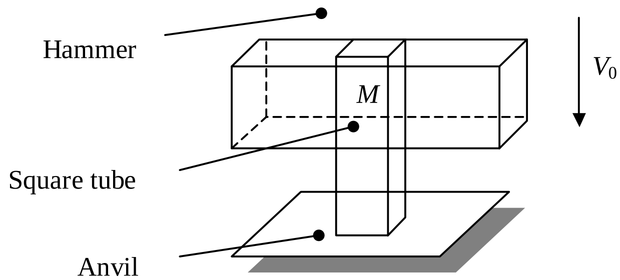


Figure 1. Crush of a Square Tube

In this paper, an overview of the progressive buckling mode of square tubes will be provided, followed by three imperfection models and their calculation results. The particular column considered has a length of l=133.0mm, side width c= $37.07^{+0.14}_{-0.15}$ mm and a wall thickness h=$1.152^{+0.028}_{-0.032}$ mm. The material is mild steel having Young's modulus E=210GPa, yield stress σ_y£½275 GPa and a hardening modulus E_t=1.0 GPa. A drop weight impact of the column by a mass G=73.6Kg, traveling with a velocity v_0 =8.963m/s is studied when both ends are subject to the same contact and friction condition. Experimental data for the square tube was supplied by Abramowicz and Jones [1].

According to Wierzbicki's theory, crushed shape of square tubes can be modeled by basic folding elements, as shown in Figure 2. Abramowicz and Jones pointed out that dynamic progressive buckling of square tubes could have four types of mode – symmetric mode, asymmetric mode A, asymmetric mode B and extensional mode. Each layer of symmetric mode consists of 4 type I elements. Each two layers of asymmetric mode A consists of 6 type I elements and 2 type II elements. Each two layers of asymmetric mode B consists of 7 type I elements and 1 type II element. And each layer of extensional mode consists of 4 type II elements. Extensional mode of square tubes is similar to axisymmetric mode or concertina mode of cylindrical tubes. As extensional energy is included in type II elements, the extensional mode absorbs more energy than any other modes.

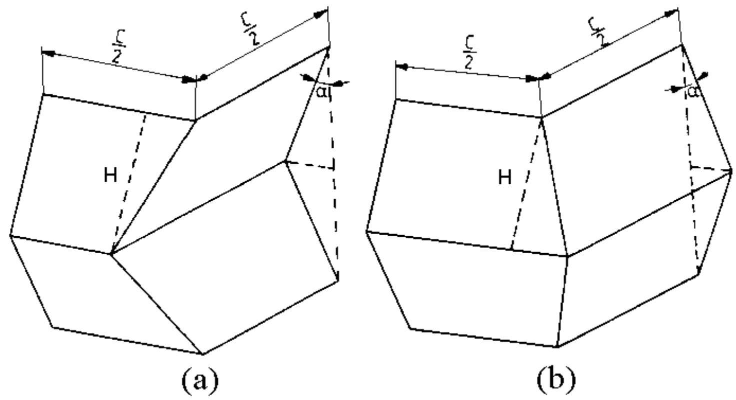


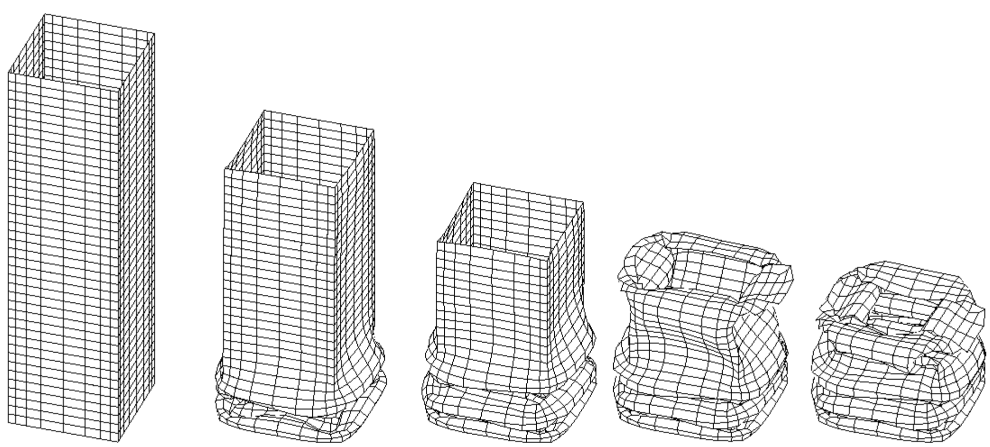
Figure 2. Basic Collapse Elements of Dynamic Progressive Buckling of Square Tubes
(a) type I, (b) type II

