An Assessment of the LS-DYNA Hourglass Formulations via the 3D Patch Test

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Abstract

The six hourglass formulations available in LS-DYNA for 8 node hexahedral elements are evaluated using the so called '3D Patch Test.'. It is demonstrated that three of the six hourglass formulations fail this patch test, including the popular default LS-DYNA viscous form of hourglass control. A detailed description of the 3D Patch Test is provided to allow readers to perform the simple test as part of their code verification.

Introduction

Single point integration 8-node hexahedral solid elements, with hourglass stabilization, are probably the most used element in the LS-DYNA (Hallquist, 2003) element library. This element also has the longest lived history in LS-DYNA reaching back to its roots in DYNA3D. During this long history, little has changed in the element's isoparametric element formulation, but considerable attention has been given to the various hourglass stabilization techniques that can be used in conjunction with the element.

During a recent Defense Threat Reduction Agency sponsored code verification exercise, one of the suggested verification problems was the so called '3D Patch Test', or 'MacNeal & Harder Patch Test,' as described by MacNeal & Harder (1985). Various patch tests have been suggested for all forms of elements, even meshfree formulations. The basic idea is to verify an element's ability to represent a constant strain/stress field, and thus ensure completeness and an ability to converge in the limit as the element size decreases.

When this patch test was attempted with LS-DYNA, using the default hourglass control, it was discovered that this most used combination of single point integration and viscous hourglass stabilization failed the patch test. While this is not a new discovery to those steeped in hourglass theory and development, it may be a quite a surprise for new, and some seasoned, LS-DYNA users.

The present work demonstrates which of the six applicable LS-DYNA hourglass formulations pass the 3D Patch Test, and provides sufficient details of the patch test to allow readers to repeat the test. A secondary purpose of this effort to underscore an emphasis on verification assessment. The burden of assuring that a given algorithm performs correctly is too often passed onto the software developer by the user community. This patch test demonstration can be viewed as an illustration of the algorithms working 'correctly,' i.e. as designed, but the user being unaware of the design and thus possibly surprised by the verification result.

Problem Description

The geometry for the 3D Patch Test is illustrated in Figure 1. A solid in the form of a regular 1x1x1 cube on the outside is modeled by seven irregular hexahedral elements. The eight exterior nodes are given a prescribed linear displacement. The interior nodes must deform in such a way that all of the elements have identical stress states to pass the patch test. That is, a 'patch' of irregular elements must be able to reproduce a uniform strain/stress state.

Units

English units (lbf-inch-sec) are used, in deference to the units used in the source MacNeal & Harder reference.

Geometry

The cube is 1"x1"x1". Specification of the node numbering for the elements is given in Figure 2 and Table 1. Initial coordinates of all of the interior and exterior nodes are given in Table 2.

Material Properties

The material is linear elastic, with $E = 106$ psi, $\nu = 0.25$, and $\rho = 2.61\times10^{-4}$ lbf-sec^2/in^4. The Young's modulus and Poisson's ratio are the values used in the MacNeal & Harder reference. Since this is intended to be a static problem, the density is arbitrary, and was chosen mainly for convenience.

