

**IMPROVED SPOTWELD SIMULATION WITH LS-DYNA –
NUMERICAL SIMULATION AND COMPARISON
TO EXPERIMENTS**

Authors:

Karl Schweizerhof, Werner Schmid, Herbert Klamser***

CAD-FEM GmbH, Stuttgart, Germany

***PORSCHE AG, Weissach, Germany,*

** also Institute of Mechanics, University of Karlsruhe, Germany*

Correspondence:

Karl Schweizerhof

LS-DYNA Group

CAD-FEM GmbH

Friedrich-List Str. 46

D-70771 Leinfelden-Echterdingen

Germany

Tel: +49-(0)711-9907450

Fax: +49-(0)0711-9907456

e-mail: kaschweizerhof@cadfem.de

Abbreviations:

FE: finite element, CD: Central differences

Keywords:

Spotweld connections, nonlinear structural mechanics, finite element modeling, crashworthiness analysis

ABSTRACT

Functioning of the body of a car structure with respect to stiffness, durability and crashworthiness is mainly dependent on the connection between the manufactured parts, in particular, if mainly shell type structural parts are used. Spot welding is the dominant technique used nowadays to connect shell type parts. Though spot welding is a fully automated process and a well known technique since many years, there is little knowledge about the behavior of the spotweld connections under single strong impulse loading. This knowledge, however, is one major ingredient for a proper representation in FE crashworthiness models.

In order to capture the correct mechanical behavior of a spotweld connection with its complex stress state, in principle a fully 3D continuum mechanical model is needed, e.g. as the welding process itself has a major influence on the material properties in the vicinity of the spotweld due to the high temperatures involved locally. In addition from experiments it is known that the failure process of a spotweld connection is strongly dependent on the form of the spotweld and local rupture takes place.

In FE crashworthiness models, however, such detailed models cannot be incorporated for the many thousands of spotweld connections due to efficiency reasons. Thus alternative models have to be developed to include the major effects of the spotweld behavior within full car crash models with only little influence on the efficiency of the FE analysis.

Taking the experiences gained from many years of modeling in the automotive industry a number of different FE spotweld models are discussed and the results of the analyses with LS-DYNA are compared to some experimental results from the investigations initiated by the FAT-working group 27 (Forschungsvereinigung Automobiltechnik e.V. AK 27).

Within the analysis particular focus is on the sensitivity of the results concerning the various spotweld models tested on single spotwelds. Finally the influence of the various models on the results for a realistic part under crashworthiness loading is shown.

INTRODUCTION – Spotweld modeling

Spotweld connections are one of the most common methods to connect structural parts of automotive bodies even for highly transient loading such as in crashworthiness situations. Then the non-continuous connection achieved even for very closely spaced spotweld has certainly some influence not only on the stiffness of the final structure but also on the local and global deformation behavior. Whereas a number of investigations on the static behavior of spotwelds up to failure can be found in the literature [2][3][4][8][9][12] there is limited knowledge only on the dynamic behavior under highly transient loading. In particular, though it is assumed that the rupture occurs in a similar fashion as in statics some rate dependency is expected with higher velocity loading.

As experiments with reasonable generality can be only performed on single spotweld connections and not on arbitrary structural parts it is of major importance to gain enough knowledge from experiments for mechanical models such as usable for FE modeling. As very detailed models are not possible in real life structures such as full car models used in crashworthiness simulations even with the high detailing found nowadays, efficient representative models have to be developed. Though a number of models is available in standard FE programs such as LS-DYNA [6][7], the investigations performed with these models suggest further development and more detailed investigations of alternative models.

Thus the Forschungsvereinigung Automobiltechnik e.V. had initiated a working group to investigate the experimental behavior of single spotwelds [5]. Parallel to this analysis models were developed in [10] and further investigated in some continuing work by CADFEM. These FE models are compared to the standard spotweld models available in LS-DYNA and their merits and limits are discussed first on single spotweld connections. Finally, the implications of the various models are compared on realistic parts with many spotwelds. Particular focus is on reliability and robustness of the models as well as on efficiency.

Experimental Investigations

A series of experimental investigations was carried out on behalf of the Forschungs-vereinigung Automobiltechnik e.V working group 27 (FAT-AK 27) in Germany mainly on single spotwelds. This is described in detail in a report by the University of Pader-born, Laboratory für Werkstoff- und Fügetechnik [5]. The investigations were performed for various loading conditions varying the loading direction of the spotweld from pure tension so-called 90°-loading to a more combined loading with increased shear thus 60°- and 30°-loading up to full shear 0°-loading, see figure 1, similar to the investigations described in [4]. In addition the velocity of the loading was increased in steps from 5 m/s, 10 m/s up to 15 m/s.

