A FAILURE CRITERION
FOR
POLYMERS AND SOFT BIOLOGICAL MATERIALS

Authors:

William W. Feng
John O. Hallquist
Livermore Software Technology Corp.
7374 Las Positas Road
Livermore, CA 94550
USA

Correspondence:

925-449-2500

wwfeng@sbcglobal.net
John@lstc.com

Keywords:
failure criterion, strain invariants, Cauchy strains, membrane tests, LS-DYNA

Abstract

A failure criterion, for polymers and soft biological materials subjected to very large deformation, is presented in this paper. The criterion is written in terms of the strain invariants in finite elasticity. Experimental tests for determining the failure criterion of a material and some numerical results from LS-DYNA are shown.

Introduction

A failure criterion for composite materials, based on finite elasticity, was published by Feng [1]. The failure criterion proved to be simple to use and accurate when compared with the experimental data. In this article, the failure criterion has been extended for polymers and soft biological materials. The criterion is written in 2terms of the strain invariants in finite elasticity given by Green and Adkins [2]. These invariants are written as functions of the Cauchy strains and, in terms, the deformation gradients. The failure criterion offers a relatively simple form. It can be applied to a wide variety of materials subjected to very large deformation. It contains three failure constants for general three-dimensional strain states. These constants can be determined from experimental data. Experimental tests and apparatus for obtaining these constants are mentioned in this paper. The failure criterion is being implemented in LS-DYNA now.

Formulation

A material particle initially at $P(X_\alpha)$, as shown in Figure 1, in the undeformed rectangular Cartesian coordinate system X_α, is moved to $p(x_i)$, in the deformed rectangular Cartesian coordinate system x_i. The indicial notation is used throughout the paper. A neighboring point $Q(X_\alpha + \Delta X_\alpha)$ in the undeformed configuration is moved to $q(x_i + \Delta x_i)$. The unknown function, x_i, is described by

$$x_i = x_i(X_\alpha)$$
(1)

For isotropic elastic continuum, there are three invariants of the strain tensor. Choosing the Cauchy strain tensor ($C_{\alpha\beta}$) as the strain measure, the invariants are:

$$\begin{aligned} I_1 &= C_{\alpha\alpha} \\ I_2 &= \tfrac{1}{2}\left[(C_{\alpha\alpha})^2 - C_{\alpha\beta}C_{\alpha\beta}\right] \\ I_3 &= \det\left(C_{\alpha\beta}\right) \end{aligned}$$
(2)

The Cauchy strain tensor is defined by

$$C_{\alpha\beta} = x_{r,\alpha} x_{r,\beta} \tag{3}$$

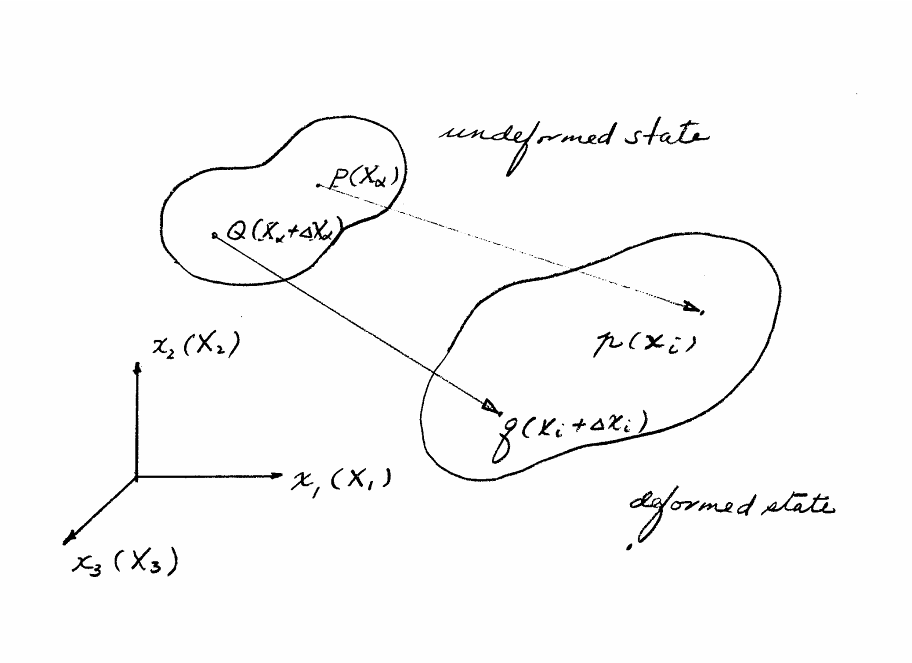

Figure 1. Large deformations of a continuum

The failure surface is based on the energy principle. When the strain energy reaches a maximum, the material fails. The most general failure criterion in terms of the strain invariants is then

$$F(I_1, I_2, I_3) = 0$$
(4)

