

ALE Modeling of Surface Waves

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Abstract

An Arbitrary Lagrange Euler formulation for the propagation of surface waves is developed in LS-DYNA®. The ALE computational time step in this code is divided in two cycles: a Lagrangian cycle in which the mesh follows the material deformation and an advection cycle in which the users through remapping algorithms control the mesh motion. The new feature presented in this paper is one of these remapping algorithms. It enables a Lagrangian behavior of free ALE mesh boundaries whereas, in the direction of the wave propagation, the ALE mesh is Eulerian to avoid distortions. Nodes on the ALE mesh borders moves with the surface waves during the Lagrangian cycle. During the advection cycle, the remap positions of these nodes are computed by interpolating the Lagrangian positions of their neighbors with biquadratic polynomials. If the wave amplitudes are too important, ALE smoothing can be used for the internal nodes and a specific smoothing is applied on the mesh surfaces.

Introduction

According to [1] the Arbitrary Lagrangian Eulerian (ALE) method is based on three domains:

- The initial configuration of the material,
- The current configuration of the material,
- The referential or ALE domain.

A map between the first and second domain defines the material motion while a map between the third and second ones determines the mesh motion. The problem is formulated in the referential domain. The ALE formulation is a combination of the Lagrangian and Eulerian methods.

In the Lagrangian description of motion there is no map: the three domains are identical and the referential domain follows the material motion, which greatly simplifies the governing equations. Lagrangian schemes have proven very accurate as long as the

mesh remains regular. However, large material deformations lead to severe mesh distortions, what involves accuracy losses and a reduction of the explicit critical time step.

With an Eulerian description of motion the first and third domain are the same: the mesh is fixed in space and the material passes through the element grid. The transport of mass between elements complicates the governing equations by introducing nonlinear transport terms. Mass conservation is not automatically satisfied. Advection algorithms need to be implemented for the mass, momentum and internal energy conservation and for the tracking of all state variables.

In the ALE formulation the first and third domain are different what introduces a mesh motion independent of the material motion. The arbitrary movement of the referential domain computed by a moving mesh algorithm or specified by the user permits to deal with moving boundaries. In LS-DYNA® the user can choose among several moving mesh methods to remap the mesh :

- A simple average of the neighboring node coordinates (AFAC in *SECTION_SOLID),
- The Kikuchi's algorithm, which is based on the volume weighted average of the surrounding mesh coordinates (BFAC in *SECTION_SOLID),
- The isoparametric method keeps the node in the middle of its neighbors (CFAC in *SECTION_SOLID),
- Equipotential algorithm developed by Winslow [1] which solves the Laplace equations for the mesh coordinates (DFAC in *SECTION_SOLID),
- Remapping algorithms used in LS-DYNA® [2] can handle more general situations as a linear combination of the simple average, volume average, isoparametric and equipotential methods:

$$\begin{aligned}
 \vec{x}_r^{n+1} = & AFAC \cdot \vec{x}_{simpleaverage}^{n+1} + BFAC \cdot \vec{x}_{volumeaverage}^{n+1} + \\
 & CFAC \cdot \vec{x}_{isoparametric}^{n+1} + DFAC \cdot \vec{x}_{equipotential}^{n+1}
 \end{aligned}$$

- *ALE_REFERENCE_SYSTEM_CURVE prescribes a rigid motion to the ALE mesh,
- *ALE_REFERENCE_SYSTEM_GROUP provides several ways to adjust the ALE mesh: smoothing, rigid motion, expansion, refinement, ...
- *ALE_SMOOTHING enables the user to keep a node at its initial parametric position between two other nodes so that the inner mesh remains regular when the boundary motion is large.
- ...

These methods generally deal with the interior nodes. The new algorithm presented in this paper remaps boundary nodes. A Simple ALE naturally manages this kind of remapping because the mesh boundary is Lagrangian and thereby follows the motion of the material frontiers. But some problems must be treated with a multi-material ALE formulation and in this case the mesh boundary can follow the global motion prescribed

by the previous remapping methods but not the local material boundary motion. The VOF method included in the multi-material ALE formulation can track the material interface but a refined mesh is required to highlight small surface waves. Thus the interest of this method is to model the propagation of perturbations on free boundaries of a multi-material ALE mesh. The nodes on these boundaries can track the material surfaces because the ALE code in LS-DYNA® divides each computational time step into two phases:

- The first step is the Lagrangian phase, during which the incremental motion of the material is computed and the material motion and the mesh motion are identical. Thus at the end of this cycle the position of the material surface is known.
- The second step is the Eulerian phase, which is referred to as the advection, or remap phase. In this step there is a transport of material between the cells because the mesh is moved independently to the material position. To keep a regular mesh the new remapping algorithm moves back the frontier nodes along the material surface determined in the Lagrangian cycle. Thus the position of the material surface is kept.

