# Optimization of Nonlinear Dynamical Problems Using Successive Linear Approximations in LS-OPT

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Abstract

*This paper focuses on a successive response surface method for the optimization of problems in nonlinear dynamics. The response surfaces are built using linear mid-range approximations. To assure convergence, the method employs two dynamic parameters to adjust the move limits. These are determined by the proximity of successive optimal points and the degree of oscillation, respectively. Three diverse examples namely in impact design, sheet metal process design and system identification are used to demonstrate the method. The methodology has been incorporated as a parallel solver in the commercial software code LS-OPT.*

## INTRODUCTION

The Response Surface Method (Myers, 1995) has become a popular method for conducting optimization involving the simulation of nonlinear dynamical problems. The purpose of the method is primarily to avoid the necessity for analytical or numerical gradient quantities as these are either too complex to formulate, discontinuous or sensitive to roundoff error. A common optimization procedure is to build a high order response surface in a region of interest in the design space and to refine the response surface in a semi-automated fashion by moving the center of the region of interest as well as reducing its size. Such an approach is suitable for making design improvements but not for problems such as the parameter identification of systems or materials where a converged result is desirable. Automated methods have therefore been formulated to address problems in rigid body dynamics (Etman, 1997) and sheet metal forming (Kok, 1998), (LSTC, 1999). The method presented here incorporates sophisticated features into these approaches. These are:

- contraction of the region of interest to a reasonable, possibly irregular design space,
- the use of *D*-optimal experimental design within an irregular design space,
- the use of move characteristics to determine the contraction rate of the region of interest and
- the identification of oscillation vs. 'panning' (translation of the region of interest in the design space) to determine the maximum shrinkage rate of the region of interest.

Nonlinear dynamic problems are particularly susceptible to random error of which the degree is difficult or impossible to determine analytically. Hence the approximation error has not been used to determine the dynamic parameters of the method.

## APPROACH

*Experimental design*
The method presented here is based on the design of experiments. For experimental design the *D*-optimality criterion is used (Myers, 1995). The advantage of this criterion is that experiments can be chosen in an irregular design space (Kaufman, 1996). This feature is advantageous for two main reasons:

- *Accuracy*. If design constraints are chosen as bounds for the region of interest, the size of the region is reduced which is likely to give a more accurate result.
- *Robustness*. Non-robust or unreasonable designs can be avoided. Examples can be found amongst designs with an unreasonably large mass or designs which may cause failure of the simulation process.

*Approximations*
In response surface methodology surfaces are fitted to the responses of the design points determined by the experimental design. A common approximation method is the fitting of polynomials although other types of surfaces can also be used. Quadratic polynomials are usually accurate for a mid-range region of the design space but because the expense is a function of $n^2$ (where $n$ is the number of design variables) they are normally avoided for large design problems. This applies particularly to nonlinear problems involving large finite element models. A possible solution is to use linear approximations. These are generally inaccurate beyond the immediate neighborhood of the design point but can be used in a successive response surface procedure (Etman, 1997). The difficulty with using successive linear approximations is that *cycling* or oscillation may occur. This phenomenon can be countered by manipulation of the size of the region of interest, a measure analogous to applying move limits in successive linear programming. Heuristic measures are typically introduced (Etman, 1997).

*Successive response surface method*
In the following procedure, two parameters have been used to drive a successive linear response surface method:

1. A *maximum contraction* parameter is determined based on whether the current and previous designs are on opposite or the same side of the region

of interest. The former case signals the onset of oscillation while the latter suggests that the optimum lies beyond the region of interest. The parameter determines the maximum shrinkage rate and should therefore be small for the oscillatory case and big for the 'panning' case.

2. An *effective contraction* parameter interpolates between the maximum contraction parameter and a constant minimum contraction parameter using the distance of the current optimum to the center of the region of interest as input.

*Software: LS-OPT*
The afore-mentioned methods have been incorporated in the program LS-OPT, a command language-based, standalone general optimization program which is closely interfaced with LS-DYNA. Access to most quantities available in the LS-DYNA database has been provided and maximum, minimum, averaged and filtered (see cylinder example) quantities can be automatically extracted. Special metal-forming quantities such as the forming limit criterion (FLD) are also available. For shape optimization (Akkerman, 2000), a preprocessor can be incorporated in the design cycle. Job execution can be conducted in parallel and in a distributed fashion (Akkerman, 2000) using an additional module. Multi-case and multi-disciplinary optimization can be conducted.