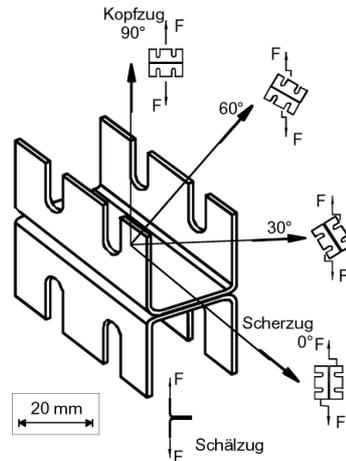


Figure 1 Specimen for single spotweld investigations under varying loading direction

The major results are that in all cases the spotweld itself did not fail but was pulled out of the neighboring material. The load deformation curves show a highly nonlinear behavior with failure after locally large deformations in the very vicinity of the spot-welds. As expected the specimen also show a quite strong velocity dependent behavior to some extent though the geometrical deformation does not show much difference. As expected in these cases the peaks of the force deformation curves occur at smaller deformations. The curves for higher velocity loading do not differ much for all three velocities for the pure tensile case (90°) and the 60° and 30° cases, but are substantially different for the pure shear case. The latter also indicates the difficulties involved in performing the experiments at higher velocity. Then often the vibration characteristics of the complete testing equipment is involved and it becomes difficult to extract the real action of the specimen. More detailed information on the experiments will be given elsewhere.

General FE Modeling of Spotwelds

In the following we focus on the spotweld connections of thin walled structures usually described by shell theory in structural mechanics. In principle a spotweld connection must be modeled by a complete three dimensional continuum in order to contain all effects involved. Within the welding process at higher temperatures locally a change in material properties is observed with the spotweld being less ductile than the surrounding material. In addition due to the transient heating and cooling process an eigenstress state [11] remains in the structure which is stronger, if the sheets are thick. Such models are appropriate for the simulation of the welding process and local static analyses [2][8][9][4], however, within a structural model for transient processes the thin walled structures are represented with the kinematical restrictions of shell theory and a two dimensional stress state plus shear stresses. The connection between two thin shell parts itself is then - within the model restrictions - only consistent via a surface type connection. If a finite element representation is chosen, then complete elements should be connected to achieve a consistent approximation.

Nodal Spotweld Models

A first rather straightforward representation is to connect two nodes with each other by a nonlinear spring with arbitrary force deformation behavior. Such models are incorporated in LS-DYNA directly as spring models with nonlinear behavior which can be defined by so-called load curves plus a number of additional features. The definition is by forces thus it is a lumped approach. Such models allow to adjust the behavior for each direction and loading condition. An advantage of this modeling is the chance to combine it with a viscous damper. Then even an almost elasto-plastic - viscoplastic behavior can be simulated and failure can be defined.

An extension compared to the lumped type model for the use with real beam elements is a specific spotweld material (type 100) with elasto-plastic behavior including the chance to define failure via plastic strains at the integration points or via damage evolution based on a failure strain for the start of damage and a rupture strain for final rupture. Furthermore brittle failure can be defined via a force based model:

$$\left(\frac{N_r}{N_{rF}}\right)^2 + \left(\frac{N_{sr}}{N_{sF}}\right)^2 + \left(\frac{N_t}{N_{tF}}\right)^2 + \left(\frac{M_s}{M_{sF}}\right)^2 + \left(\frac{M_t}{M_{tF}}\right)^2 + \left(\frac{T}{T_F}\right)^2 - 1 \geq 0$$

In order to account for possible high oscillations in the forces due to numerical noise also filtering of the forces used for the failure criterion can be defined. This reduces an artificial failure due to overstiff parts resp. problems with fairly coarse meshes.

Further nodal based models are the constrained connections which can either be defined via a constrained definition or via a contact definition. In the constrained definition in LS-DYNA a nodal rigid body is assumed with the original distance between both connected nodes resp. further connected nodes if a multiple layered spotweld is defined. Failure of such a spotweld is possible either via a force based criterion taking the normal and the shear force into account. This is similar to the quadratic criterion for the specific spotweld material (type 100):

$$\left(\frac{N_t}{N_{tF}}\right)^n + \left(\frac{N_{Shear}}{N_{Shear,F}}\right)^m - 1 \geq 0$$

In addition a plastic strain based failure is possible using the strains of the elements assigned to the connected nodes. If the contact definition is used to connect nodes, a penalty based approach - tie-break-nodes - is taken and failure is defined in the same fashion. For connected surfaces a similar approach - tie-break-surfaces - can be used with the same failure criterion but based on stresses in the contact surface.

