A simplified approach to the simulation of rubber-like materials under dynamic loading

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abstract

The simulation of rubber materials is becoming increasingly important in automotive crashworthiness simulations. Although highly sophisticated material laws are available in LS-DYNA to model rubber parts, the determination of material properties can be non-trivial and time consuming. In many applications, the rubber component is mainly loaded uniaxially at rather high strain rates. In this paper a simplified material model for rubber is presented allowing for a fast generation of input data based on uniaxial static and dynamic test data.
A simplified approach to the simulation of rubber-like materials under dynamic loading

Implementation of MAT_181 in LS-DYNA

Paul A. Du Bois
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Mechanical behaviour of rubber

- Nearly incompressible
- Hyperelastic under quasistatic loading
- Highly rate-dependent under dynamic loading
Numerical simulation

- Quasistatic hyperelastic response: best fit for the Ogden functional based on uniaxial tension, simple shear and biaxial testing
- Dynamic viscoelastic response: best fit for a generalized Maxwell model
- Example of implementation: MAT_77 (Ogden or general hyperelastic) in LS-DYNA

Practical problems:

- Very often, only uniaxial tensile and/or compressive test results are available
- Parameter fitting can be difficult and time consuming
- Sometimes dynamic response cannot be fitted by a generalized Maxwell model
MAT_SIMPLIFIED_RUBBER

- A pragmatic and simplified alternative is proposed
- Ogden functional is computed from uniaxial tensile and compressive data only (fit is exact)
- Viscoelastic approach is replaced by rate-dependent hyperelasticity
- Incompressibility is assumed

MAT_SIMPLIFIED_RUBBER

- Implemented in LS-DYNA v970 as MAT_181 in December 2002
- Tested extensively in a number of industrial simulation projects since
MAT_181 : user input for quasistatic response

<table>
<thead>
<tr>
<th></th>
<th>Specimen gauge length</th>
</tr>
</thead>
<tbody>
<tr>
<td>SGL</td>
<td>Specimen width</td>
</tr>
<tr>
<td>SW</td>
<td>Specimen thickness</td>
</tr>
<tr>
<td>ST</td>
<td>Load curve or table ID, defining force versus actual change in gauge length</td>
</tr>
</tbody>
</table>

If $SGL=1$ and $SG=ST=1$ then engineering stress/strain curves are input.

MAT_181 : user input

User must provide full range of data taking incompressibility into account:

$$
\varepsilon_{\text{min}} = \min \left( \varepsilon_{\text{ulim}}, -\frac{1}{\sqrt{\varepsilon_{\text{ulim}} + 1}} - 1 \right)
$$

$$
\varepsilon_{\text{max}} = \max \left( \varepsilon_{\text{ulim}}, -\frac{1}{\sqrt{\varepsilon_{\text{ulim}} + 1}} - 1 \right)
$$

Typical strain range from -0.8 to 1.2

$$
\sigma = \frac{f}{A_0}
$$

$$
\varepsilon_0 = \frac{l - l_0}{l_0}
$$

Tension test

Compression test

$\varepsilon_{0T}$

$\varepsilon_{0C}$
MAT_181 : user input

To avoid localisation, negative slopes in the true stress versus true strain curve should be avoided, thus:

\[
\frac{\partial \sigma}{\partial \varepsilon} = \frac{\partial \sigma_0 (1 + \varepsilon_0)}{\partial \varepsilon} = \frac{\partial \sigma_0}{\partial \varepsilon} (1 + \varepsilon_0) + \sigma_0 > 0
\]

\[
tension: \quad \frac{\partial \sigma_0}{\partial \varepsilon} > 0
\]

Some theory:

- MAT_SIMPLIFIED_RUBBER will reproduce the quasistatic uniaxial tension and compression tests exactly, no fit is done
- Under quasistatic arbitrary 3D loading the response of MAT_SIMPLIFIED_RUBBER is identical to MAT_OGDEN based on parameters that would allow an exact fit of the uniaxial test results
Some theory: Ogden model

\[ W = \sum_{i=1}^{3} \sum_{j=1}^{n} \frac{\mu_j}{\alpha_j} \left( \lambda_i^{*\alpha_j} - 1 \right) + K (J - 1 - \ln J) \]