## EXAMPLES

In the present study, the iterative solver within LS-OPT has been applied to the optimization of a diversity of problems in nonlinear dynamics using LS-DYNA. The purpose, in each example, is to use a remote, often unreasonable initial design to test the robustness and efficiency and to assess the requirement for a good starting design. A final tolerance of 1% has been set on the objective function.

Three examples are used to illustrate the methodology implemented in LS-OPT. A fourth example, that of a vehicle crashworthiness optimization appears elsewhere in these proceeedings (Akkerman, 2000).

*Airbag system identification*
Five system parameters, representing the leakage properties of a deploying and impacted airbag (Figure 1) are determined from the displacements, velocities and accelerations produced by two separate physical experiments. The leakage properties are represented by a leakage vs. pressure curve defined by five unknown ordinates. The two experiments are distinguished by the velocity of impact namely 4 m/s and 5 m/s. Altogether 54 responses, 9 per time history curve, are used in the regression. This represents a monitoring increment of 5 ms.

The design requires a *monotonic* leakage curve. Some designs resulting from a standard experimental design may not have this property and may therefore cause the simulation to fail. Monotonicity of the experimental design can be enforced by using the constraints on the design variables:

$$x_{k+1} > x_k, \ \ k = 1, 2, \ldots, 9 \qquad (1)$$

to relocate points in a new, irregular region. A *D*-optimal experimental design, using 17 points, is thus determined within a so called "reasonable design s-pace" (Kaufman, 1996) defined by the monotonicity constraints. *D*-optimality is chosen as one of few possibilities for determining an experimental design in an irregular design space. To test the robustness of the algorithm, a constant number, 0.6, was chosen to represent the starting design for all variables. The bounds [0.2,4] were applied uniformly with an initial range of 1.0. This starting point causes two designs to abort. In the first iteration the procedure therefore relied on oversampling of the design space to simulate enough designs (8 out of 10).

The objective of the problem is to minimize the discrepancy between the computational and experimental results, thereby effectively calibrating the computational model. The RMS residual

$$R = \sqrt{\sum_{j=1}^{54} \left[ \frac{f_j(x) - F_j}{\Gamma_j} \right]^2} \qquad (2)$$

is used as a measure of calibration accuracy. Since all quantities are in different units, they have been normalized using suitable $\Gamma_j$. The symbols $F_j$ represent the target values of the responses and $f_j$ the computational responses.