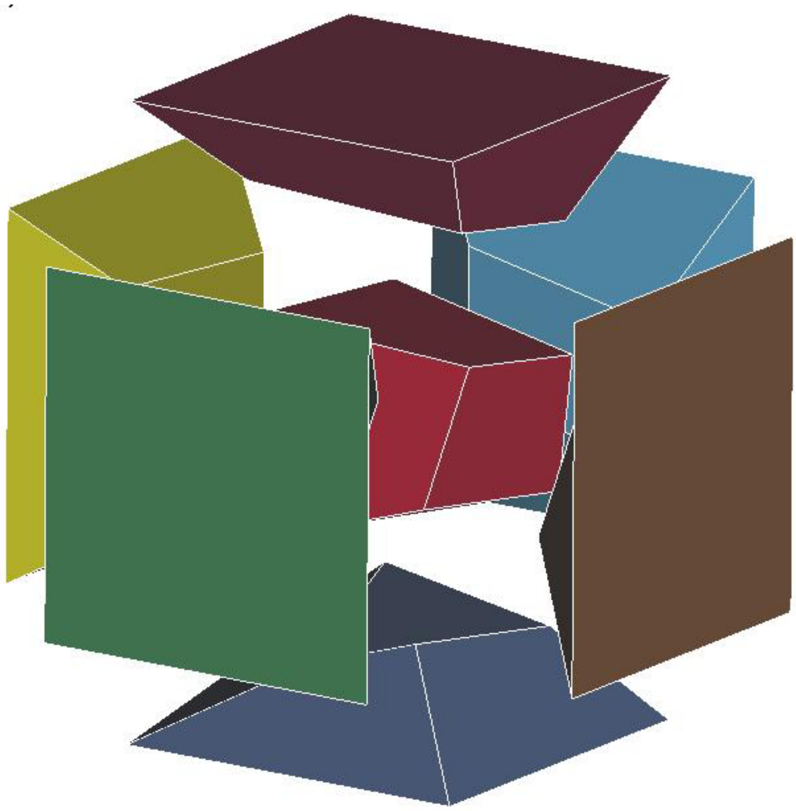


Figure 1 Exploded view of seven irregular elements comprising the 3D Patch Test geometry.

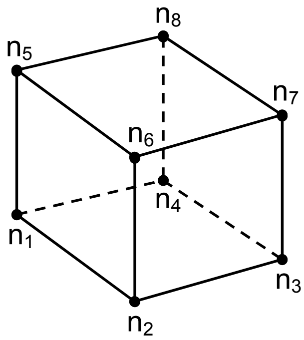


Figure 2 Numbering convention for elements.

Table 1 Element node numbering.

Element	Nodes								Comment
	n_1	**n_2**	**n_3**	**n_4**	**n_5**	**n_6**	**n_7**	**n_8**	
1	1	2	3	4	5	6	7	8	Interior
2	9	10	11	12	1	2	3	4	Back
3	5	6	7	8	13	14	15	16	Front
4	12	16	13	9	4	8	5	1	Left
5	3	7	6	2	11	15	14	10	Right
6	4	8	7	3	12	16	15	11	Top
7	9	13	14	10	1	5	6	2	Bottom

Loading

The loading consists of prescribed displacements applied to the exterior nodes. The problem is intended to be static, and the displacements are applied gradually, up to specified steady-state values, which are given in Table 2. The temporal dependence of the applied displacements should be as follows:

$$u_{n\alpha}(t) = \bar{u}_{n\alpha}\, f(t) \tag{1}$$

where $u_{n\alpha}(t)$ is the displacement of node *n* in direction α at time *t*, $\bar{u}_{n\alpha}$ is the final, steady-state displacement, and *f(t)* is a load curve given by

$$f(t) = \begin{cases} 0.5\,\left(1-\cos\left(\pi t/t_r\right)\right) & 0 \le t \le t_r \\ 1 & t > t_r \end{cases} \tag{2}$$

where $t_r = 1$ ms is the rise time. The problem should be run to a simulation time of 1.2 ms, which is sufficient to minimize dynamics and provide essentially static results.

Table 2 Nodal point coordinates and steady-state displacements

node	type	coordinates (in)			steady-state displacement (10^{-3} in)		
		x	*y*	*z*	$\overline{u}_x$	$\overline{u}_y$	$\overline{u}_z$
1	Int	0.249	0.342	0.192	–	–	–
2	Int	0.826	0.288	0.288	–	–	–
3	Int	0.850	0.649	0.263	–	–	–
4	Int	0.273	0.750	0.230	–	–	–
5	Int	0.320	0.186	0.643	–	–	–
6	Int	0.677	0.305	0.683	–	–	–
7	Int	0.788	0.693	0.644	–	–	–
8	Int	0.165	0.745	0.702	–	–	–
9	Ext	0.0	0.0	0.0	0.0	0.0	0.0
10	Ext	1.0	0.0	0.0	1.0	0.5	0.5
11	Ext	1.0	1.0	0.0	1.5	1.5	1.0
12	Ext	0.0	1.0	0.0	0.5	1.0	0.5
13	Ext	0.0	0.0	1.0	0.5	0.5	1.0
14	Ext	1.0	0.0	1.0	1.5	1.0	1.5
15	Ext	1.0	1.0	1.0	2.0	2.0	2.0
16	Ext	0.0	1.0	1.0	1.0	1.5	1.5

Results

Analytical Solution

The analytical solution is obtained by substituting the displacement boundary conditions

$$\begin{aligned} u &= 0.5(2x+y+z)10^{-3} \\ v &= 0.5(x+2y+z)10^{-3} \\ w &= 0.5(x+y+2z)10^{-3} \end{aligned} \tag{3}$$

into the strain-displacement relations to obtain

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 10^{-3} \tag{4}$$

and then using Hooke's Law to obtain the corresponding stresses

$$\begin{aligned} \sigma_x &= \sigma_y = \sigma_z = 2000 \text{ psi} \\ \tau_{xy} &= \tau_{yz} = \tau_{zx} = 400 \text{ psi} \end{aligned} \tag{5}$$

