Acoustic and Vibroacoustic Modeling in LSDYNA Based on Variational BEM

Authors:

Ahlem Alia, LML, USTL
Mhamed Souli, LSTC

Correspondence:

Laboratoire de Mécanique de Lille, LML, USTL
Bd Paul Langevin Cité Scientifique
Villeneuve d'Ascq 59655, France
Phone: (33)03.20.33.71.76
Fax:(33)03.20.33.71.53
E-mail: ahlem.alia@ed.univ-lille1.fr
Mhamed.souli@univ-lille1.fr

Keywords:
Vibroacoustics, weak coupling, temporal structural analysis, FFT, Variational BEM

ABSTRACT

The paper concerns the vibroacoustic simulation based on the acoustic Variational Indirect Boundary Element Method (VIBEM) recently implemented in LSDYNA. In this formulation, which assumes a weak acoustic-structure interaction, the transient structural response is computed first. By applying the FFT, it is transformed into a frequency response. The obtained results are taken as boundary conditions for the acoustic BEM. Consequently, the radiated noise at any point into space can be calculated. The efficiency of the present method is checked for both pure acoustic and vibroacoustic problems. The obtained results are in agreement with the analytical solutions.

INTRODUCTION

Vibroacoustics consists of the interaction between elastic and acoustic waves. In such interaction, the acoustic pressure exerts a force on the structure whereas the structural motion produces an effective fluid load. Many numerical formulations have been applied to simulate the vibroacoustic interaction [1]. The Finite Element Method (FEM) [1], the Boundary Element Method (BEM) [2], Statistical Energy Analysis (SEA) [1]…can be successfully used for this purpose. FEM and BEM are employed for low frequency modeling whereas SEA is recommended for high frequency range.

In recent years, the BEM is used in many technological fields, especially, in many problems of radiation and diffusion in acoustics. The important advantage of the BEM is the discretization. While in FEM the complete domain has to be discretized, the BEM discretization is restricted only to the boundary. The BEM, which satisfies implicitly the Sommerfeld radiation condition via Green's function [2], addressed the difficulty of the modeling of the open boundary problems by the FEM in which absorbing boundary conditions must be used. In fact, BEM is considered as the best method for the analysis of unbounded problems since they are treated in the same way as interior problems without any additional effort. Unlike the collocation method, the Variational Boundary Element Method (VBEM) leads to symmetric matrices due to the double surface integration. However, both of them yield to full matrices which constitutes the principal drawback of these methods. In addition, VBEM suffers from the singularity problem which occurs when the double integral surface involves the same element. In literature, many solutions have been proposed to overcome this problem.

For many problems, it is not necessary to consider full fluid structure interaction. For instance, in the vehicle box case, many authors have reported fully coupling modelling by assuming uncoupled interaction. In this formulation, the structural response in vacuum is computed first. The obtained results are taken as boundary conditions for the acoustic part of the problem. Hence, the structural vibrations excite the fluid whereas the structure is never influenced by the acoustic waves propagating through the fluid.

Obviously, the uncoupled approach is more simple to be implemented than the coupled one, since two smaller models are to be computed one after the other [3]. However, it is limited for only heavy structures and light fluid (air acoustics). For example, in underwater acoustics, the vibration behavior of the submerged structures is influenced by the surrounded water because of the important value of its normal acoustic impedance [4]. In this case, strong coupling may be taken into account.

In this paper, both Finite and Boundary Element Methods are employed to simulte, in low frequency range, noise radiated in acoustic and vibroacoustic problems. The transient response of mechanical system is computed first using LSDYNA, an explicit finite element code for general structure and fluid-structure interaction problems. The FFT allows its transformation into the frequency domain. In the other hand, a variational BE code, developed for quadrilateral elements in LSDYNA, is based on the velocity information issue from the FFT results. This weak structural acoustic coupling is presented by assuming that the acoustic pressure does not affect the structural vibration [5].

Vibroacoustic problem

In this paper, the interaction of an elastic structure Ω_s with a compressible, isotropic, homogeneous and non-viscid fluid Ω_f is considered [Fig.1]. The interface fluid-structure is represented by S_{sf}.