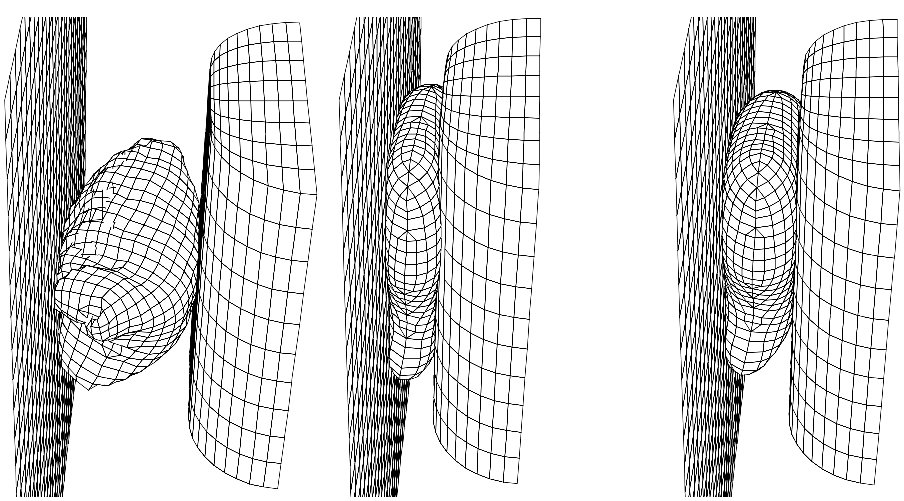


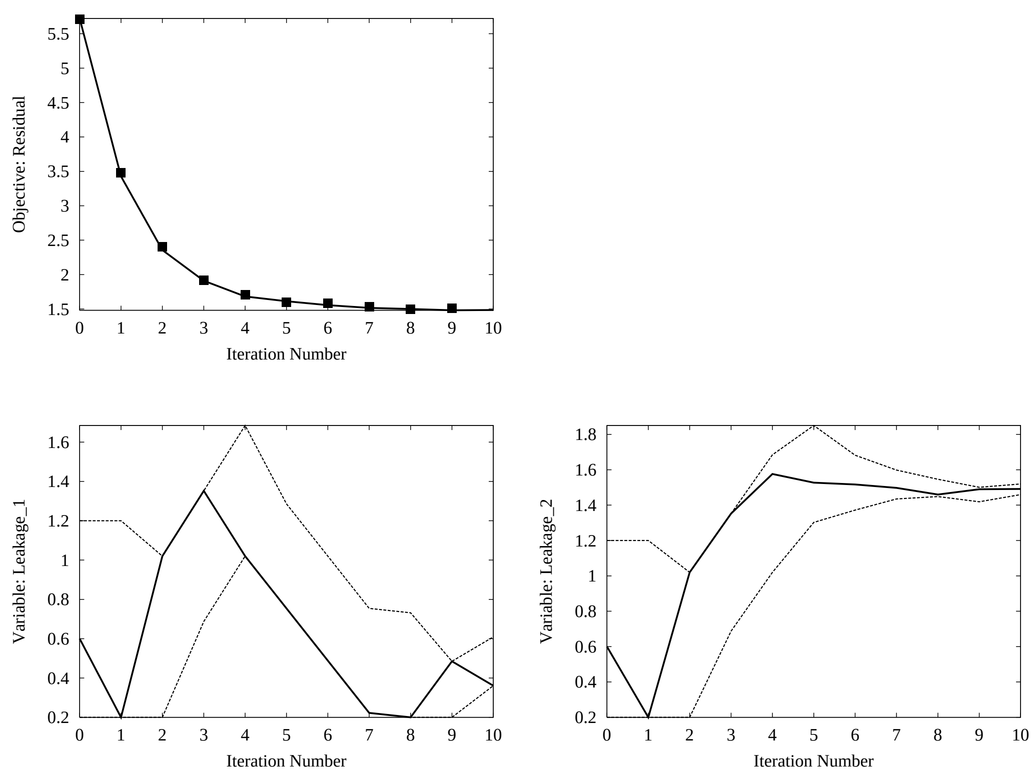
Figure 1: Airbag: Deployment, impact (33 ms) and rebound (50 ms)

The optimization history (Figure 2) and the optimal design (Table 1) show that although there are 5 design variables, some of them are constrained by the monotonicity, with the result that there are only three independent variables.

Table 1. Optimal leakage ordinates vs. pressure

| Variable | Pressure | Leakage |
|---|---|---|
| $x_1$ | 2.0 | 0.361 |
| $x_2$ | 2.25 | 1.492 |
| $x_3$ | 2.5 | 1.492 |
| $x_4$ | 2.75 | 1.492 |
| $x_5$ | 3.0 | 5.25 |

The first variable oscillates while the last has not converged yet. However, judging by the convergence properties of the residual and by the design sensitivities (not shown here) the smaller sensitivities of these two variables appear to render them insignificant in the neighborhood of the optimum.