The outline of this paper is as follows. In the first section an overview of the governing equations in the ALE description is presented along with the LS-DYNA® keywords. The next section will describe the new remapping technique and input deck keywords. In the last section the numerical modeling of an extrusion problem is presented, illustrating the abilities of the new remapping method.

ALE Modeling

The application described in this paper is the extrusion of an aluminium “billet”, which is the metal piece resulting from the process. The billet motion is solved by using an ALE formulation [3].

General description

Let $\Omega \in R^3$, represent the domain occupied by aluminium, which is assumed elastoplastic (*MAT_PLASTIC_KINEMATIC), and let $\partial\Omega$ denote its boundary. This domain is meshed by solid elements (*ELEMENT_SOLID) based on a ALE formulation (*SECTION_SOLID_ALE). The equations of mass, momentum and energy conservation in a general ALE formulation are given by:

$$\frac{\partial \rho}{\partial t} + \rho \text{div}(\vec{v}) + (\vec{v} - \vec{w}) \text{grad}(\rho) = 0 \quad (1)$$

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho(\vec{v} - \vec{w}) \cdot \overrightarrow{\text{grad}}(\vec{v}) = \overrightarrow{\text{div}}(\overline{\overline{\sigma}}) + \vec{f} \quad (2)$$

$$\rho \frac{\partial e}{\partial t} + \rho(\vec{v} - \vec{w}) \cdot \overrightarrow{\text{grad}}(e) = \overline{\overline{\sigma}} : \overrightarrow{\text{grad}}(\vec{v}) + \vec{f} \cdot \vec{v} \quad (3)$$

where \vec{v} and \vec{w} are the fluid and mesh velocity fields respectively. If $\vec{v} = \vec{w}$, the equations (1)-(3) give the Lagrangian formulation. In the Eulerian formulation $\vec{w} = \vec{0}$. The equations (1)-(3) are solved by the split approach detailed in [3],[4] and implemented in LS-DYNA®. The ALE formulation is input by the following keywords:

```
*ALE_MULTI-MATERIAL_GROUP
$ id, idtype
  3,      1
  4,      1
*PART
ALUMINIUM
$ pid, secid, mid
  3,      2,   1
*PART
VOID (initially)
$ pid, secid, mid
  4,      2,   4
*SECTION_SOLID
$ secid, elform
  2,      11
*MAT_PLASTIC_KINEMATIC
$ mid,      ro,      E,      pr,      sigy,      etan
  1, 2.70000+3, 70.000+09, 0.3300000, 240.0+6, 1060.0+6
*MAT_VACUUM
$ mid
  4
*ELEMENT_SOLID
$ eid, pid, n1, n2, n3, n4, n5, n6, n7, n8
```

Equations (6)-(8) are completed with appropriate boundary conditions. The part of the boundary at which the velocity is specified is denoted by $\partial\Omega_1$:

$$\vec{v} = \vec{g}(t) \quad \text{on} \quad \partial\Omega_1 \quad (4)$$

In the application the nodes on the mesh boundaries of pid=3 are blocked in the normal direction to get a sliding condition while the ones of pid=4 are free. The inflow boundary condition is applied on the inlet nodes by the following keyword:

```
*BOUNDARY_PRESCRIBED_MOTION_NODE
$ nid, dof, vad, lcid
```

The traction boundary condition is imposed on the remaining part of the boundary of Ω .

$$\vec{\sigma} \cdot \vec{n} = \vec{h}(t) \quad \text{on} \quad \partial\Omega \quad (5)$$

In the application the traction boundary vanishes on the free surface of pid=4. On this boundary, the surface waves take place during the extrusion.

Multi-Material formulation

Surface wave problems are more easily modeled by a Lagrangian or Simple ALE method. However if an analysis like the extrusion problem involves large material deformation, the distortion of the mesh makes such a method difficult to use. The Simple ALE would require a specific relaxation technique or many re-meshing steps as for the Lagrangian approach. Another alternative is the multi-material ALE formulation (*ALE_MULTI-MATERIAL_GROUP_PART and elform=11). Thus in the application a part (pid=4) is created and the dummy material is void. Actually the remapping technique presented in this work is mainly designed for the multi-material formulation because the Simple ALE approach can intrinsically handle the boundary motion. The multi-material ALE formulation is a method that allows more than one material in each cell. It is powerful by the fact that complex geometries can be described without an element grid matching the material interface. The multi-material concept is used in many hydrocodes, but its implementation is more complex than a single material formulation. Also the memory requirement is higher, since each cell must be prepared to store more than one set of state variables. The multi-material method involves dealing with two new issues: the interface tracking and the advection of fluid materials across element boundaries.