For incompressible materials, I_3 is one; therefore, the most general failure criterion for incompressible materials is

$$F(I_1, I_2) = 0$$
(5)

If we expand this failure criterion and retain only those terms up to the quadratic terms of strain state $C_{\alpha\beta}$, the failure criterion for hyperelastic solids is

$$\Phi = F_1(I_1 - 3) + F_{11}(I_1 - 3)^2 + F_2(I_2 - 3) + \alpha$$
(6)

Any strain state satisfying $\Phi < 0$ is considered as below the failure state. A stress-free state should be below the failure stress state; thus $\alpha < 0$ is required. Without losing generality, let us say . The failure surface is described by the strain state in which $\Phi = 0$, i.e.,

$$F_1(I_1-3)+F_{11}(I_1-3)^2+F_2(I_2-3)-1=0$$
(7)

or

$$(I_1-3)+\Gamma_1(I_1-3)^2+\Gamma_2(I_2-3)-\mathrm{K}=0$$
(8)

The strain invariants can also be written in terms of the three principal stretch ratios, λ_1, λ_2 and λ_3.

$$I_1=\lambda_1^2+\lambda_2^2+\lambda_3^2$$

(9)

$$I_2=\lambda_1^2\lambda_2^2+\lambda_1^2\lambda_3^2+\lambda_2^2\lambda_3^2$$

There are three material failure constants Γ_1, Γ_2 and K in equation (8). They are to be determined from test data for each material. Hence, the minimum number of tests required for determining the failure criterion is three.

The failure surfaces for hyperelastic solids, based on this failure criterion, are shown in Figure 2. In the Figure the values of Γ_1, and Γ_2 are 0.0 and 0.02 respectively. Three values of K = 10, 20 and 30, are shown in the Figure. There are two lines. The homogeneous biaxial stretching state is indicated by the line

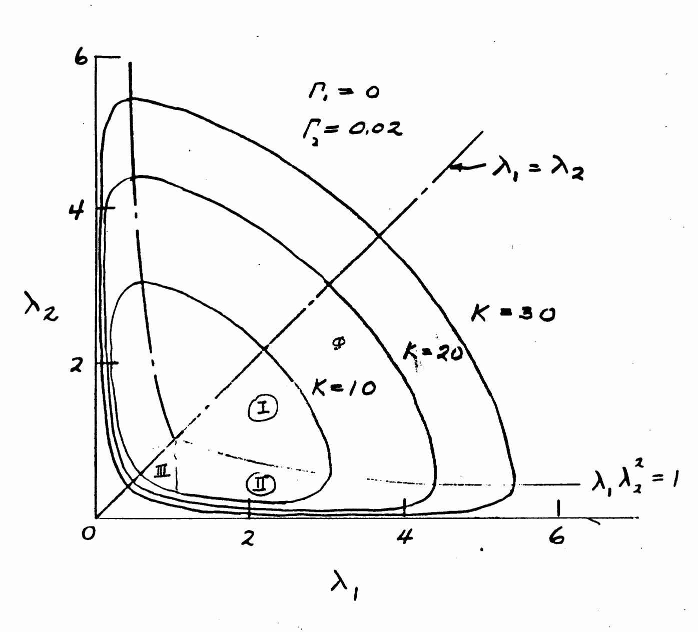

Figure 2. Failure surface for polymer

$\lambda_1=\lambda_2$. The failure surface is symmetric to this line. The uniaxial stretching is shown by the line $\lambda_1\lambda_2^2=1$. There are three regions bounded by these lines and the failure surface: the tension-tension region (I), the tension-compression region (II) and the compression-compression region (III)

Experimental tests for determining Γ_1, Γ_2 and K

The tests for determining Γ_1, Γ_2 and K can be uniaxial stretching tests, biaxial stretching tests with various stretch ratios, inflation of a circular membrane, inflation of a circular membrane with rigid inclusion, inflation of a rectangular membrane, torsion of a circular cylinder, or any other tests for which the strain states are known.