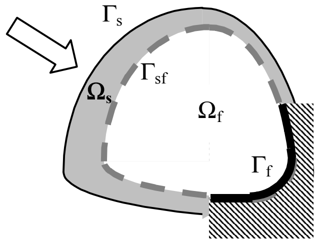


Figure 1. Vibroacoustic problem

Structural analysis

The structure occupies the bounded domain Ω_s and n represents the external normal to the boundary of Ω_s. In the case of elastic, linear and isotropic structure without any initial stress or strain in absence of body forces, the displacement u satisfies the following elasto-dynamic equation:

$$\sigma_{ij,j}(u) - \rho \frac{\partial^2 u_i}{\partial t^2} = 0 \qquad in\ \Omega_s \tag{1}$$

where u_i is the displacement in the i^{th} direction, ρ is the density. $\sigma_{ij}(u)$ is the depended displacement stress tension. For a given displacement u_s on S_f (u_s=0 for rigid surface), we have the following boundary condition:

$$u = u_s \qquad on\ S_f \tag{2}$$

For a prescribed surface force density f on S_s, the corresponding boundary condition can be written as follows:

$$\sigma_{ij}(u) n_j = f_i \qquad on\ S_s \tag{3}$$

Finally, initial conditions can be written as:

$$\begin{cases} u(x,0) = 0 \\ \dfrac{\partial u(x,0)}{\partial t} = 0 \end{cases} \quad in\ \Omega_s \tag{4}$$

In almost all studies, the structure simulations have been done using FE models. The detailed description of the FE algorithm of LS-DYNA is not the aim of this paper.

Generally, the discretization of the variational form involving any mechanical system response using the FEM reduces into the following equation [1]:

$$[K]u + [M]\frac{d^2u}{dt^2} = \{F\} \tag{5}$$

where t designs the time, [M] and [K] represent, respectively, the mass and the stiffness matrices of the structure and {F} is the mechanical load vector. The centred second order scheme in time of LSDYNA yields to the nodal displacement vector of the vibrating structure. At each time step, the nodal normal velocity at the interface fluid-structure is curried out:

$$V_n(x,t) = \vec{V}.\vec{n} = \frac{\partial \vec{u}(x,t)}{\partial t}.\vec{n} \tag{6}$$

The FFT is applied to the normal velocity of the elements which constitutes the boundary condition of the BEM.

Acoustic analysis

The BEM is used to evaluate the pressure response in the acoustic domain from the structure velocity results deduced from equation (6). Consider a boundary surface S enclosing a volume Ω_f filled and surrounded by an ideal and homogeneous fluid medium [Fig.2].

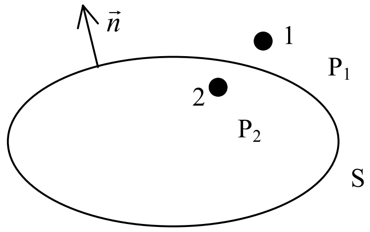


Figure 2. Acoustic domain and its boundary

For an harmonic disturbance of frequency f in an ideal fluid medium, the Helmholtz equation is given by [2]:

$$\Delta p + k^2 p = 0 \,, \tag{7}$$

By using Green's theorem, the corresponding integral equation can be written as [2]:

$$C(r)p(r) = \int_{S_y} \left(g(r,r_y)\frac{\partial p(r)}{\partial n_y} - p\frac{\partial G(r,r_y)}{\partial n_y} \right) dS_y \tag{8}$$

This equation allows the calculation of sound pressure at any point of the acoustic domain. In equations (7) and (8), "k=ω/c" denotes the wave number, c is the sound velocity, "ω=2πf" is the pulsation, p(r) is the pressure at any field point "r", $G(r,r_y) = e^{-ik|r-r_y|} / 4\pi|r-r_y|$ is the Green's function, "ry" is the position vector of a source point located at acoustic domain boundary and C is the jump term resulting from the treatment of the singular integral involving Green's function.The Indirect Boundary Element Method (IBEM) defines the primary variables as the jump in the pressure ($\mu = p_1 - p_2$) and the jump in the normal gradient of the pressure ($\sigma = \partial p(r_{y1})/\partial n_y - \partial p(r_{y2})/\partial n_y = i\omega\rho(V_n(r_{y1}) - V_n(r_{y2}))$,) between the two sides of the boundary element model. For Neumann problem, in which only velocity is prescribed on the acoustic boundary, the following integral equation is obtained:

$$p(r) = \int_{S_y} \frac{\partial G(r, r_y)}{\partial n_y} \mu(r_y) dS_y \tag{9}$$

On the surface of the acoustic boundary, the pressure is related to the structural velocity by:

$$\nabla p = -i\omega\rho v \tag{10}$$

Hence, equation (9) can be written as following:

$$-i\omega\rho V_n(r_x) = \int_{S_y} \frac{\partial^2 G(r, r_y)}{\partial n_x \partial n_y} \mu(r_y) dS_y \tag{11}$$