There are several methods to treat the free surface in a fluid problem; the common one is the VOF (Volume Of Fraction) method, which is attractive for solving a broad range of non-linear problems in fluid and solid mechanics, because it allows arbitrary large deformations and enables free surfaces to evolve. The Lagrangian phase of the VOF method is easily implemented in an explicit ALE finite element method. Before advection, special treatment for the partially voided element is needed. An element containing a material interface is partially filled and the volume fraction

satisfies $V_f \leq 1$. In order to compute accurately the position of the material interface, interface-tracking algorithm is performed before the remesh process and advection phase. A possible way of tracking interfaces is the use of the volume fractions of the elements, or the Young method (Young, 1982). In this method, the material layout is described solely by the volume fraction of the fluid material in the element. Specifically, a plane approximates the interface in the cell. Then nodal volume fractions are computed to each node based on the fraction volumes of elements that share the same node. The nodal volume fractions determine the slope of the material interface inside the element. The position of the interface is then adjusted so that it divides the element into two volumes, which correctly matches the element volume fraction. The interface position is used to calculate the volume of the fluid flowing across cell sides. As the X-advection, Y-advection and Z-advection are calculated in separate steps, it is sufficient to consider the flow across one side only by taking the interface position into account. The fluxes of conservative variables for each material are computed by evaluating the volumes given by the intersection of the material interface and the total volume of the flux transported to the adjacent element.

As already stated the multi-material ALE mesh can be moved, expanded, refined, ... thanks to the global remapping techniques presented in the introduction but it can not afford a local motion of the mesh boundaries like the Simple ALE method. Thus a new remapping method was required to model the surface waves.

Remapping technique

The new remapping technique implemented in LS-DYNA® involves *ALE_REFERENCE_SYSTEM_GROUP. This paragraph will present this technique through the description of the time step split. The computational time step is divided in two phases: the Lagrangian phase during which the physical problem is solved and the advection phase during which the remap technique is applied. This split permit to adjust the mesh during the second phase without which large deformations could jeopardize the run.

Lagrangian phase

In the Lagrangian phase of a cycle, the solution is advanced from t^n to t^{n+1} with a second order accurate central difference scheme. The mass is assumed concentrated at the nodes. This leads to a diagonal mass matrix, which greatly simplifies the relation between nodal forces and accelerations. For numerical stability reasons, the magnitude

of the time step, Δt^{n+1} , is limited by the highest eigenfrequency of the system or by flux limits in the advection algorithm. To save memory, a staggered scheme in time is used. Velocities are computed at $t^{n+1/2}$ and accelerations and coordinates at t^{n+1} .

Knowing the mass m^n of the node and the force \vec{f}^n acting on it, the acceleration is easily computed by Equation (6):

$$\vec{a}^n = \frac{\vec{f}^n}{m^n} \quad (6)$$

Once the acceleration is computed, the material velocity can be updated by Equation (7):

$$\vec{v}^{n+1/2} = \vec{v}^{n-1/2} + \frac{\vec{a}^n}{2} (\Delta t^n + \Delta t^{n+1}) \quad (7)$$

Subsequently the Lagrangian node coordinate at t^{n+1} is computed by Equation (8):

$$\vec{x}^{n+1} = \vec{x}_r^n + \vec{v}^{n+1/2} \Delta t^{n+1} \quad (9)$$

\vec{x}^{n+1} is the updated Lagrangian node coordinate at t^{n+1} and \vec{x}_r^n is the remapped node coordinate after the advection phase at t^n . Note that, in a pure Lagrangian formulation there is no modification of the node coordinate during the mesh remapping and $\vec{x}_r^n = \vec{x}^n$.