Drop tests on square tubes has been performed excessively. In Abramowicz, Jones [1] and Wierzbicki's [2] among many others' work, extensional mode wasn't observed in experiments. To the author's knowledge, extensional mode of hollow squared metal tubes is only reported once by Langseth and Hopperstad[3].

APPROACH

Square Tube Modeling and Discussion

The square tube is first modeled as an ideal straight square cross-sectioned hollow tube. Numerical calculation is then performed. The result, as shown in Figure 3, is a typical extensional mode that 4 sidewalls buckle outward or inward simultaneously. This is quite different from experimental result, as shown in Figure 4, which is a typical symmetric mode.



0ms 5ms 10ms 15ms 20ms

Figure 3. Dynamic Progressive Mode of an Ideal Square Cross-Sectioned Tube

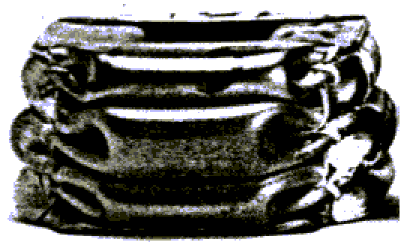


Figure 4. Final Deformed Shape of Specimen Number I5 [1]

In a real world, manufacturing deviations are inevitable, and hence random geometrical imperfections are quite natural in structures. As for a square tube, the length of each side of cross-section could not be exactly the same and the wall thickness could fluctuate slightly near the mean value. This may render the progressive buckling mode to change. For the square tubes discussed in this paper, as we assume that both side length deviations and wall thickness deviations are subject to "6σ criteria", we get σ_c=0.05mm and σ_h =0.010mm. To simplify the problem, we build two simple models to simulate side length and wall thickness imperfection.

A rectangle section tube with the length of section side equals $c+\sigma_c$ and the width equals $c-\sigma_c$ is considered for the side length imperfection model, as shown in Figure 5.

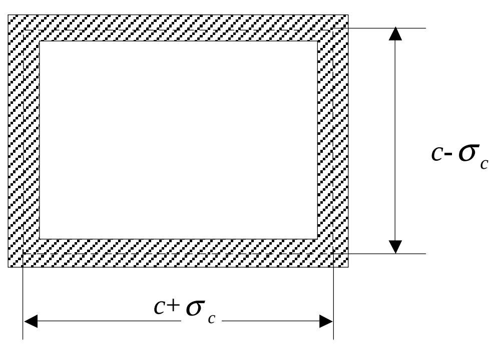


Figure 5. The Imperfect Model for Side Length Deviation

The thickness of the wall thickness imperfect model is considered to be h+σ_h. The thicknesses of the opposite sides are equal, as shown in Figure 6. The central line of the wall thickness imperfect model is a square.

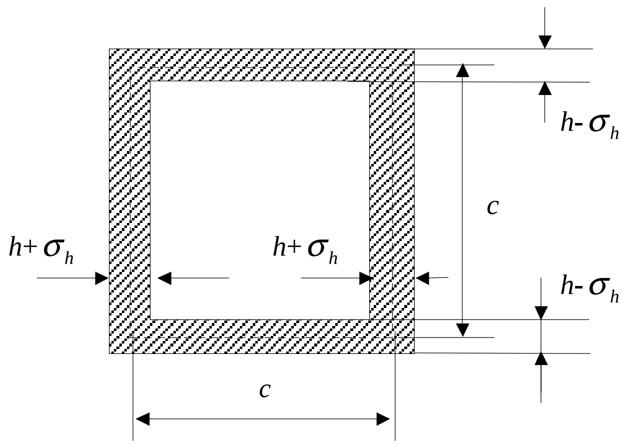


Figure 6. The Imperfect Model for Wall Thickness Deviation

Numerical calculations are then performed. The simulation results of the side length imperfection model which is mainly extensional mode, as shown in Figure 7, is similar to that of ideal square tubes. There are 2.5 layers of extensional mode and 1.5 layers of symmetric mode. Furthermore, even when the imperfection amplitude is assumed to be 10 times of real σ_c, i.e. 0.5mm, the simulation result is almost the same.