A polymer is subjected to uniaxial tension or compression test. The force is applied along the 1-axis. The principal stretch ratio in the 1-axis is λ_1 and the principal stretch ratios in the 2- and 3-axes are λ_2 and λ_3, respectively. The principal stretch ratios in the 2-axis and 3-axis are the same

$$\lambda_2 = \lambda_3 . \tag{10}$$

For incompressible materials, $\lambda_1\lambda_2\lambda_3 = 1$; therefore, we have

$$\lambda_1\lambda_2^2 = 1 \tag{11}$$

The uniaxial tension or compression test will provide one data point on the failure surface. It is the simplest and most common test.

For homogenous biaxial tension or compression test, the polymer is subjected to two forces acting on both 1- and 2-axis. For homogenous deformation we have,

$$\lambda_1 = \lambda_2 \tag{12}$$

The homogenous biaxial tests will yield another data point on the failure surface.

For another point on the failure surface, one can vary the forces on the 1- and 2-axis. The deformation produces different stretch ratios in the 1-axis and 2-axis; we have

$$\lambda_1 = \beta\lambda_2 \tag{13}$$

The ratio of λ_1 and λ_2 is β.

These three tests should provide enough data for determining the three failure constants.

The biaxial tests usually will use large specimens to eliminate the edge effect. Consequently, the test apparatus is very large.

Other tests can be used to determine the failure constants also. For example, the inflation of a circular membrane can be used instead of the homogenous biaxial tension test. Due to symmetry, λ_1 equals λ_2 at the pole.

The value for λ_1 and λ_2 at the pole is a function of the height of the inflated membrane, which is a function of the inflating pressure. The relationship can be obtained numerically as shown by Yang and Feng [3]. Later, an approximate relationship was obtained by Feng and Christensen [4].

The non-homogenous state of stretch ratios as shown in equation (13) can also be obtained by the inflation of a circular membrane with a rigid inclusion. The ratio β depends on the radius of the rigid inclusion and the radius of the circular membrane.

Besides these tests, any combination of three different kinds of tests can be used to determine the failure constants Γ_1, Γ_2 and K. The data are usually scattered. Therefore, statistical methods should be used in data analysis.

An analysis of a test

There are three failure material constants Γ_1, Γ_2 and K in equation (8). These constants can be determined from test data and numerical minimization

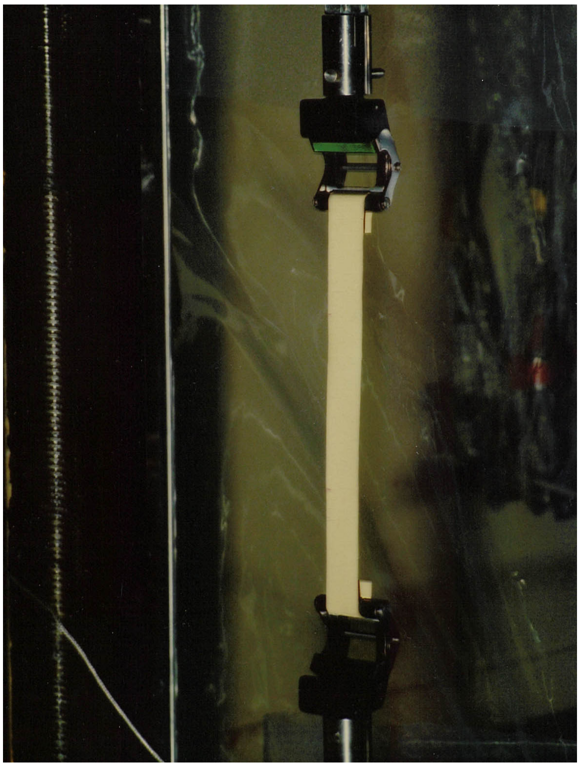

Figure 3. Uniaxial extension test

analysis. Six uniaxial failure data, obtained from uniaxial extension tests shown in Figure 3, are 8.3, 8.4, 8.45, 8.5, 8.9, and 9.0. Six equal biaxial tension failure stretching data, obtained from the inflation of a plane circular membrane shown in Figure 4, are 5.2, 5.6, 5.75, 5.90, 5.94, and 6.05. With these experimental data and the numerical analysis, the best-fit constants for K, Γ_1 and Γ_2 are 71, 0.0 and 0.006 respectively. The failure surface, as well as the test data, is shown in Figure 5.