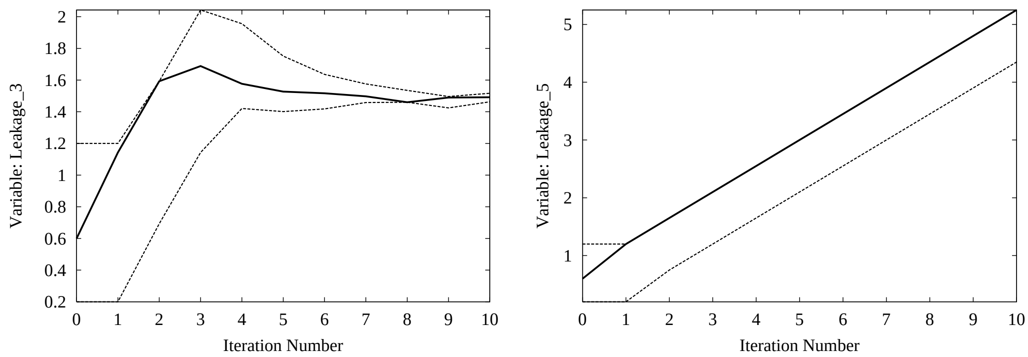
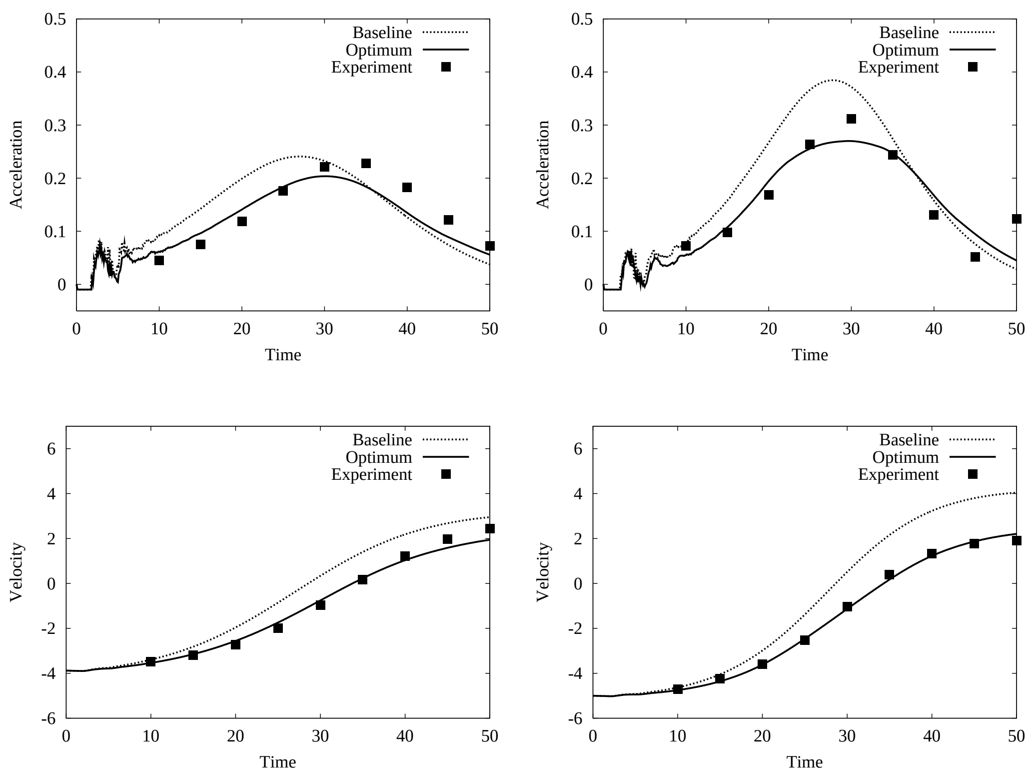



Figure 2: Airbag: Optimization History

Figure 3 shows that there is a major improvement in the model as a result of the optimization process.



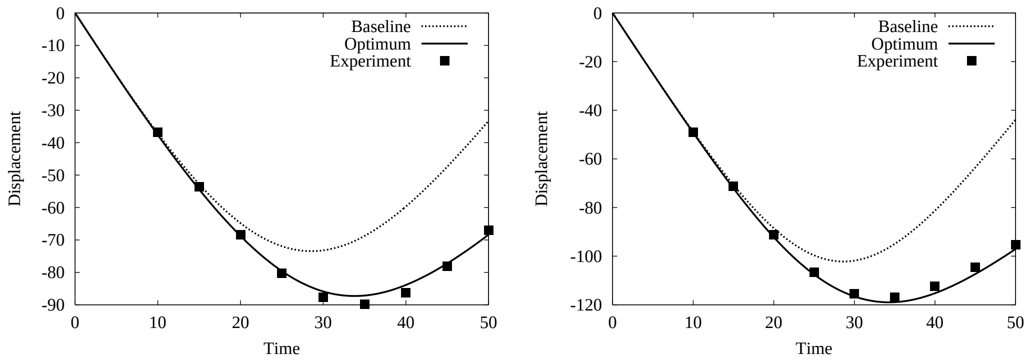



Figure 3: Airbag: Comparison of computational and experimental results

*Sheet metal form design*
A sheet metal problem (Figure 4) is presented in which the *maximum* radius of the cross-sectional die geometry has to be minimized.