Ogden functional depends on principal stretch ratios

\[ \lambda_i^* = \lambda_i J^{-1/3} = \frac{\lambda_i}{J^{1/3}} \quad J = \lambda_1 \lambda_2 \lambda_3 = \frac{V}{V_0} \]

\[ \sigma_i = \sum_{j=1}^{n} \frac{\mu_j}{J} \left[ \lambda_i^{*\alpha_j} - \sum_{k=1}^{3} \frac{\lambda_k^{*\alpha_j}}{3} \right] + K \frac{J - 1}{J} \]

true stress

Expression for true stress:

\[ \sigma_i = \sum_{j=1}^{n} \frac{\mu_j}{J} \left[ \lambda_i^{*\alpha_j} - \sum_{k=1}^{3} \frac{\lambda_k^{*\alpha_j}}{3} \right] + K \frac{J - 1}{J} \]

Generalisation:
f need not be polynomial

\[ f(\lambda_i) = \sum_{j=1}^{n} \mu_j \lambda_i^{*\alpha_j} \]

Polynomial function

\[ \sigma_i = \frac{1}{J} \left( f(\lambda_i) - \frac{1}{3} \sum_{j=1}^{3} f(\lambda_j) \right) + K \frac{J - 1}{J} \]
Some theory: simplified model

\[ \varepsilon_{oi} = \lambda_i - 1 \]

\[ f(\lambda_i) = \lambda_i \sigma_0 (\varepsilon_{oi}) + \sum_{n=1}^{\infty} \left( \lambda_i^{(-1/2)^n} - 1 \right) \]

for \[ \left| \lambda_i^{(-1/2)^n} - 1 \right| \leq 0.01 \]

Principal strain follows from principal stretch ratio
f is evaluated from the tabulated uniaxial engineering stress/strain data

Comparison of material laws for rubber in LS-DYNA:

Single element tests for 4 basic loadcases
compare MAT_77, MAT_77 with fit and MAT_181
Comparison of material laws for rubber in LS-DYNA:

Uniaxial response is identical for all 3 models.

Least squares fit is also very close.

Comparison of material laws for rubber in LS-DYNA:

Hydrostatic and shear response of MAT_181 are equivalent to the Ogden model.
MAT_181 follows test curve:

Some test curves may be hard to fit with an Ogden-type functional

MAT_181: user input for dynamic response

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TENSION</td>
<td>0=rate effects only in loading</td>
</tr>
<tr>
<td></td>
<td>1=rate effects in loading and unloading</td>
</tr>
<tr>
<td>RTYPE</td>
<td>0=true strain rate</td>
</tr>
<tr>
<td></td>
<td>1=engineering strain rate</td>
</tr>
<tr>
<td>AVGOPT</td>
<td>0=simple average</td>
</tr>
<tr>
<td></td>
<td>1=running average</td>
</tr>
</tbody>
</table>
Treatment of rate effects:

5 brick elements of different height are compressed about 50% using the same loading velocity.

Effect of TENSION:

TENSION=0  
rate effect only in loading

TENSION=1  
rate effect also in unloading

rate dependent hyperelasticity shows no exponential stress relaxation.
Effect of AVGOPT:

- AVGOPT=0
  - simple 12 point average of strain rate
- AVGOPT=1
  - running average of strain rate

Effect of RTYPE:

- RTYPE=0
  - true strain rate
  - test results hard to obtain
- RTYPE=1
  - engineering strain rate
  - test results at constant speed
Practical choices:

- Rate dependent hyperelasticity is not as physical as viscoelasticity
- some formulation choices must be made by the user
- in our applications, we have used TENSION=0, RTYPE=1 and AVGOPT=1

Applications:

- Application examples include:
  - assembly adhesives (rubber-based)
  - MVSS-201 head impactor skin
  - pedestrian head impactor skin
Future developments:

- Implementation of MAT_181 for shell elements
- Application on a PVB windshield interlayer, previous simulation work regarding this material has been presented in the 11th international workshop of computer aided mechanics of materials, September 2002

Conclusions:

- With MAT_181 no parameter identification is necessary if uniaxial test results are available
- Highly nonlinear rate effects can be considered in the model
- Elastic oscillations sometimes cause instabilities, viscous hourglass control is recommended
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