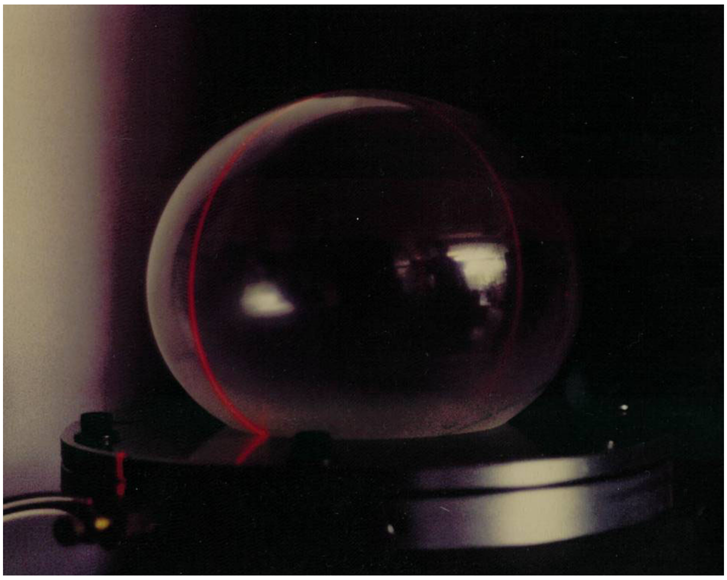

Figure 4. Inflation of a circular plane membrane

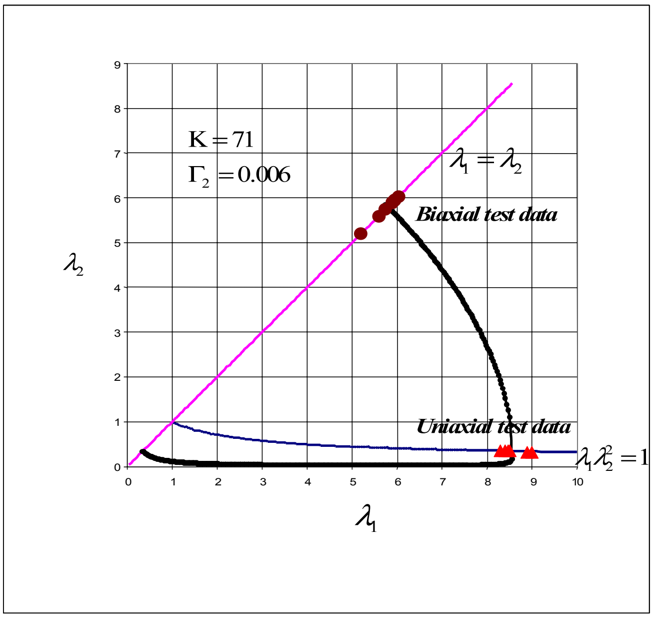

Figure 5. Test data and the failure surface for a polymer

The effect of Γ_2

The effect of Γ_2 on the failure surface is shown in Figure 6. The values for Γ_2 are 0.0, 0.02, 0.1 and 0.5. The value for K is 25. It can be seen that Γ_2 is more sensitive to the biaxial test data than the uniaxial test data. Therefore, while the value for K can be determined relatively accurately by the uniaxial test, the value for Γ_2 must be obtained from the biaxial test data.