As a demonstration of the LS-DYNA 8-node hexahedral solid element's ability to produce the analytical solution, the baseline case uses the fully integrated selected/reduced solid element formulation, i.e. ELFORM=2. This fully integrated formulation does not require hourglass control as there are no spurious energy modes. Figure 3 shows the x-component stress history for all seven elements in the patch test unit cube; it is left to the reader to confirm that all the other stress components also agree with the analytical solution.

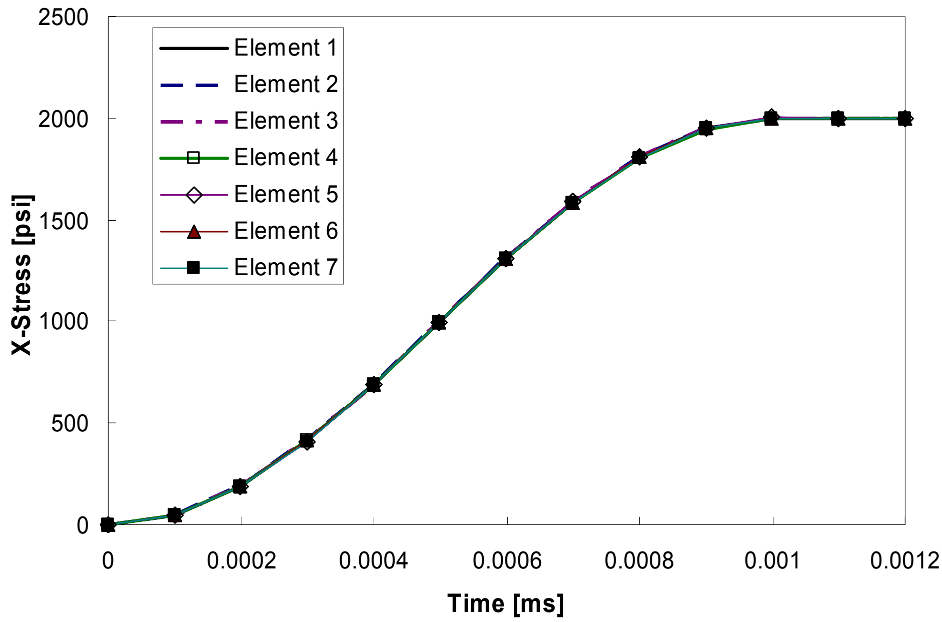


Figure 3 Demonstration of analytical solution obtained for x-stress component using fully integrated S/R (ELFORM=2) 8-node hexahedral solid element.

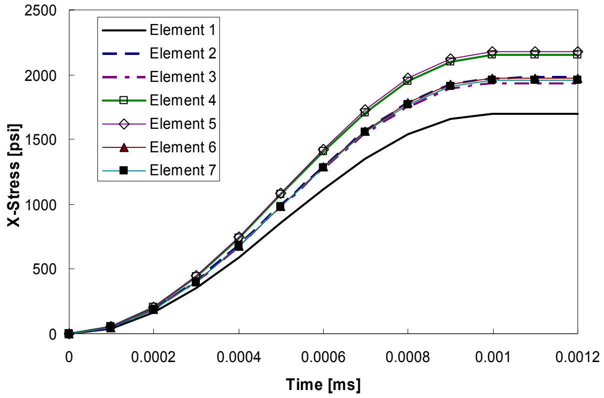
Hourglass Control Solutions

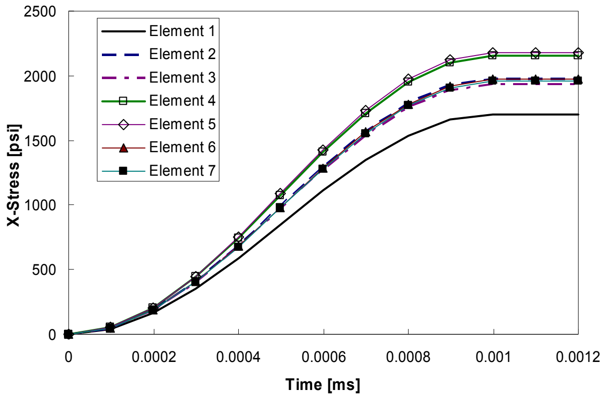
LS-DYNA has six[1] forms of hourglass control applicable to solid elements as summarized in Table 3. The parameter IHQ, of the Keyword *Hourglass, determines the hourglass type. By default, if no value of IHQ is specified by the user, the value IHQ=1 is assigned. Citations to the work of Flanagan & Belytschko (1981) and Belytschko & Bindeman (1993) are provided in the Reference section.