The disadvantage of the brittle failure behavior is that it introduces artificial stress waves into the structure which in many cases cause further unrealistic responses.

The rigid body connections of the constrained and contact based approach often may also lead to a somehow locally artificially stiffened structure; thus using real beam elements with specific spotweld material appears to be the better choice. The disadvantage of the latter is the short length of the beam which results in a high frequency. The latter can be reduced by mass scaling which mostly does not affect the real inertial behavior of the global structure.

Single nodal connections, however, are in principle not possible in continuum theories and in addition often lead to an excitation of the hourglass modes which are possible with the mostly used efficient hourglass-controlled elements [1]. If with local refinements of the meshes the real extension of the spotweld is taken into account by connecting more nodes, then both objections are removed.

Mesh Independent Connection and Mesh Dependency of Results

An important feature for the flexible connection of nodes is the feature to remain mesh independent to some extent concerning the location of the connection by using the connection between nodes which are not mesh nodes but constrained to elements of the surfaces. This feature may reduce the hourglass excitation, if the nodes are more in the interior of the elements describing the surfaces. In addition, by constraining the connection nodes to the interior of the specimen elements the size of the spotweld is artificially increased to almost the size of the connected surface elements. Thus a major mesh dependency - dependency of the location of the node within the elements - of the results is obtained, unless rather fine meshes are chosen. This effect is studied in detail in the numerical examples section below.

Torsional Resistance

In standard shell theory there is naturally no resistance against rotations around the normal. However, by the single beam connection of the nodal spotwelds rotations around the normal occur, and it is desirable that the single beam - a model for the 3D spotweld - should also carry over a torsional moment. This can be achieved by adding further beams between the connected nodes on each surface to the corresponding nodes of the connected elements, see figure 2. Then all rotations of the beams can be coupled and via the spatial action the torsion is carried over. The disadvantage is the high stiffness introduced into the structure and the mesh dependency which is particularly large for coarser meshes. In addition the efficiency in modeling and analysis is substantially affected, if such a model would be used for real structures with many thousands of spotwelds. As an alternative, for smaller mesh sizes multiple beams - as shown in figure 2 - may be used to model one spotweld. However, though this may provide some resistance against torsion it is a rather inefficient model.

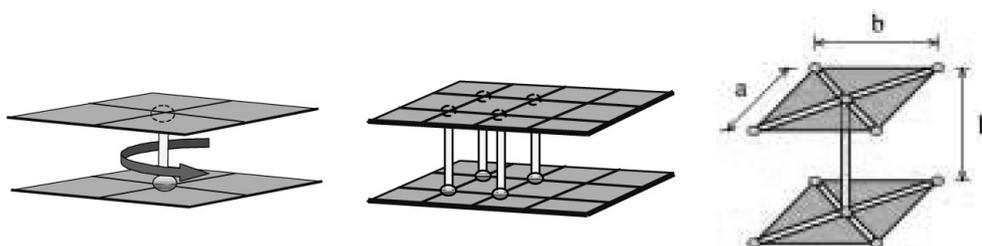


Figure 2 Standard nodal connection (left), multiple beam connection (center), single beam with torsional resistance by cross beams (right)

An approach to avoid both - artificial stiffening as well as high modeling effort - is contact type s7 (SPOTWELD WITH TORSION), where some torsional resistance can be defined for the nodes by forces acting in the plane of the elements. However, such forces are hard to control resp. validate and it has shown to be fairly difficult to achieve a certain robustness within more sophisticated models.

Contact Shell Set

In order to achieve an almost mesh independent spotweld connection with uniform size for the spotweld a so-called contact shell set was developed in [10]. The set, see figure 3, consists of two flexible shell elements and one beam element connecting the two shell elements with each other. Two nodes, each of them constrained to the corresponding element are used to fix the beam. The nodes of the shells are con-strained to the respective shell surfaces. The size of the shell elements of the contact shell set are chosen equal to the diameter of the real spotweld. The properties of the beam element are identical to the properties of the real spotweld. The only quantity not represented is the torsional stiffness which is not available in shell theory. How-ever, the use of the special spotweld with torsion is possible as discussed above. Due to the beam model mass scaling may be necessary to keep the time step size for the complete car model within an efficient level.