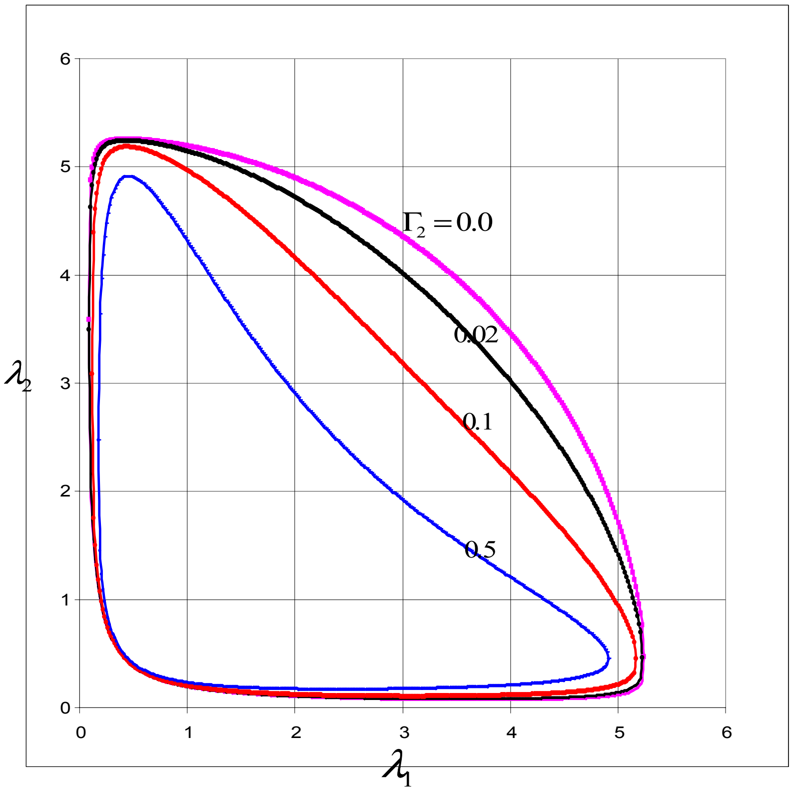

Figure 6. The effect of Γ_2 on the failure surface.

Results from LS-DYNA

The failure criterion has been implemented into LS-DYNA. The failure constants used in the input file are $\Gamma_1 = 0.0$, $\Gamma_2 = 0.02$ and $K = 1.0$. With these constants, the failure uniaxial stretch ratio is 1.67 and the failure equal-biaxial stretch ratio is 1.355, as shown in Figure 7. The LS-DYNA result, Figure 8, shows this phenomenon. Other results from LS-DYNA are shown in Figure 9. The failure constants used in the input file are $\Gamma_1 = 0.0$, $\Gamma_2 = 0.02$ and K=5.0, 1.0, 0.75 and 0.5. The first bar is not subjected to the failure study; therefore, it remains intact.

In the future, the failure criterion will be implemented into other material models as well.

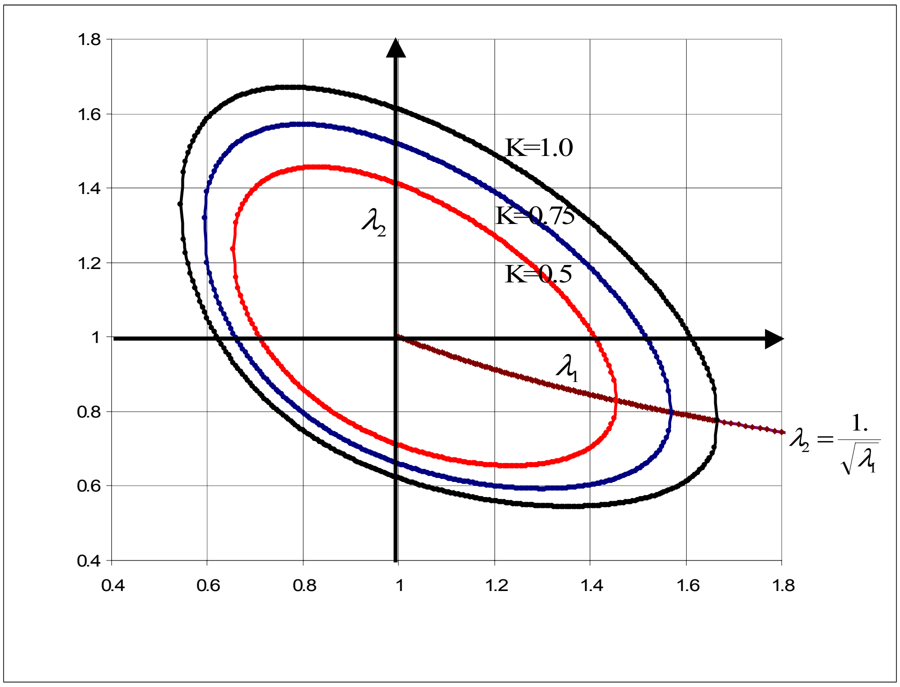

Figure 7. The failure surfaces for $\Gamma_1 = 0.0$, $\Gamma_2 = 0.02$ and K=1.0, 0.75 and 0.5.

