Numerical Modelling and Biaxial Tests for the Mullins Effect in Rubber

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ABSTRACT

The formulation, testing and numerical study of the Mullins effect on rubber are presented. Ogden first modelled the Mullins effect for studying the unloading in filled rubber. It has been extended here to include the Mullins effect on both unloading and subsequent loading.

To demonstrate the Mullins effect experimentally, a new biaxial test, inflation of a plane circular membrane, is used. Some experimental test data are presented.

An approximate solution, a relation between the inflation pressure and the displacement at the centre for the inflation of a plane circular membrane is presented. The test data and the approximate solution are used to determine the Mullins damage material constants. For more accurate study, the material constants can also be obtained through the combination of LS-DYNA, LS-OPT and the test data.

The test data, analytical results and numerical results from LS-DYNA are shown. They agree with one another.

Keywords:

Mullins effect, Rubber, Strain-energy-density function, Damage, Biaxial test, LS-DYNA

INTRODUCTION

Rubber has been used in many structures for years. But the mechanical properties of rubber, other than the elastic properties, are still unknown; hence, studies of these structures are generally based upon their elastic properties only. In reality, rubber often exhibits the Mullins effect, viscoelastic, and chronorheological behaviour. The magnitudes of these properties are often large enough that they cannot be neglected. To date, very few analytical and experimental studies for determining these properties have been attempted. This is because the study of mechanics for rubber must consider both geometric and material nonlinearities. The additional effects and the lack of constitutive equations that describe these phenomena make the analytical and experimental studies very difficult. In this paper we address the Mullins effect both from the constitutive equation and the experimental test point of view. For experimental study, tests can be uniaxial or biaxial. The uniaxial test is easier to perform but less relevant to actual designs; the biaxial test is harder to perform but more relevant to actual designs. In this paper we developed a new biaxial test method that is easy to perform and uses very little laboratory space.

In the formulation, Ogden [1] first modelled the Mullins effect to study the unloading in filled rubber. It has been extended here to include the Mullins effect on both unloading and subsequent loadings. The new constitutive equation simulates real rubber behaviour better.

Based on the experimental biaxial test results and the new constitutive equation for the Mullins effect, the material constants are determined. In turn, these new constitutive equations and the material constants are implemented in LS-DYNA. Hence, the Mullins effect for the mechanical parts with rubber can be studied numerically.

BIAXIAL TEST

In the experiment a flat circular membrane of rubber was clamped between two plates as shown in Figure 1. The membrane is inflated by air or liquid from a reservoir at a constant temperature. The height of the deformed membrane at the pole is measured by a LVDT. The pressure is measured with a pressure transducer. A data acquisition system and a computer gather the data from the LVDT and pressure transducer and are shown in Figure 2.

The pressure-height relationship is measured, as shown in Figure 3. The radius of the membrane is 2 inches and the thickness is 0.015 inches. In the experiment, the membrane is inflated monotonically to 0.8 inches then deflated to the flat surface. It is inflated again to 0.8 inches then deflated again for three cycles. At the end of the third cycle, the membrane is inflated again but the height inflated to past 0.8 to 1.6 inches before unloading again. This loading and unloading was again repeated for three cycles. During the loading and unloading, at the pole, the deformation is in a uniform biaxial stress state.