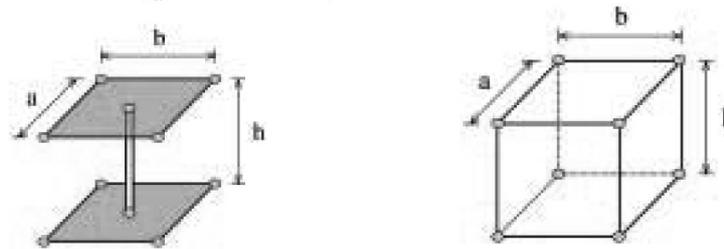


Figure 3 Contact shell set for spotweld modeling (left), Continuum element as spot-weld (right)

Continuum Elements as Spotwelds

For the rather fine meshes used in the automotive industry with shell element size of about 4 mm or less, it appears to be an efficient alternative to use continuum elements directly as spotwelds. One element appears to be sufficient for the representation of some quantities, such as the stiffness in normal and shear direction and the general geometrical extension necessary to provide the correct resistance e.g. within a folding process. However, for bending it is certainly not sufficient, neither torsion is well represented. As bending is not really a problem for realistically loaded spotwelds it must be noted that for proper action in torsion some further adjustment may be needed. Clearly fully underintegrated elements with one point integration only lead often to strong hourglassing thus the selectively reduced integrated solid elements are suggested and are used in all examples below. With the use of anisotropic material a fairly good adjustment to many experiments is possible. In the limit – with further mesh refinement - an automatic mesh size independence is achieved, as the side length of the continuum element is chosen identical to the diameter of the spot-weld. Mass scaling may be advisable for coarser meshes.

Numerical Investigations on Single Spotweld Connections

The general model is taken as given in figure 4 left; the vertical walls of the specimen are taken as rigid, whereas the horizontal part is flexible. The loading is by a velocity profile; in order to avoid artificial impulses a smooth transition from zero to the final loading velocity is achieved over a short period of 0.1 ms. Unless further indicated the final loading velocity is rather low with 1 m/s.

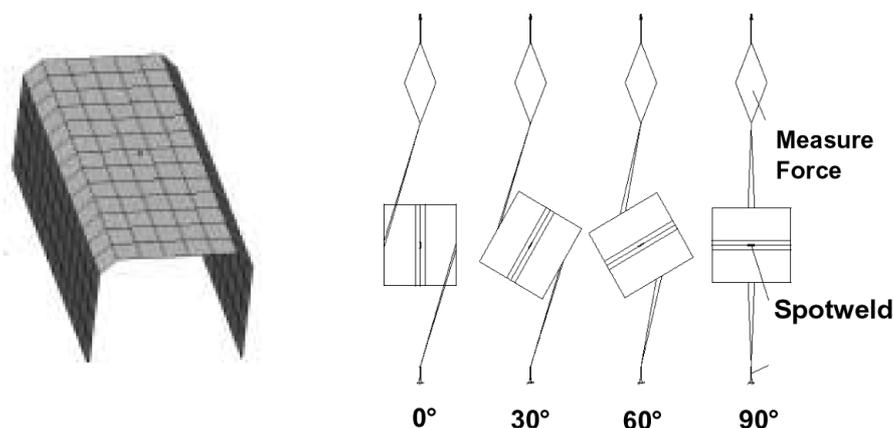


Figure 4 Numerical model for single spotweld connection; FE mesh (left) and general loading model (right) with variation of loading angle

Variation of mesh density and relative location of nodes

In order to show the mesh and location dependency of the various models for single spotwelds a number of parametric studies with different locations of a node in the element as indicated in figure 5 is performed. In addition the element size is varied. Further studies include a change in the element type from the hourglass controlled elements with full underintegration [1] to the assumed-shear strain elements [7] in

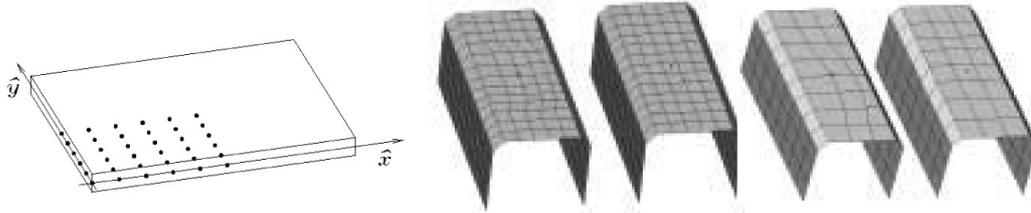


Figure 5 Variation of mesh density and location of connection nodes within elements

order to check the hourglass sensitivity. The deformation pattern - not shown - is in general almost completely identical for both element types, however, the hourglass energy is - as expected - visibly larger, as the hourglass modes carry some loading.

The forces obtained for some different locations of the spotweld are depicted in figure 6 for the nodal spotweld connection with a beam and specific spotweld material 100 assuming fully elastic material, standard steel and a diameter of 3.5 mm. The mass density is slightly increased due to mass scaling. The forces depicted are the external forces measured outside of the specimen.