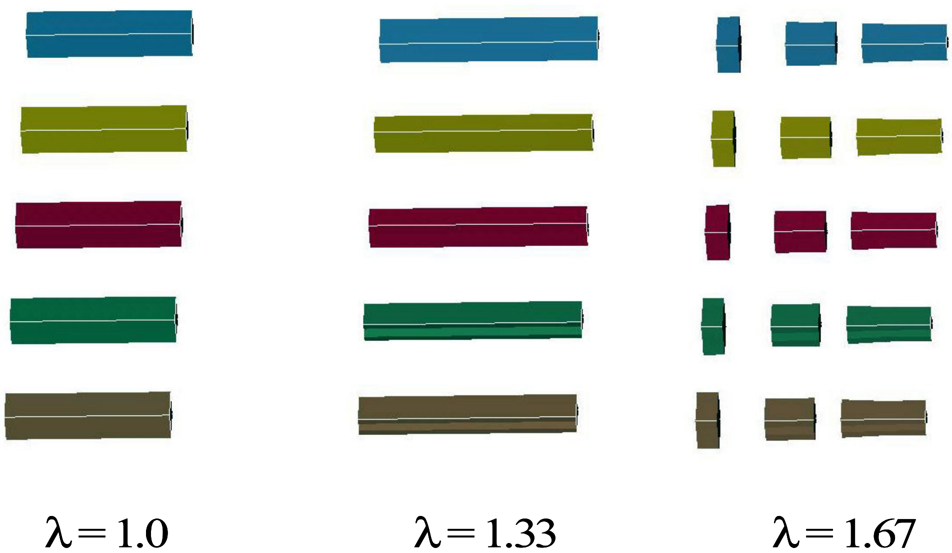

Figure 8. The results from LS-DYNA; the failure constants are $\Gamma_1 = 0.0$, $\Gamma_2 = 0.02$ and $\mathrm{K} = 1.0$

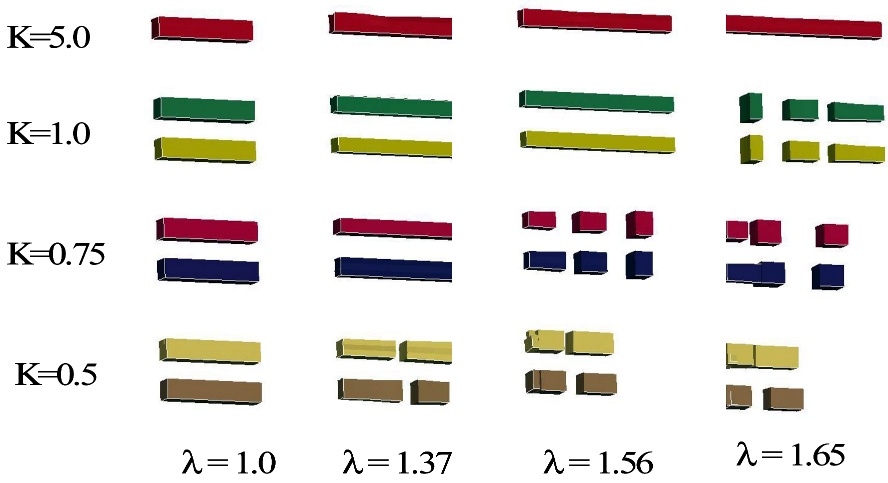

Figure 9. The results from LS-DYNA; the failure constants are $\Gamma_1 = 0.0$, $\Gamma_2 = 0.02$, K=5.0, 1.0, 0.75 and 0.5.

Conclusions

A failure criterion for polymer and soft biological materials, subjected to very large deformation, has been presented in this paper. The failure criterion can be applied to small deformation as well as to finite deformation problems. There are three failure material constants. These constants can be determined with three kinds of experimental data. Some simple experimental tests for obtaining these constants are mentioned. The new criterion has been implemented into LS-DYNA for numerical simulations and practical applications.

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