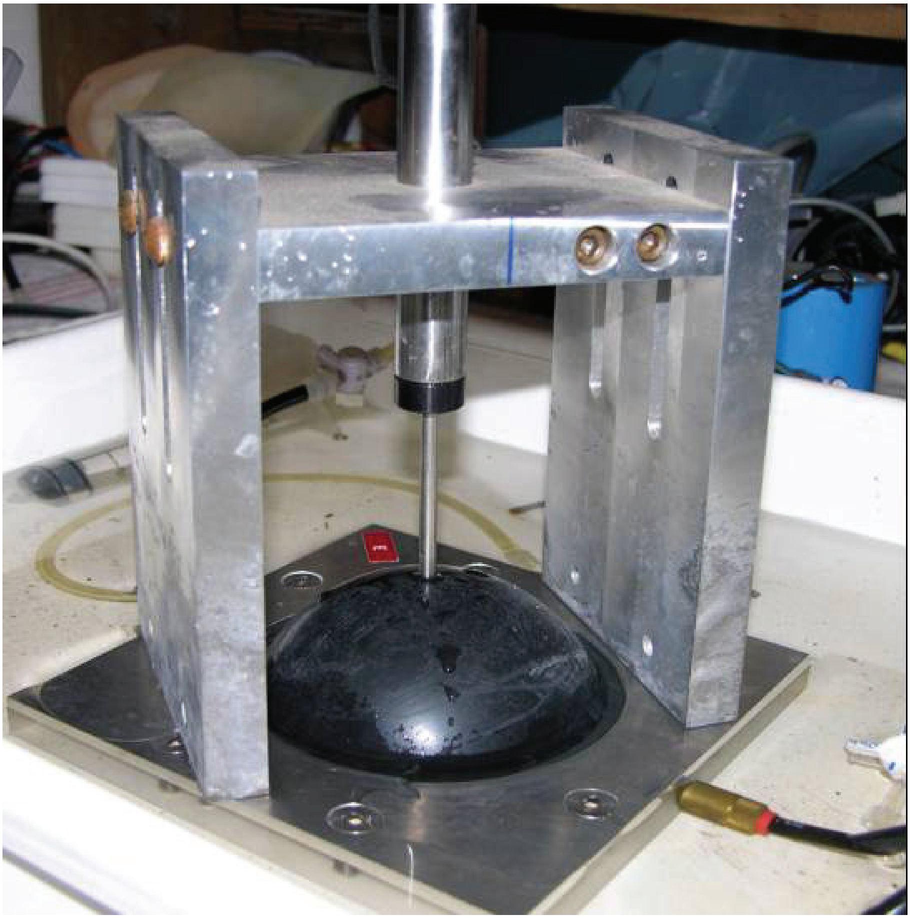

Figure 1. The apparatus

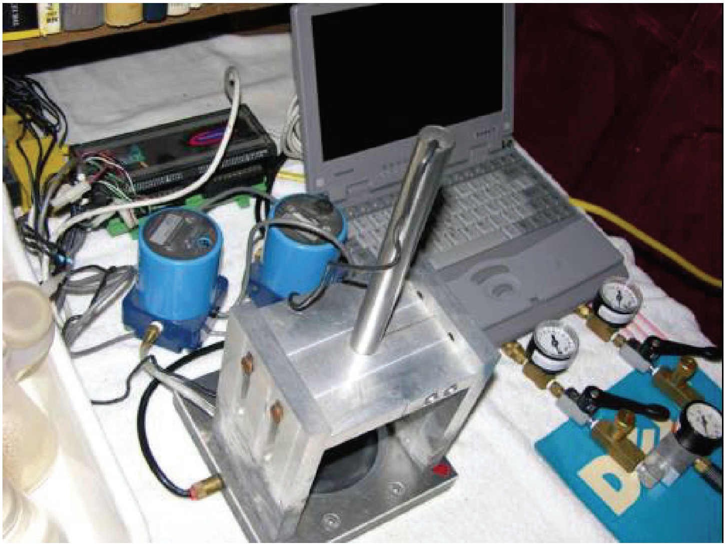

Figure 2. Laboratory set-up for the biaxial test

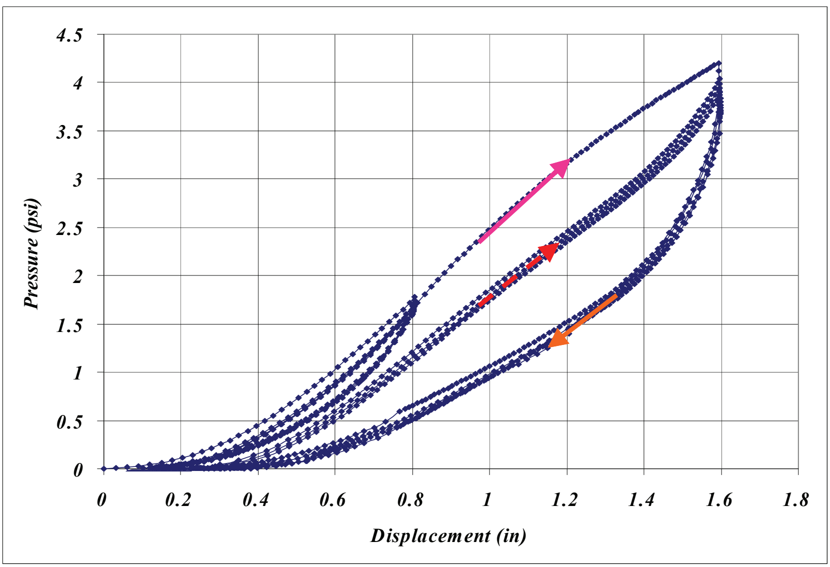

Figure 3. The results of a biaxial test

ANALYTICAL STUDY

Ogden and Roxburgh [1] proposed that for the Mullins effect on rubbers the loading and unloading follow a different curve, but the loading and subsequent loading follow the same path as shown in Figure 4 for an uniaxial test.