The simulation result of wall thickness imperfection model does not change a lot. There are 3 layers of extensional mode and 1 layer of symmetric mode, as shown in Figure 8.

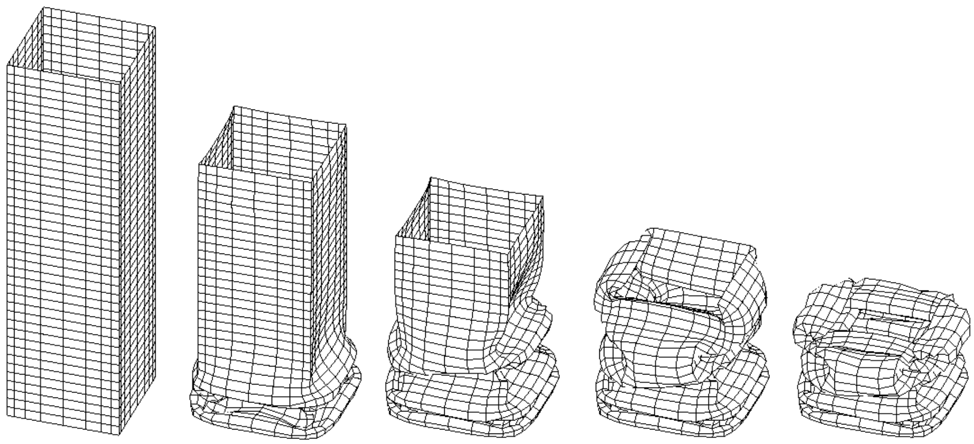


Figure 7. Dynamic progressive buckling of section length imperfection square tube

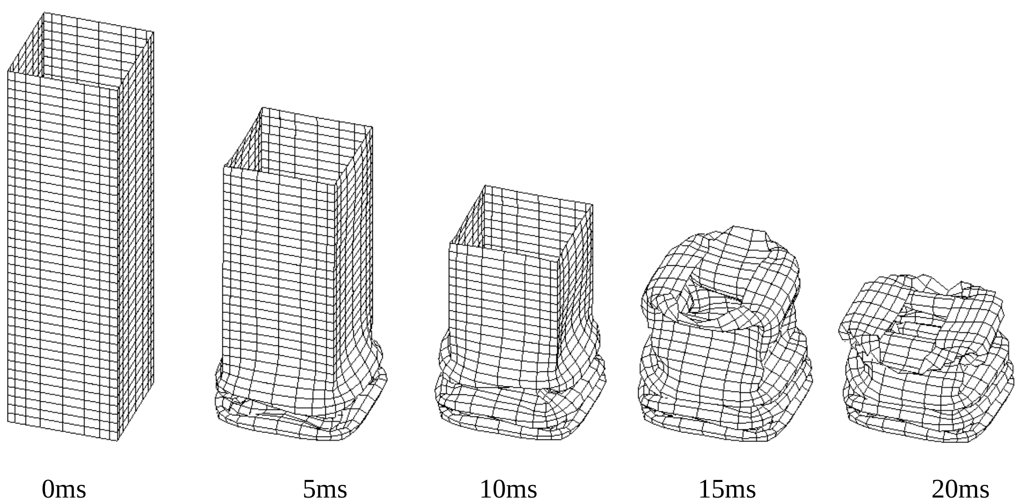


Figure 8. Dynamic Progressive Mode of Imperfect Wall Thickness Model

None of the above models includes wall unflatness. The sidewall may have an initial deflection and it may not perpendicular to the impact end. The initial deflection can be described as following progression:

$$y_0 = \sum_n f_n \sin\left(\frac{n\pi y}{l}\right) = f_1 \sin\frac{\pi y}{l} + f_2 \sin\frac{2\pi x}{l} + ... \qquad (1)$$