Equation (11) can be solved using the variational principle to the integral equation. In fact, it permits to reduce the hypersingular integrals to a less singular form. In addition, the variational indirect boundary element method yields to symmetric fully populated matrices.By using the variational method, the last equation can be rewritten as:

$$-i\omega\rho \int_{S_x} V_n \mu(r_x) dS_x = \int_{S_x}\int_{S_y} \frac{\partial^2 G(r, r_y)}{\partial n_x \partial n_y} \mu(r_x)\mu(r_y) dS_y dS_x \tag{12}$$

where μ(x) represents the test function of the variational method. The solution of the problem can be obtained by minimising the following functional F of the Variational Indirect Boundary Element Method (VIBEM):

$$F(\mu) = 2\int_{S_x} j\rho\omega\mu(x) V_n(x) dS_x + \int_{S_x} \left(\mu(x) \int_{S_y} \left(\frac{\partial^2 G(x, y)}{\partial n_x \partial n_y} \mu(y) \right) dS_y \right) dS_x \tag{13}$$

The discretized form of the functional F noted F^h may then be written as:

$$F^h = \sum_i \sum_j \mu_i a_{ij} \mu_j - 2\sum_i \mu_i b_i, \Rightarrow F^h = \mu^T A\mu - 2\mu^T B \tag{14}$$

F^h is a bilinear function of the unknown nodal potentials. Imposing stationary condition on F^h with respect to unknown primary variables μ, leads to the following system of equations:

$$\frac{\partial F^h}{\partial \mu} = 0 \Rightarrow A\mu = B \tag{15}$$

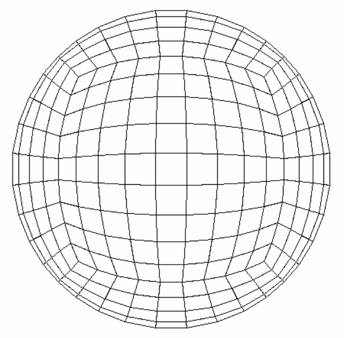
Form equation (12), the double potential layer is calculated. Finally, the pressure at any point of the field can be computed via equation (9).

Numerical results

Since the analysis will be carried out using the FE element code of LSDYNA, verification of the structural response is beyond the scope of this paper. However, the accuracy of the BEM should be checked. To that end, the radiated pressure is calculated using the presented BEM analysis for simple acoustic problems (radiation from a pulsating sphere and propagation inside a rigid parallelepiped box) without any coupling with a structure in order to validate the VBE code. In a second step, a linear problem consisting of simulation of a forced vibroacoustic response of a structure-cavity system is treated. This problem has been largely studied analytically [6] and numerically [7].

Pulsating sphere

To demonstrate the accuracy and the efficiency of the present approach, radiation of a unit pulsating sphere is analysed [Fig. 3.a]. The example of the sphere is selected because analytical solution is available. The sphere has a radius of "$a = 1\,m$". It's excited by a unit velocity at frequency $f = 100Hz$. It's surrounded by air ($\rho = 1.21\,Kg/m^3$, $c = 343\,m/s$). Figure. 3.b depicts, the variation of the radiated pressure with the radial distance computed analytically and by using VBEM. A good agreement is observed.



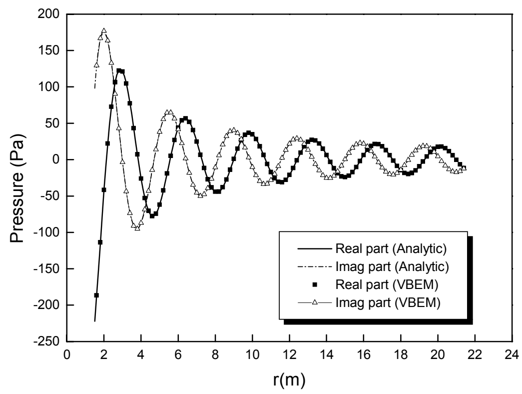


Figure 3. Pulsating sphere
(a) Sphere Be model (486 quadrilaterals), (b) radiated pressure

For external acoustic problems, the BEM involves at the eigen-frequencies of the associated internal problem, ill-conditioned system and therefore non uniqueness solution. This problem known as of irregular frequencies occurs because of the system rank deficiency. From figure. 4, many peaks occur which coincide with the eigen-frequencies of the inner volume of the sphere. If we refine the mesh, the solution still not converge and we will get another wrong solution. These peaks do not have any physical meaning at the considered frequencies. To overcome this problem, it is necessary to descritise some additional elements inside the sphere on which the impedance boundary condition is prescribed in order to eliminate the highly resonant interior effects from the solution [Wu, Von].