Advection phase

For some problems, the element grid after the Lagrangian phase is rather distorted and a remapping is necessary to prevent a dropping time step size and, eventually, a negative Jacobian of some elements. Working with an Eulerian description of motion, the remapping becomes trivial. It is a special case of the ALE formulation, where the mesh

is moved back to its initial configuration. That is $x_r^{\rightarrow n+1} = x^{\rightarrow 0}$. In the case of the remapping technique presented in this paper a new $x_r^{\rightarrow n+1}$ for a node (i,j) on the mesh surface is determined according to the value of `prtype` in `*ALE_REFERENCE_SYSTEM_GROUP`. If `prtype=10`, the user selects a 3D remapping and if `prtype=11`, the remapping is 2D. The new position $x_r^{\rightarrow n+1}(i,j)$ is interpolated from $x^{\rightarrow n+1}(i,j)$ determined in the previous phase and the Lagrangian positions of its eight neighbors $x^{\rightarrow n+1}(in,jn)$ $i-1 \leq in \leq i+1$ $j-1 \leq jn \leq j+1$ with a biquadratic spline. The ALE remapping involves a mesh motion independent of the material motion. Thus during the advection phase, a hyperbolic or transport problem is solved, for which the variables are density, momentum and internal energy per unit volume. Details of the numerical method used to solve the equations are described in detail in [5], [6], for which first order Donor Cell method and second order Van Leer algorithm [7] are used.

This remapping technique is called by the following keywords:

```
*ale_reference_system_group
$ sid, stype, prtype
   3,      1,      0

*ale_reference_system_group
$ sid, stype, prtype
   4,      1,     10
```

The new `prtype` (10 in 3D or 11 in 2D) is created to distinguish the parts, which the new feature is applied on from the other parts. Thus `pid=4` is the ALE part with the moving boundaries and `pid=3` is the Eulerian part where aluminium is initially set. The following section details the application.

Extrusion problem

The purpose of this part is not to model exactly a real extrusion process but to show the capabilities of the new remapping method in handling surface wave motions and large section variations. Thus a hot process usually produces extruded shapes. In this application the billet, which is the product of the extrusion is directly compressed by a prescribed inflow which represents the hydraulically operated ram. However the inflow velocity is enough large to highlight section variations on Fig.1. This figure shows a funnel-shaped die having a square section of 10mm at the end (`pid=3`). Sliding

conditions are applied on the mesh boundaries of the die. The metal squirts out of this opening in the meshed part (pid=4) of the model as a continuous bar having the same cross-sectional shape as the die opening. The boundaries of the meshed part are free and Fig.1 shows the propagation of surface waves with important amplitudes, which can squeeze the meshes as showed on Fig.2a. Thus a specific smoothing is applied on the boundaries along with the ALE_SMOOTHING for the inner nodes only. The mesh boundary smoothing is called automatically if ALE_SMOOTHING is employed.