The initial deflection is assumed to have the save wave shape as the final buckled shape. The buckling mode number equals five based on the experimental observation. Consider $f_i << f_5$ (i=1,2,3,4,6,7,….) and two and a half waves over the full length of tube, the unflatness function can be written as

$$y_0 = \pm\lambda_w \sin(5\pi y / L) \tag{2}$$

where, λ_w is the amplitude of the initial deflection, L is the length of the tube, ± stands for outward or inward deflection.

The imperfection model is shown in Figure 9, where

$$\theta = \pm tg^{-1}(5\pi\lambda_w / L) \tag{3}$$

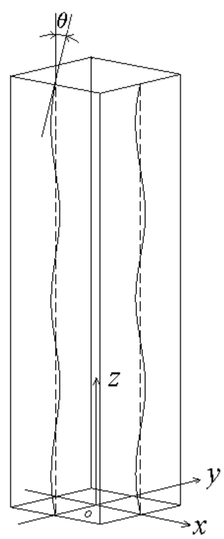


Figure 9. Imperfection Model for Initial Deflection of Sidewall of Square Tube

The initial imperfections are assumed to be sufficiently small in order not to affect the deformed shape. Thus, it is assumed that $\lambda_w = 0.01h$, where h is the column thickness. For this particular tube, λ_w=0.01$h \approx$0.01mm. This imperfect model is calculated and the simulation result is shown in Figure 10.

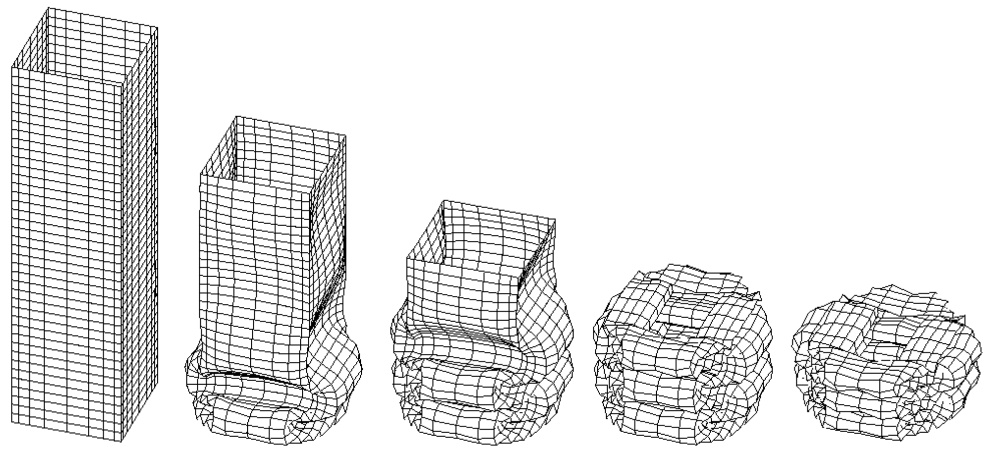


Figure 10. Crushing Mode of the Wall Deflection Model of Square Tube

Compared Figure 10 with Figure 4, good agreement can be found. For all the four numerical models, similar finite element meshes were build in order not to affect the calculated result. The load-displacement curves of the above four models are shown in Figure 11 and some important characters are listed in table 1.