Table 3 Summary of LS-DYNA hourglass forms for solid elements.

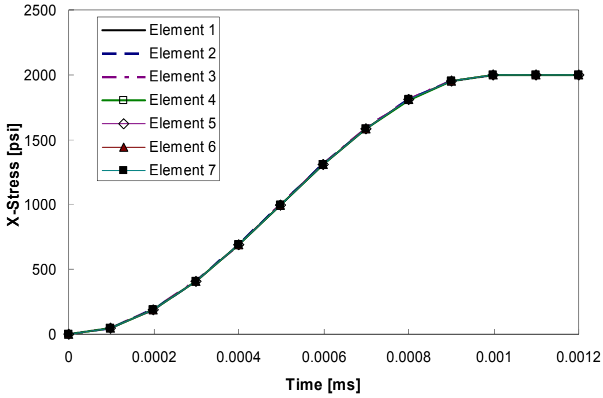
IHQ	Description
1	Standard LS-DYNA Viscous Form.
2	Flanagan-Belytschko Viscous Form.
3	Flanagan-Belytschko Viscous Form with Exact Volume Integration.
4	Flanagan-Belytschko Stiffness Form.
5	Flanagan-Belytschko Stiffness Form with Exact Volume Integration.
6	Belytschko-Bindeman Assumed Strain Co-Rotational Stiffness Form

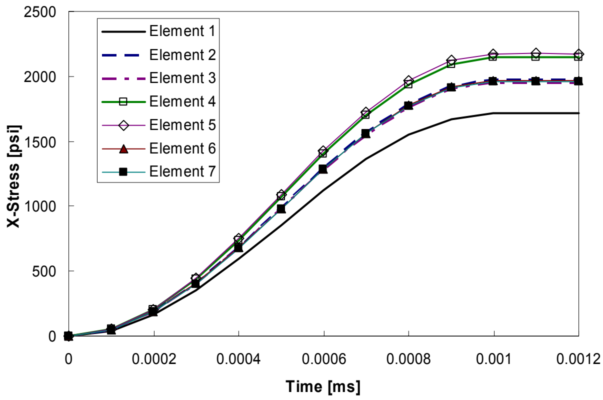
[1] LS-DYNA Version 971 adds a seventh hour glass form and is briefly described in the appendix.



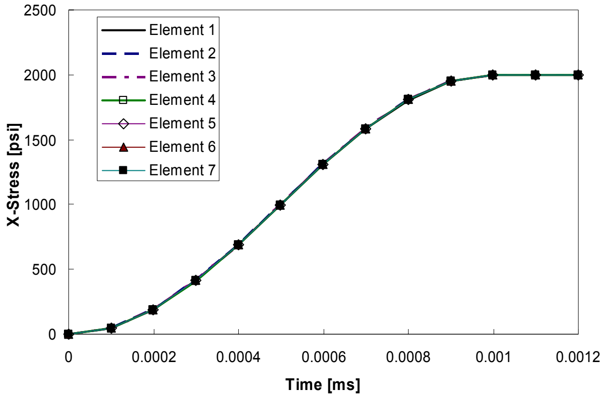


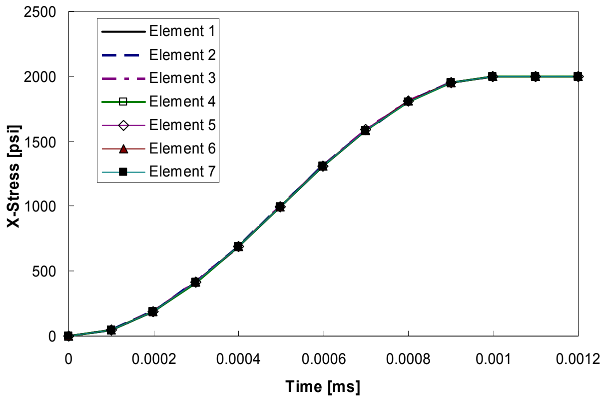
Standard LS-DYNA Viscous Form (IHQ=1) Flanagan-Belytschko Viscous Form (IHQ=2)





F-B Viscous Form Exact Volume Integration (IHQ=3) Flanagan-Belytschko Stiffness Form (IHQ=4)





F-B Stiffness Form Exact Volume Integration (IHQ=5) B-B Co-Rotational Stiffness Form (IHQ=6)

Figure 4 Summary of x-component stress histories for six forms of hourglass control.