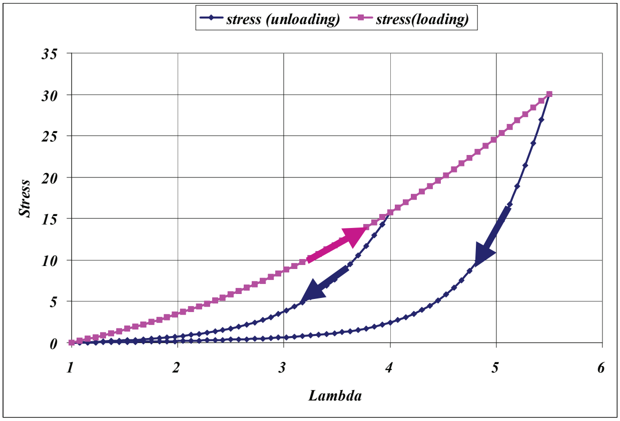

Figure 4. The Mullins effect based on Ogden formulation for uniaxial loading, unloading and subsequent reloading.

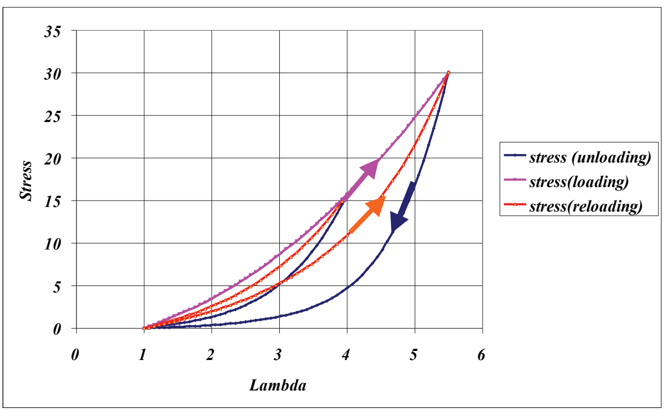

Figure 5. The Mullins effect based on the current formulation for uniaxial loading, unloading and subsequent reloading.

However, for most materials the initial loading and subsequent unloading and reloading follow a different path, as shown in Figure 3 for the biaxial test. The new formulation presented below solves the difference between the Ogden model and real material behaviour. The Mullins effect based on the new formulation for uniaxial initial loading, unloading and subsequent reloading is shown in Figure 5.

In this paper, the formulation of Ogden has been extended to consider both the Mullins effect on unloading and subsequent reloadings. We introduce a damage function that has four material constants: two for unloading and two for subsequent reloading. The strain-energy density function with Mullins damage function of a rubber is $\tilde{W}(\lambda_i)$

$$\tilde{W}(\lambda_i) = \eta W(\lambda_i) \qquad \text{(Eq. 1)}$$

where $W(\lambda_i)$ is the strain-energy density function based on the initial loading, and η is a damage function for the Mullins effect.

For initial loading $\eta = 1$

For unloading $$\eta = 1 - \frac{1}{r_1}\tanh\left[\frac{1}{m_1}(W_m - W)\right] \qquad \text{(Eq. 2)}$$

For subsequent reloading $$\eta = 1 - \frac{1}{r_2}\tanh\left[\frac{1}{m_2}(W_m - W)\right]$$

$W_m(\lambda_i)$ is the maximum strain-energy density function before unloading. r_1, r_2, m_1 and m_2 are the material constants for the Mullins effect damage function. With this damage function, the loading and subsequent unloading follow different paths as shown in Figures 3 and 5. The material constants r_1 and r_2 are greater than one. For a loading with a value of the strain-energy density function greater than $W_m(\lambda_i)$, the process repeats again.

The strain-energy function $W(\lambda_i)$ can be in any form. For example, it can be neo-Hookean, Mooney, Ogden incompressible or Ogden compressible. When Ogden compressible material is considered, all others are special cases.