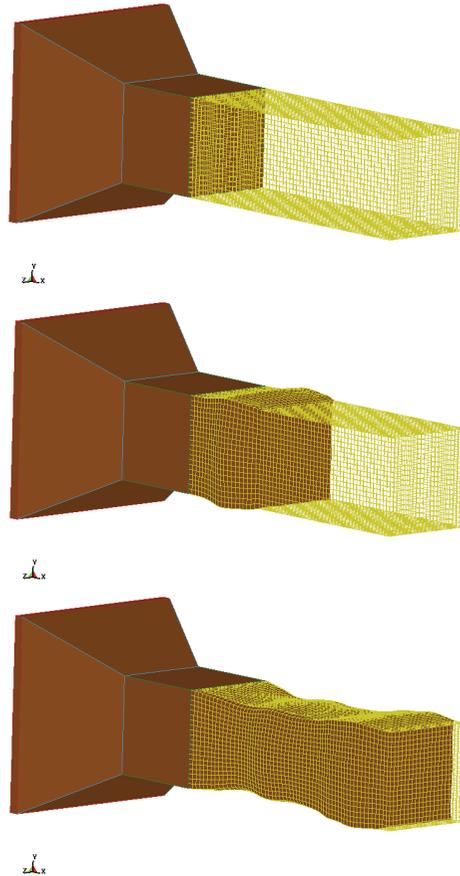


Figure.1 EXTRUSION PROCESS AT DIFFERENT TIMES

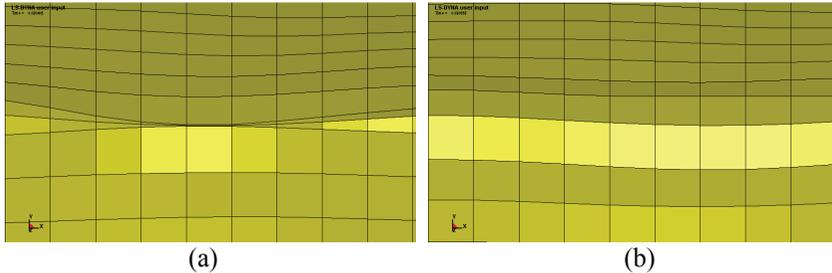


Figure.2 MESH WITH (a) AND WITHOUT (b) A MESH BOUNDARY SMOOTHING

Figures 3 and 4 show the nodal displacement histories on the free surface for different mesh sizes. The plotted displacements are normal to the free surface at its initial position. The different refinement are cubic meshes with a size of 1mm, 0.5mm and 0.25mm. The plots indicate the finer the mesh is the higher the amplitude of the wave is. The displacement histories for 0.5mm and 0.25mm are close each other what reveals a convergence of the numerical solution for the surface wave motion.

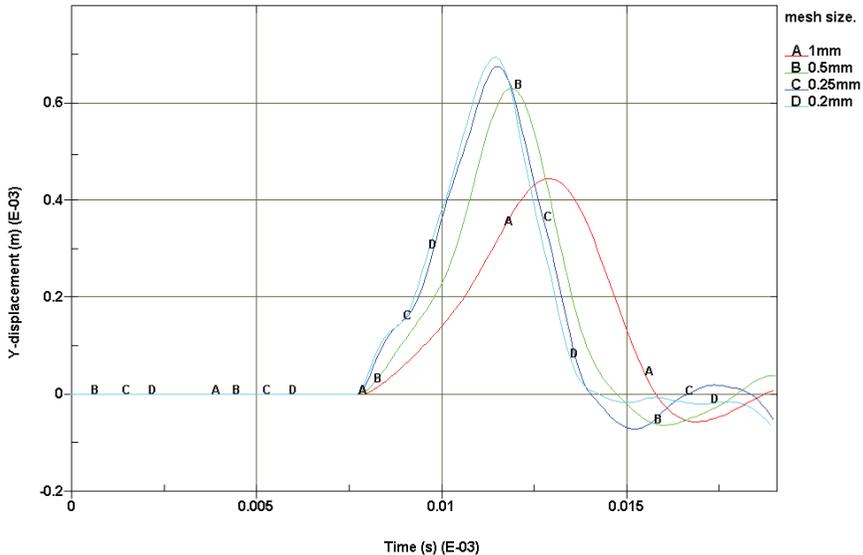


Figure.3 Y-DISPLACEMENT FOR DIFFERENT MESH SIZES

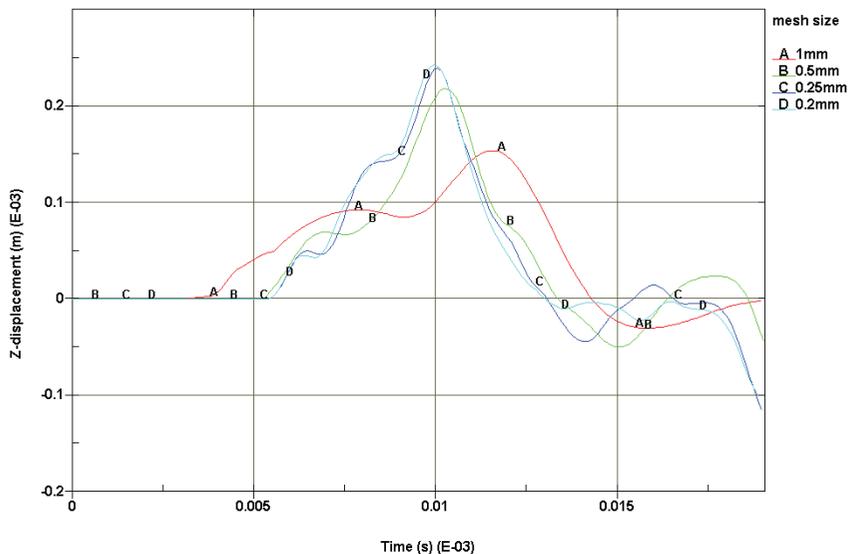


Figure.4 Z-DISPLACEMENT FOR DIFFERENT MESH SIZES

Conclusion

This paper has described an ALE method to solve the surface wave propagation during an extrusion process. The numerical results converge to a solution along with the mesh refinement. The prospective goal of this ongoing research will be to implement the MPP version of this code. The remapping technique along with the ALE smoothing feature can be applied to several industrial problems with large section variations: laminating process, metal rolling process, blood artery modeling...

References

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