Figure 4 shows the x-component stress history of all seven elements in the patch test unit cube for the applicable six forms of hourglass control in LS-DYNA. Note: default values of the hourglass coefficient (QM) were used for each of the six hourglass forms. The results indicate that only three of the six forms of hourglass control reproduce the analytical stress results, i.e. pass the hourglass patch test. The three forms that pass the hourglass patch test all share the common feature of providing for the 'exact volume integration' in their formulations.

Elaborating on the work of Flanagan & Belytschko (1981), Key (2003) provides that the difference between the 'exact volume integration' approach and 'one-point quadrature' is that the latter method effectively neglects terms in the gradient operator. For a parallelepiped, the relative nodal coordinates, used in the gradient operator, contain no component of the hourglass base vectors, and consequently, only one term is non-zero, by inspection, in evaluating the gradient operator, and volume. In such a case, one-point quadrature is equivalent to the mean quadrature. However, for a general hexahedron, as in the present 3D patch test, one-point quadrature does not correctly assess a state of uniform strain.

There are two components to developing a constant strain/stress 8-node hexahedral element that passes the 3D patch test:

1. The method by which the gradient/divergence operator is calculated, i.e. integration method.
2. The care with which the ad-hoc hourglass resisting forces are computed in response to the development of non-constant deformation.

If either component is not calculated carefully, the resulting integration-hourglass formulation will fail the 3D patch test.

The two integration methods have the following characteristics:

- A constant strain/stress gradient/divergence operator obtained by a one-point quadrature, at the center of the element, fails the 3D patch test with zero hourglass control.
- A constant strain/stress gradient/divergence operator obtained by an exact volume integration passes the 3D patch test with zero hourglass control.

When hourglass control is non-zero, even for a exact volume integration 8-node hexahedron (that otherwise passes an 3D patch test), a failed 3D path test can result if the hourglass control is not carefully crafted to act exclusively on the non-constant deformation. That is the hourglass control must produce resisting forces that are orthogonal to the non-constant deformations.

Perhaps unfortunately, the LS-DYNA input parameter for hourglass control combines both the integration method and the hourglass formulation. Table 4 present the LS-DYNA hourglass types in a manner that indicates the integration method and hourglass formulation. The hourglass types listed in **bold** pass the 3D patch test.

Table 4 LS-DYNA hourglass types identified by integration method and hourglass formulation.

Hourglass Formulation	Integration Method	
	One-Point	Exact Volume
Standard LS-DYNA Viscous (Default)	IHQ=1	n/a
Flanagan-Belytschko Viscous	IHQ=2	**IHQ=3**
Flanagan-Belytschko Stiffness	IHQ=4	**IHQ=5**
Belytschko-Bindeman Assumed Strain Co-Rotational Stiffness	n/a	**IHQ=6**

Conclusions

As demonstrated in the present work, LS-DYNA users should consider using hourglass control forms other than the LS-DYNA default form. While most mesh generators provide initial solid element meshes with nearly regular hexahedra (parallelepipeds) and the default hourglass control will perform well, i.e. accurately represent constant strain fields, with these initial meshes. However, once the mesh begins to deform, and especially severe deformations typical in LS-DYNA applications, the resulting irregular hexahedral meshes, coupled with the default LS-DYNA hourglass control, do not provide an accurate description of constant strain fields.

Acknowledgements

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Appendix

Lee Bindeman of LSTC has added a seventh form of hourglass control that is available in LS-DYNA Version 971. The new hourglass form is a variation on Type 6 and has worked well with tire models. It is a completely linear hourglass control that uses a total measure of hourglass deformation rather than the incremental measure used in the other forms. The advantage of the new hourglass form is that it can go through millions of cycles with no growth in error. With the incremental forms, the nonlinearity of the hourglass control can cause tire models to become ‘bumpy’ over time due to permanent hourglass offset after the load is removed. Type 6 hourglass control is most prone to this problem because it is the most sensitive to changes in element dimensions, and therefore the most nonlinear. The new linear form, Type 7, always springs back to the original element shape, but it is more CPU intensive than the Type 6 hourglass control. Hourglass control Type 7 passes the 3D patch test.