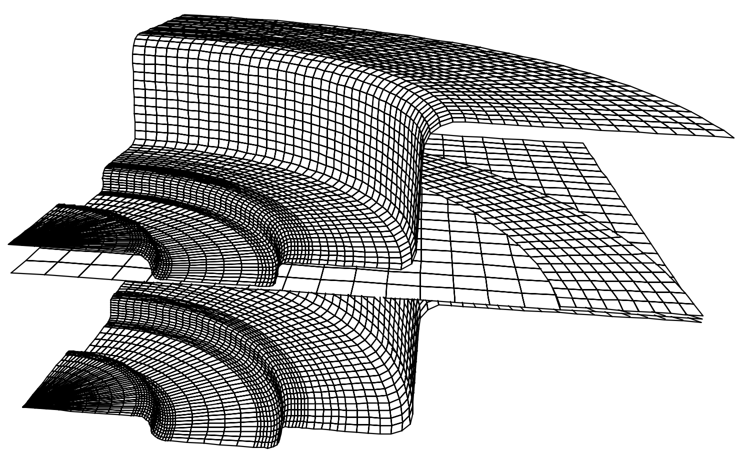


Figure 4: Finite Element model of tools and blank

Three design variables, the outer three radii of the cross-section of the die, have been chosen. The constraints are the forming limit criterion (zero is the bounding value) and the maximum thinning of 20%. Mesh adaptivity is used during analysis to improve the curvature of the deformed model (shown with a coarse mesh in Figure 4). A detailed description of the problem can also be found in the LS-OPT User's Manual (LSTC, 1999).

The initial radii are chosen as 1.5mm uniformly and the final results are shown in Figure 5. The history results show that the thinning and FLD responses converge in about 2 iterations. Two or three further iterations are required to minimize the maximum of the three radii. A violation of the bounds of the region of interest occurs in the first iteration because a feasible design could not be found and therefore the bounds are compromised by the core optimization solver. Figure 6 shows the baseline and optimal flow limit diagrams with the degree of violation clearly visible for the baseline case.