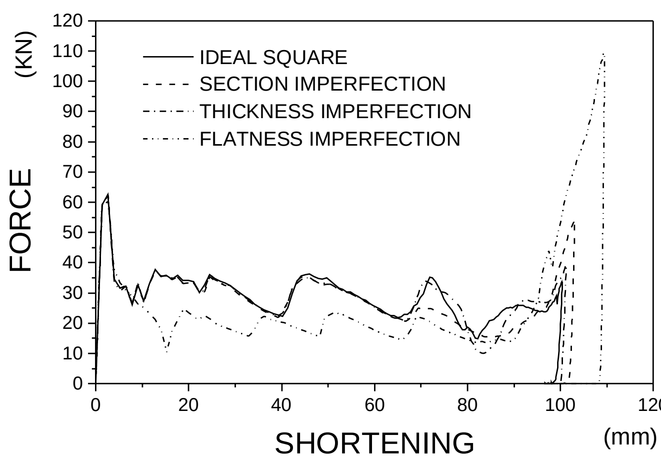


Figure 11. Loading-Shortening Curves of the Above Four Square Tube Models

Table 1. Comparison of Four Square Tubes' Load-Shortening Behavior

Models	Mean Crushing Force	Final Crushed Distance	Error(%)	Crushing Mode*
Ideal sectioned	29.6201	97.8898	-8.8549	E
Imperfect section length	28.4970	101.524	-5.4711	E

Imperfect wall thickness	29.1565	99.4696	-7.3840	E->S
Wall deflection	26.6383	108.146	0.6946	S
Experimental result	27.5	107.4	/	S

* Note: "E" - Extensional mode, "S" – Symmetric mode, "E->S" – Transition mode from extensional mode to symmetric mode.

According to Wierzbicki's theory, Abramowicz and Jones found that extensional mode is of higher energy absorbing character than either symmetric or asymmetric mode when c/h is large. Dimensionless mean crushing forces P_m for extensional mode and symmetric mode are respectively written as equation (4) and (5) and are shown in Figure 12.

$$\frac{\overline{P}_m^{\,E}}{M_0} = 36.83(c/h)^{1/2} + 10.39 \qquad (4)$$

$$\frac{\overline{P}_m^{\,S}}{M_0} = 52.22(c/h)^{1/3} \qquad (5)$$

where, $M_0 = \sigma_0 h^2 / 4$, c is the length of side and h is wall thickness.

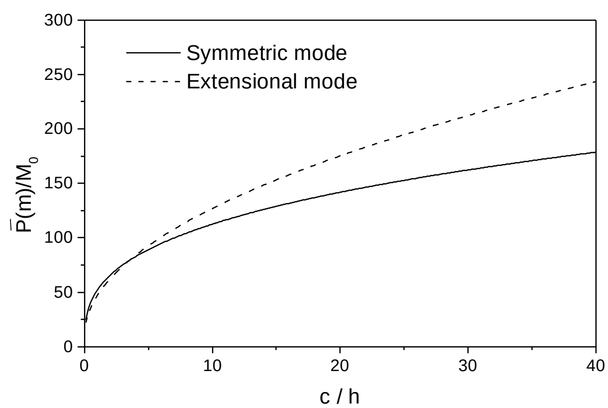


Figure 12. Relationship Between Dimensionless Mean Crushing Load and c/h

From Figure 12, one gets:

- When 0<c/h<3.5, extensional mode may have lower energy absorbing character theoretically. In such case, the hollow square tube is nearly a solid bar. Overall bending should be predominant buckling mode.

- When c/h>3.5, extensional mode has higher energy absorbing character. In most case for square tubes, c/h is above 10. For the particular tube discussed above, c/h=32. According to the lowest energy principle, extensional mode is not likely to happen since geometrical imperfection and perturbation are omnipresent.

DISCUSSION OF RESULTS

Deflection Effects

Effects of wall deflection on initial buckling and progressive buckling will be discussed in the following section. In the pervious model with deflection, we take liberty to quantify the deflection amplitude as 0.01*h*, but in a real world it is hard to quantify the amplitude. Many characteristic values for the crushing process of the square tube is a function in respect to the amplitude of imperfection. The deflection model shown in Figure 9 will be applied to calculate the characteristic value of square tube collapse, while λ_w is the only variant.

According to the stress wave propagation theory, the first impact force peak should be:

$$\sigma = \rho_0 C_0 v \tag{6}$$

where, ρ is the density of the material, C_0 is the sonic velocity of the material and v is the impact velocity. While the impact velocity is over the yield velocity, plastic wave will be propagated in the tube.

A series of calculations on different amplitude are performed. Figure 13 shows the relationship between the initial impact force peak and the amplitude of the deflection. Figure 14 shows the relationship between the initial impact force peak time and the amplitude of the deflection.