For highly compressible materials the strain-energy density function for Ogden constitutive equation is:

$$W=\sum_{j=1}^{m}\frac{C_j}{b_j}\left[\lambda_1{}^{b_j}+\lambda_2{}^{b_j}+\lambda_3{}^{b_j}-3+\frac{1}{n}\left(J^{-nb_j}-1\right)\right] \qquad \text{(Eq. 3)}$$

here C_j , b_j and n are material constants and $J=\lambda_1\lambda_2\lambda_3$ represents the ratio of deformed to undeformed volume. When $n=\infty$, Equation (3) is the Ogden incompressible material. When $n=\infty$, $m=2$, $b_1=2$ and $b_2=-2$, Equation (3) is the Mooney material. When $n=\infty$, $m=1$ and $b_1=2$, Equation (3) is the neo-Hookean material. In this paper, for simplicity in formulation, only neo-Hookean strain-energy

$$W(\lambda_i)=C_1\left(\lambda_1^2+\lambda_{21}^2+\lambda_3^2-3\right) \qquad \text{(Eq. 4)}$$

is considered for demonstration. The Ogden compressible constitutive equation is programmed in LS-DYNA.

DETERMININATION OF DAMAGE MATERIAL CONSTANTS

The damaged material constants can be determined from the biaxial test. An approximate relationship between the inflating pressure (P) and the deformation at the pole (Δ) has been obtained by Feng and Christensen [2].

$$P=\frac{2C_1H}{R}\left\{\frac{1}{\left(\frac{\Delta}{R}+\frac{R}{\Delta}\right)}\left[1-\frac{1}{\lambda^6}\right]\right\} \qquad \text{(Eq. 5)}$$

The relationship between λ and Δ is

$$\lambda=\frac{\left(\frac{\Delta}{R}+\frac{R}{\Delta}\right)}{2}\sin^{-1}\left(\frac{2}{\left(\frac{\Delta}{R}+\frac{R}{\Delta}\right)}\right) \qquad \text{(Eq. 6)}$$

where R is the radius of the circular membrane and H is the thickness of the circular membrane. Similar expressions can be obtained for Mooney and other constitutive equations. The exact numerical solution, approximate solution obtained from the above equation, and test data for inflating a circular thin disk, shown in the Figure 7, are for the Mooney material with $C_2 / C_1 = 0.02$. The C_2 / C_1 is denoted by g_1 / g_0 in Figure 7. The approximate solution is very good.

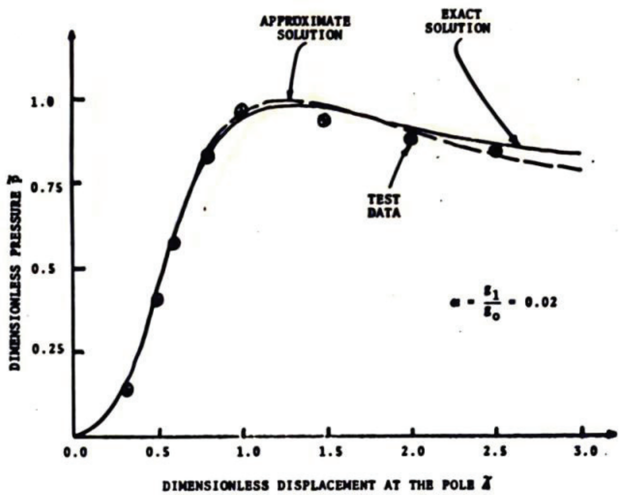

Figure 6. The approximate solution, exact solution and test data for an inflated membrane.

When the damage function for the Mullins effect is considered, Equation 5 becomes

$$P = \frac{2C_1 H}{R} \left\{ \frac{1}{\left(\frac{\Delta}{R} + \frac{R}{\Delta} \right)} \left[1 - \frac{1}{\lambda^6} \right] \right\} * \eta \qquad \text{(Eq. 7)}$$

With the experimental biaxial test data and the above equation, the damage constants in Equation (2) can be obtained. The damage material constants r_1 and m_1 can be obtained from the pressure-deformation unloading curve. The damage material constants, r_2 and m_2 can be obtained from the pressure-deformation reloading curve. These constants are all obtained from the least-square curve fit.