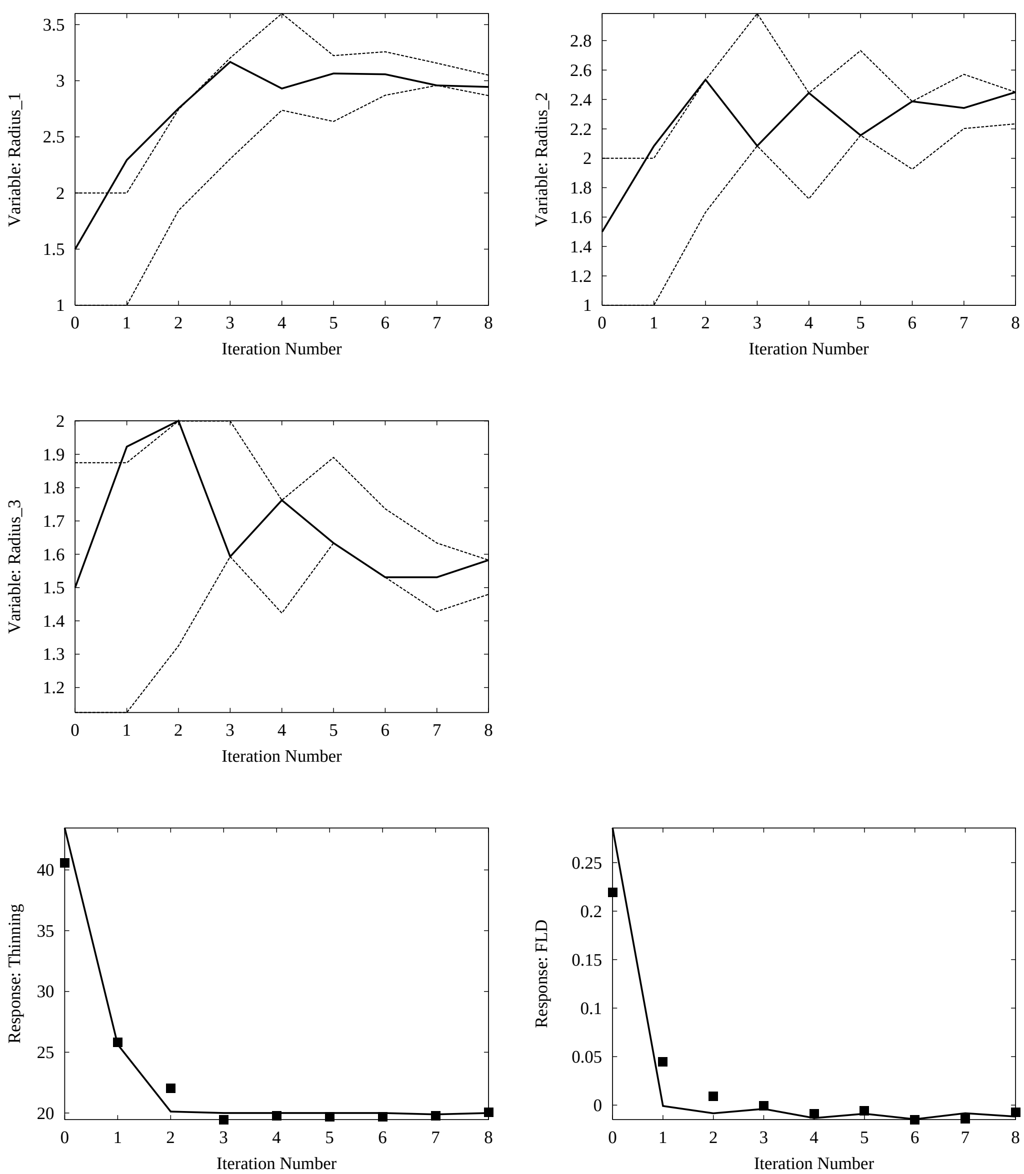



Figure 5: Metal forming: optimization history

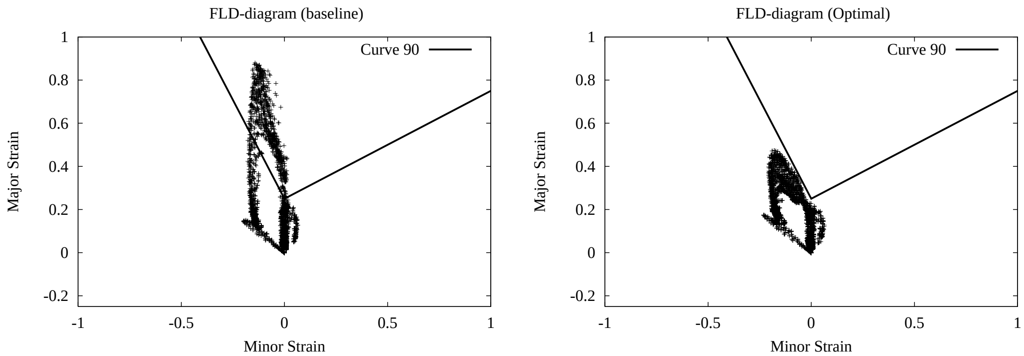



Figure 6: Baseline and optimal flow limit diagrams

*Impact optimization*
The problem, based on (Yamazaki, 1997), consists of a tube impacting a rigid wall as shown in Figure 7. The energy absorbed is maximized subject to a constraint on the maximum rigid wall impact force. The cylinder has a constant mass of 0.54 kg with the radius and thickness as design variables. The length of the cylinder is dependent on the design variables because of the mass constraint. A concentrated mass of 500 times the cylinder weight is attached to the end of the cylinder not impacting the rigid wall. The deformed models are shown in Figure 8.

The optimization problem is stated as:

$$\text{Maximize } E_{\text{internal}}(x_1, x_2)|_{t=0.05}$$

subject to

$$F_{\text{normal}}^{\text{wall}}(x_1, x_2)|_{\max} \leq 100{,}000$$

$$l(x) = \frac{0.52}{2\pi\rho x_1 x_2}$$

where the design variables $x_1$ and $x_2$ are the radius and the thickness of the cylinder respectively. $E_{internal}(x)|_{t=0.05}$ is the objective function and constraint functions $F_{normal}^{wall}(x)|_{max}$ and $l(x)$ are the maximum normal force (filtered with SAE 300Hz) on the rigid wall and the length of the cylinder respectively.