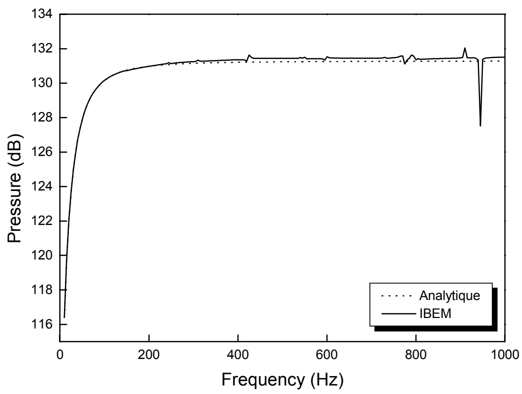


Figure 4. Sound pressure radiated by pulsating sphere (r=4m)

Parallelepiped box

A parallelepiped box of $(1 \times 0.2 \times 0.2)$ m^3 containing air was considered as a test example for the interior problem. The VBEM was made of 160 linear elements and 162 nodes [Fig. 5.b]. It was assumed that an end surface at x=0 was vibrating as a rigid piston with an harmonic amplitude velocity of 1m/s whereas all other walls were rigid. Figure. 5.b shows the calculated field pressure on the longitudinal axis when the excitation frequency f=100Hz. Similar to the previous exterior problem, VBEM result coincides with the exact theory.

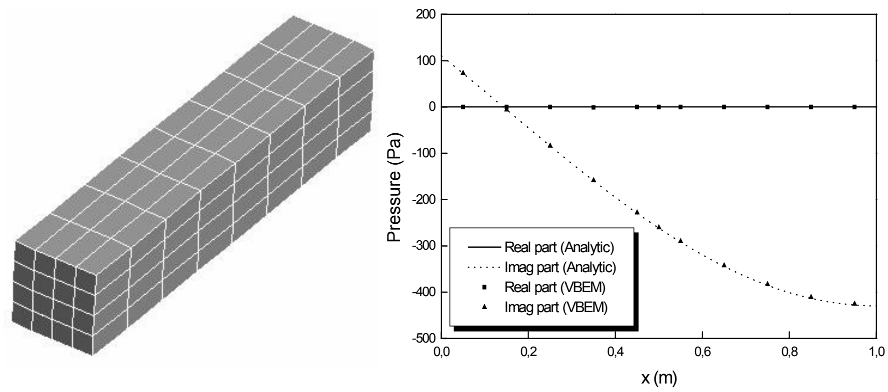


Figure 5. Parallelepiped box: (a) Box Be model, (b) radiated pressure inside box

In the last examples, velocity (V=1 m/s) has been considered in the frequency domain. However, in order to check the used FFT, we imposed V=sin(ωt) and converted it from the temporal domain into the frequency one. It is obvious that using the FFT for a signal containing multiple number of periods leads to very accurate results. However, when a non periodic sinusoidal velocity signal is applied (signal duration =100.75 periods, for example (Table1)), a relative error in Pa of 25 % occurs (corresponding to a difference of about 3 dB). To reduce this difference, we have used the Hanning window which imposes the signal periodicity. In this case, the relative error in Pa decreases to 10 % (i.e. a difference of about 1 dB).

Position (m)	Analytic	Rectangular window	Hanning window
0.05	128.03	125.57	127.05
0.15	106.39	103.94	105.42
0.25	129.26	126.80	128.28
0.35	134.80	132.35	133.82
0.45	137.96	135.51	136.98
0.50	139.10	136.65	138.12
0.55	140.04	137.59	139.06
0.65	141.48	139.04	140.50
0.75	142.46	140.01	141.48
0.85	143.07	140.62	142.09
0.95	143.36	140.91	142.38

Table 1. Pressure (dB) obtained by considering the FFT of non periodic temporal velocity

Structure-cavity system

Consider a rigid cavity ($0.2 \times 0.2 \times 0.2$) m^3 with one simply supported flexible plate (0.2×0.2) m^2 made of brass with the following properties: Young modulus E=103 GPa, ρ_s=8500 kg/m^3 , Poisson ratio ν=0.34 and thickness t= 0.9144mm [Fig. 6.a]. The cavity contains air of density ρ_f =1.21 kg/m^3 and sound velocity c=343m/s. The plate is subjected to a uniformly distributed harmonic pressure load (1kPa).