Since (Eq. 7) is an approximate solution, the material constants determined are also approximate values. For more accurate results, LS-DYNA with LS-OPT can be used. LS-DYNA can be used to determine the inflation of a circular membrane with the appropriate material model. LS-OPT and the experimental test data can be used to determine the material constants for the Mullins damage functions.

LS-DYNA ANALYSIS

The formulation presented in this paper applies to one-, two- and three-dimensional problems. For general three-dimensional problems the mathematical formulation has been implemented in LS-DYNA. In LS-DYNA the formulations and applications can be extended to various rubbers with strain-energy density represented by various constitutive equations such as: neo-Hookean, Mooney, Ogden incompressible, and Ogden compressible materials. It can also been extended to viscoelastic materials, as modelled by the Feng-Hallquist [3] constitutive equation for compressible and incompressible viscoelastic materials subjected to very large deformation.

The result of a cube of 0.5x0.5x0.5 subjected to uniaxial extension is obtained from LS-DYNA. The displacement at one end and the stress in the cube are shown in Figure 7. The stress-displacement plot is shown in Figure 8. The displacement can be converted to the stretch ratio λ. The results shown in Figure 5 from the analytical calculations, and Figure 8 from LS-DYNA are the same.

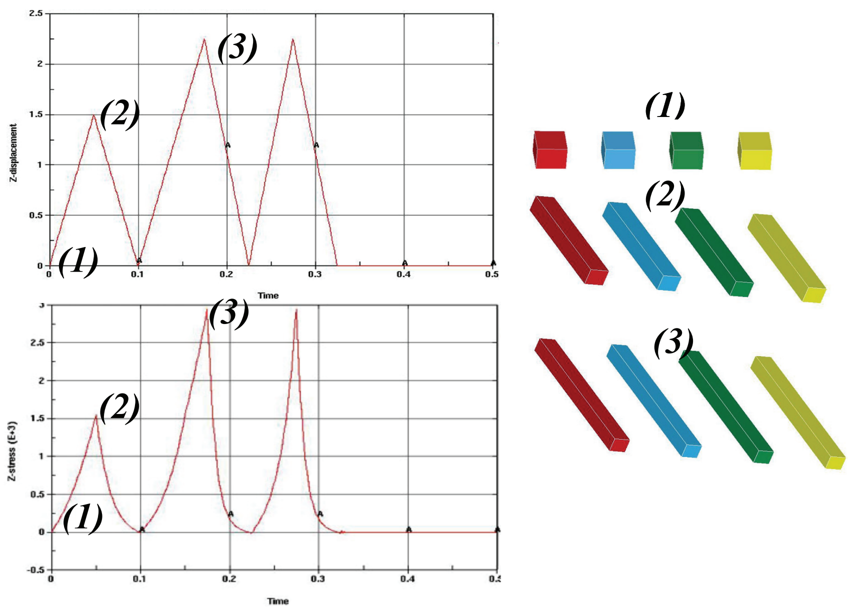

Figure 7. The results from LS-DYNA.

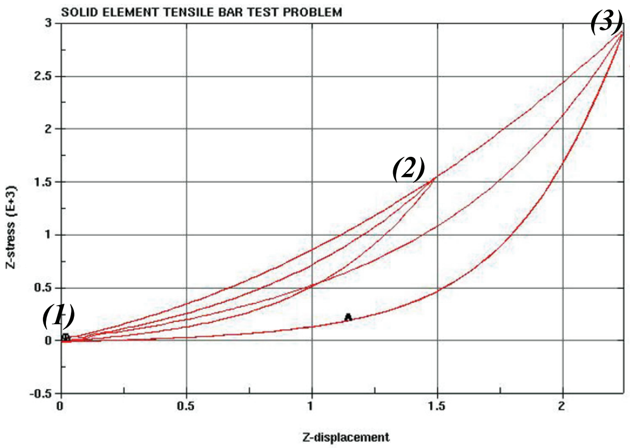

Figure 8. The stress-displacement plot from LS-DYNA.

SUMMARY AND CONCLUSIONS

Since the application of rubber in engineering design are all two- or three- dimensional problems, the biaxial test results in better simulation for real applications than the uniaxial test. With some small modifications, the design for the biaxial test apparatus can be used for all materials. For example, if the material is stiffer and the inflating pressure is too high, one can simply increase the radius of the membrane or use a thinner membrane to lower the pressure.

The biaxial tests, constitutive equation with four Mullins damage constants, the damage constants determination, and the implementation of these features into LS-DYNA are all new.

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