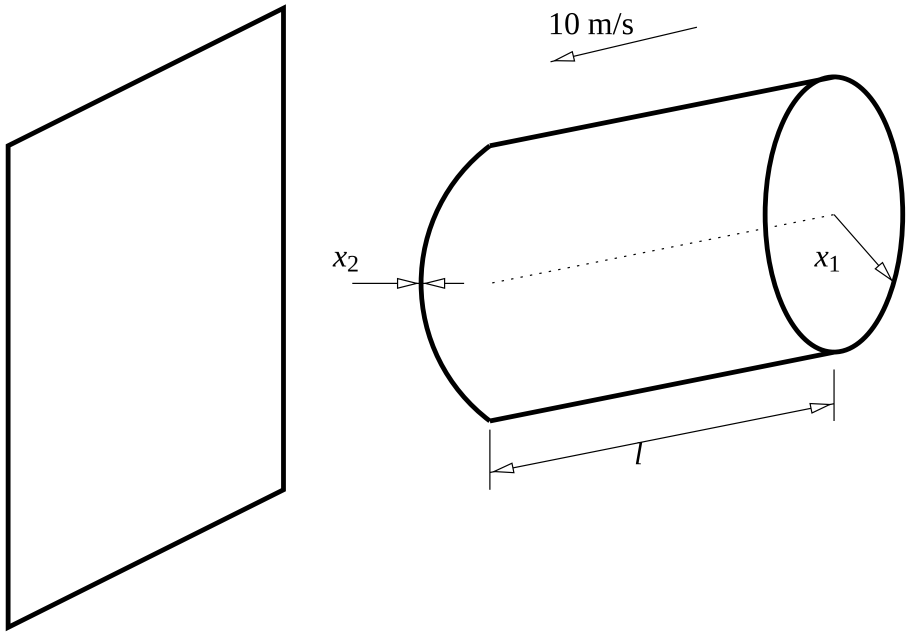



Figure 7: Impacting cylinder

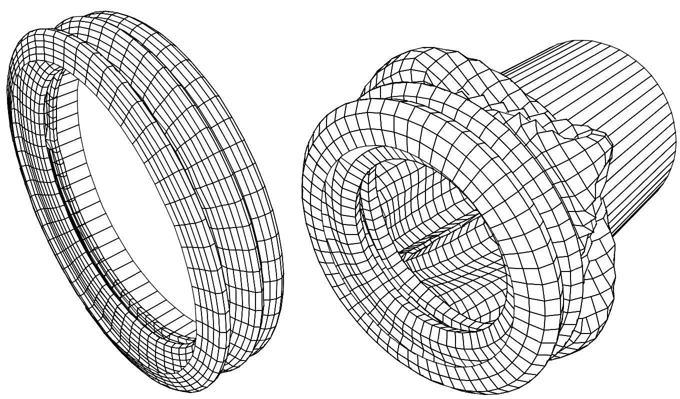


Figure 8: Deformed configurations: (a) Baseline ($t = 50$ms) and (b) Optimal ($t = 50$ms)

The optimization history (Figure 9) shows that the initial design is severely infeasible but that the design evolves to be feasible after two iterations. The reduction of the force coincides with an increase in absorbed energy. The data of the optimal design are given in Table 2.

Table 2. Starting values, bounds and optimal values

| Variable | Start | Lower Bound | Optimum | Upper Bound |
|---|---|---|---|---|
| Radius | 75 | 30 | 30 | 100 |
| Thickness | 3 | 2 | 4.7 | 6 |
| Internal energy | 12,490 | | 13,360 | |
| Peak Wall force | 1,544,000 | | 109,100 | 100,000 |

Figure 10 confirms the feasibility of the design with three of the force peaks being active. It is apparent that the baseline design is too soft, causing a sudden large force peak upon contact of the trailing mass with the wall. The optimal design has a more evenly distributed force with a small violation of about 9%.