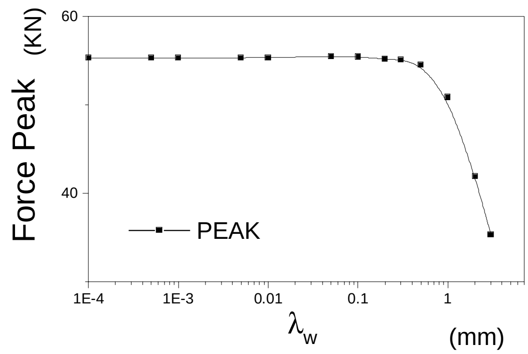


Figure 13. Relationship Between the Initial Impact Force Peak and the Amplitude

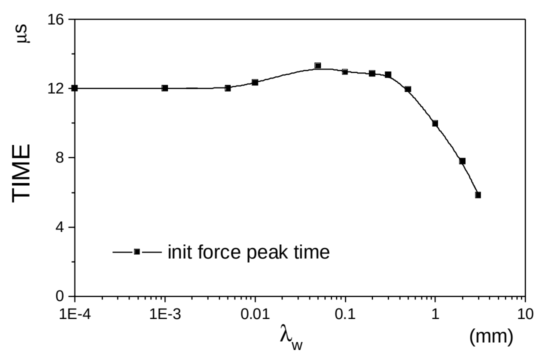


Figure 14. Relationship Between the Initial Force Peak Time and the Amplitude

Figure 13 and Figure 14 indicate that when the amplitude is less than a critical value ($\lambda_w < 0.2$), the axial force and its time are nearly the same. That means the first force peak is determined by the wave propagation. While the amplitude is relatively large ($\lambda_w \geq 0.2$), the initial axial force peak drops quickly when the amplitude increases and equation (6) is no longer valid for such kind of tubes.

The results of critical buckling force and initial buckling time are shown in Figure 15 and Figure 16. It appears that the critical buckling force decreases slightly when amplitude is under 0.2mm, when amplitude is above 0.2mm, critical buckling force drops quickly. The initial buckling time begins to drop at 1e-3mm. When amplitude is above 0.2mm, it seems the buckling take place at the impact time. This means the square tube buckles in elastic range even when the impact velocity is low. Now, a critical value for this particular tube can be determined to be 0.2mm.

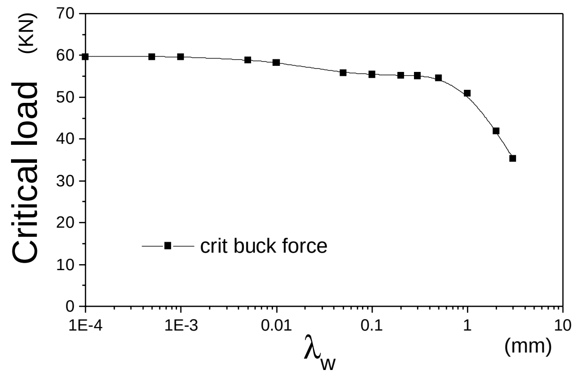


Figure 15. Relationship Between the Critical Buckling Force and the Amplitude

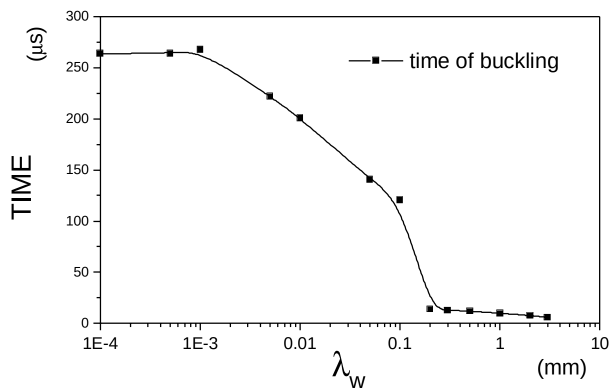


Figure 16. Relationship Between the Initial Buckling Time and the Amplitude

From Figure 15 and Figure 16, the energy absorbed before the buckling takes place can be determined, as shown in Figure 17.