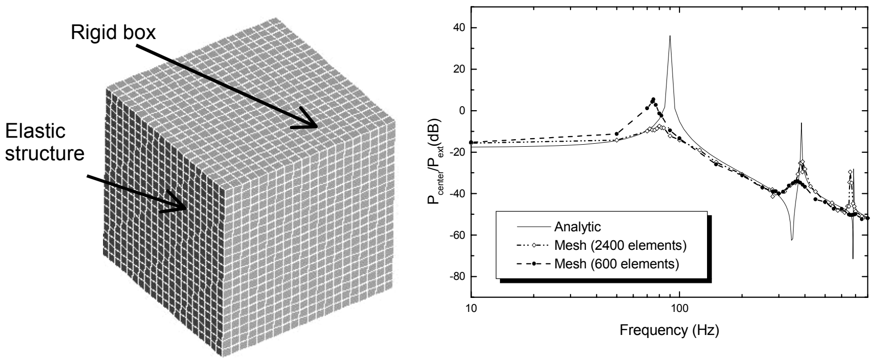


Figure 6. structure-cavity system
(a) FE-BE model (mesh 2), (b) Radiated pressure at the box centre

In figure. 6.b, the predicted pressure at the box centre (0.1,0.1,0.1)m for two different meshes (mesh1(600 elements) and mesh2 (2400 elements)) is compared to the analytical solution given by Guy et al [6]. It shows that the plate-cavity resonant frequencies, calculated analytically, are 90, 390 and 680 Hz. They correspond to the first three plate natural frequencies.

Off resonance, the predicted pressure is in good agreement with analytical solutions. In addition, the numerical resonant frequencies correspond well to the analytical ones. However, at the resonance, a difference in amplitude can be observed between numerical and analytical solutions. This can be explained by the fact that the FFT reduces the velocity amplitude and consequently it affects the BEM estimated pressure which is very sensitive to any change in the velocity magnitude.

Summary and Conclusions

In this paper, the acoustic pressure has been calculated by using the structural velocity, obtained from LSDYNA, as boundary condition for BEM. The FFT allows to transform these temporal velocities from the temporal domain into the frequency domain. In order to preserve the periodicity of the velocity signals, Hanning window is employed instead of the rectangular one. The BE code has been checked for a pulsating sphere and parallelepiped box. In addition, the presented structure-acoustic interaction simulation has been verified for a classical vibroacoustic problem. ompared to the analytical results, the presented method gave good results.

We have seen that the VBEM as implemented in LSDYNA, is not applicable to the external problems for some irregular frequencies. Special numerical treatment is to be developed in order to overcome the non uniqueness solution problem.

It is to be emphasise that the method presented in this paper still limited only for gaseous fluids. For example, in underwater acoustics, the vibration behaviour of the submerged structures is influenced by the surrounded water because of the important value of its normal acoustic impedance. In this case, strong coupling must be taken into account.

Although VBEM model has fewer elements than the FEM one, it is computationally very expensive because it involves full matrices in which the components are computed from double surface integration. To become more efficient, the code can be improved by adopting any acceleration algorithm for the double surface integration [8] and solving the linear system by iterative solvers.

References

1. Ohayon R. and Soize C., 1998, "Structural acoustics and vibration", Academic press, London.

2. Wu T.W., 2000, " Boundary element acoustics: Fundamentals and computer codes", Witpress (ed.); Southampton: Boston.

3. Marburg S., 2002, " Developments in structural-acoustic optimization for passive noise control", Archives of Computational Methods in Engineering, **9** (4), pp-291-370.

4. Estorff O. Von, 2000, "Boundary elements in acoustics: Advances and Applications", Witpress (ed.); Southampton: Boston.

5. Kim N.H., Choi K.K., Dong J. and Pierre C., 2003, "Design optimisation of structural acoustic problems using FEM-BEM". *AIAA*, pp. 1630.

6. Guy, R.W, Bhattacharya, M.C. 1973. The transmission of sound through a cavity backed finite plate. *Journal of sound and vibrations,* **27**, pp-207-223.

7. Lee C. M., Royster L. H. and Ciskowski R. D., 1995," Formulation for an FE and BE coupled problem and its application to the earmuff-earcanal system". Engineering Analysis with Boundary Elements, **16**, pp-305-315.

8. Tournour M., 1999, "Modélisation numérique par élément finis et éléments finis de frontière du comportement vibroacoustique de structures complexes assemblées et couplée à une cavité", Ph.D. thesis, Université de technologie de Compiègne, France.