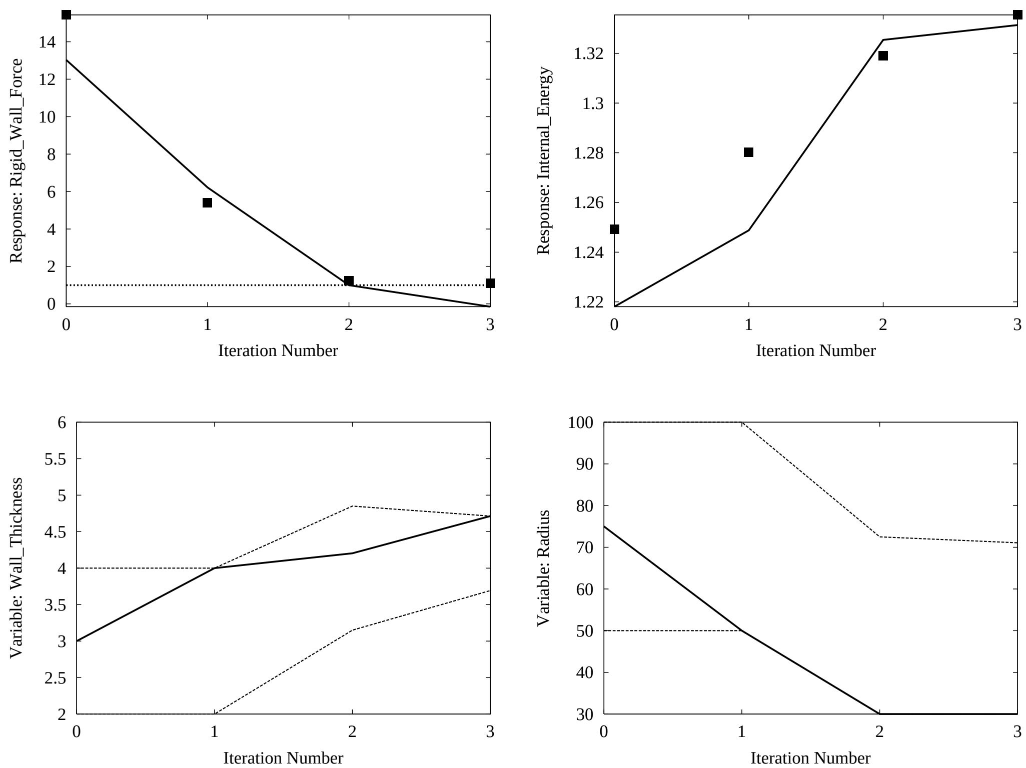



Figure 9: Cylinder: Optimization History

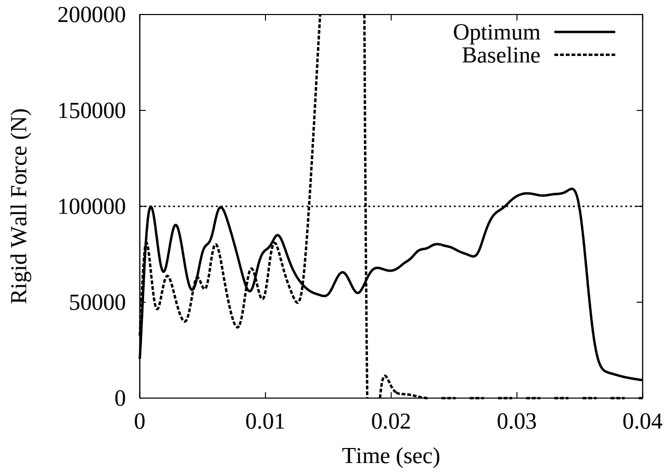



Figure 10: Cylinder: Constrained rigid wall force: $F(t) \leq 100,000$ (SAE 300Hz filtered)

## SUMMARY

The computational effort of the above examples is summarized in Table 3. The numbers are based on the stopping criterion of a 1% change of the objective function. Because of the direct use of the design variables in the objective of the sheet metal problem, the tolerance has been relaxed to 3%. The final check simulation and case multiplicity (airbag problem) are included in the numbers of the last column.

Table 3. Summary of Computational Data

| Problem type | Variables | Simulations/it. | It. | Simulations |
|---|---|---|---|---|
| Airbag | 5 | 10 | 10 | $101 \times 2$ |
| Sheet-metal Die | 3 | 7 | 8 | 57 |
| Cylinder | 2 | 5 | 3 | 16 |

The methodology has also been validated by the larger problem with 11 design variables (Akkerman, 2000). That problem requires 3 iterations employing 58 simulations in total (19 each).

## CONCLUSIONS

An adaptive successive response surface method which employs the convergence properties in conjunction with a 'reasonable' experimental design procedure is shown to provide a high degree of accuracy, robustness and efficiency for optimization. Starting from a remote and often unreasonable initial design, an optimal design of reasonable engineering accuracy can be obtained rapidly. Linear approximations make the approach effective for a large number of design variables. The methodology is suitable for a wide range of problems in nonlinear dynamics and is highly accurate, as is often required for parameter identification problems.

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