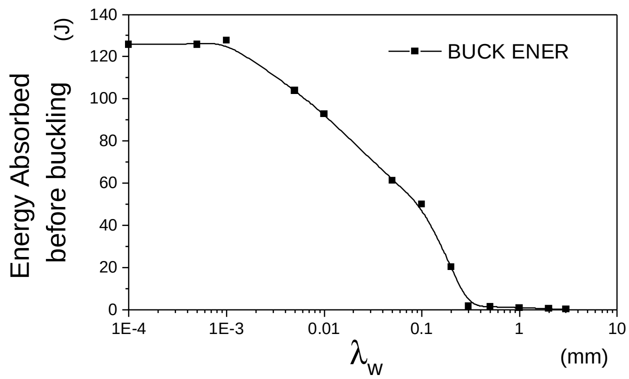


Figure 17. Relationship Between the Energy Absorbed Before Buckling and the Amplitude

The initial axial force peak and the critical buckling have only theoretical meaning for car crash analysis, because the energy absorbed in this period is relatively small. The mean crushing force is important for energy absorbing. To make the problem comparable, the first 80mm crushing distance is taken into consider. The result is shown in Figure 18.

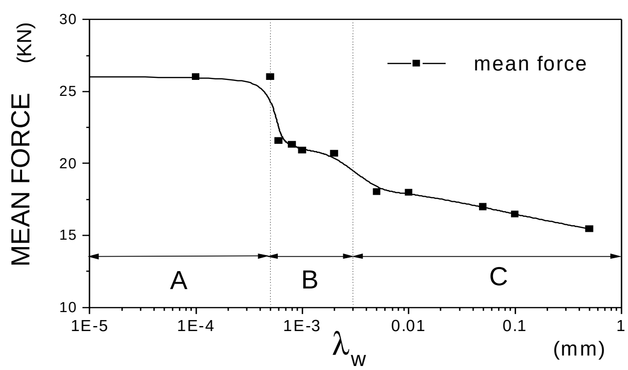


Figure 18. Relationship Between the Mean Crashing Force and the Amplitude

It is interesting to note the transition from the extensional collapse mode to symmetric mode as the random imperfection amplitude increases. The whole range of amplitude can be divide into three sub-ranges: the extensional mode range, the symmetric mode range and a transition range.

Range A

The mean crushing force is about the same (26KN). The extensional mode is predominant. The extensional layers of final deformed shape varies from 3 to 5.

Range B

The mean crushing force is about 21 KN and tends to decline. The crushing mode is mainly symmetric mode except the first layer is extensional mode.

Range C

The mean crushing force is even low. It is around 17KN. The crushing mode is symmetric mode only.

The theoretical analysis pointed out that symmetric mode absorbs more energy than extensional mode. When the amplitude of deflection increases, symmetric mode is predominant. Hence, the mean crushing force drops simultaneously. It should be point out that the amplitude of wall deflection should be very small to obtain a extensional mode. For this particular tube, the amplitude should less than 1 μm. Also, Figure 18 indicates that in range B and C the mean crushing force drops when the amplitude increases, while progressive buckling mode does not change.

CONCLUSION

The dynamic progressive buckling is sensitive to the geometrical imperfections. Experimental studies shows symmetric mode is predominant in square tube crushing. The geometrical imperfection is introduced to the numerical model for square tubes in this paper. The variant includes the imperfection of the length of the side of the square cross-section, the imperfection of the wall thickness and the imperfection of the wall deflection. The ideal square tubes tend to crush in extensional mode, while imperfect ones tend to symmetric mode or transitional mode. The imperfection is compared with manufacturing deviation. The calculation results show the length of the side deviation and wall thickness has only limited affects on crushing mode, while the wall deflection has great influence on square tube crushing mode. Some characteristic values with respect to the amplitude of wall deflection for dynamic progressive buckling are achieved. A critical value of the amplitude is emerged. When the amplitude is less than this value, the buckling force doesn’t changes a lot. When the amplitude is above this value, the buckling force drops quickly.

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