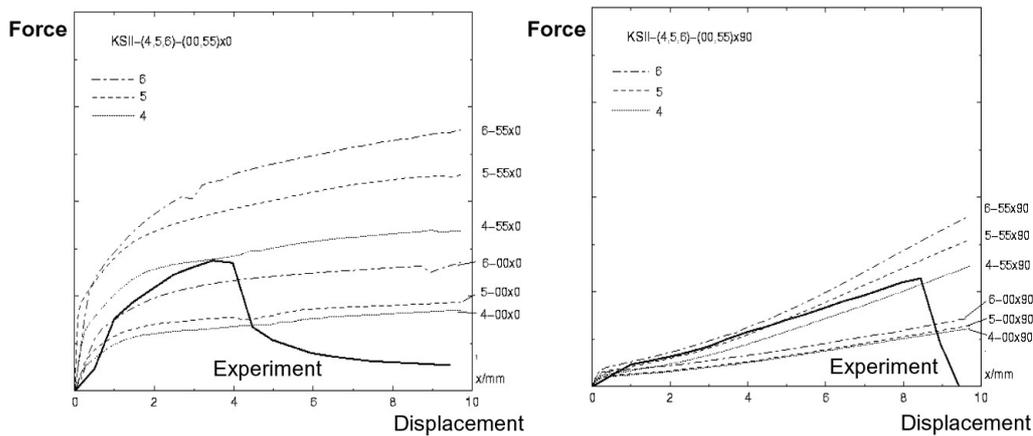


Figure 6 Measured spotweld forces vs. displacement for varied mesh density and varied location of the spotweld nodes for nodal connections; pure shear (0°) and pure tension (90°)

The effect of the varying spotweld location on the deformation of the shell model is depicted in figure 7 for 30° and 90° loading and the Belytschko-Tsay elements [1].

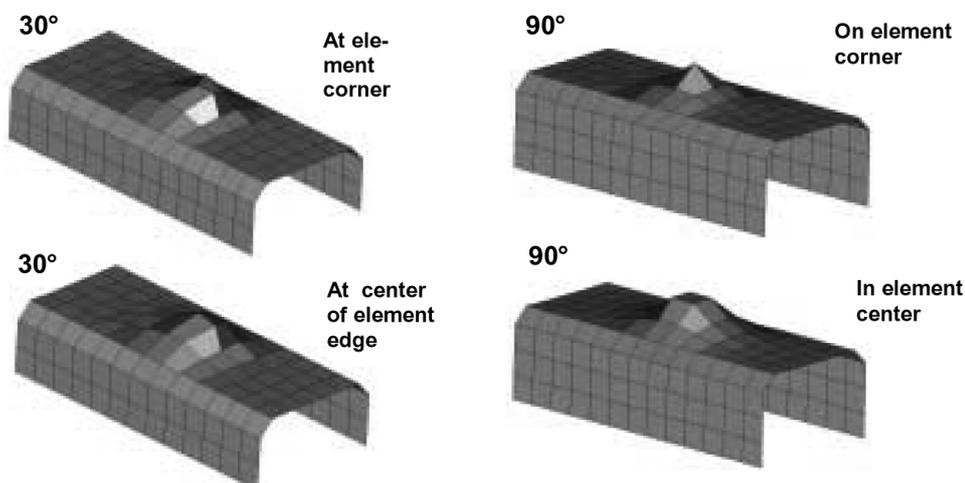


Figure 7 Deformation of single spotweld connection for nodal spotweld model; variation of relative location of connection node in element; 30° (left) and 90° (right, pure tension) loading

For the same meshes of the specimen the so-called contact shell set was also used with a side length of the shell elements of 3.5 mm. The location was varied with the same meshes for the specimen as used for the nodal spotwelds. The curves in figure 8 show that the results are far less sensitive against the mesh size than the nodal spotweld. Some dependency of the results – expressed here in considerably larger force values - concerning the location is unavoidable, as e.g. with a regular mesh either one element only is involved or four elements in the extreme cases. In all cases investigated also some sharp peaks with a sudden drop of the force level are visible in the load deformation curves. This is due to the effect that the contact algo-rithm may fail due to an overly large counter deformation; then the contact surface is bending and the contact shell element with a linear deformation mode is not capable to follow and develops large forces against the deformation. If the distance between the nodes and the contact surfaces is too large, contact may be released then. If such a failure occurs, the assumed shear strain elements have proven to be more robust in the following due to their higher stiffness than the fully underintegrated Belytschko-Tsay elements with stiffness control.

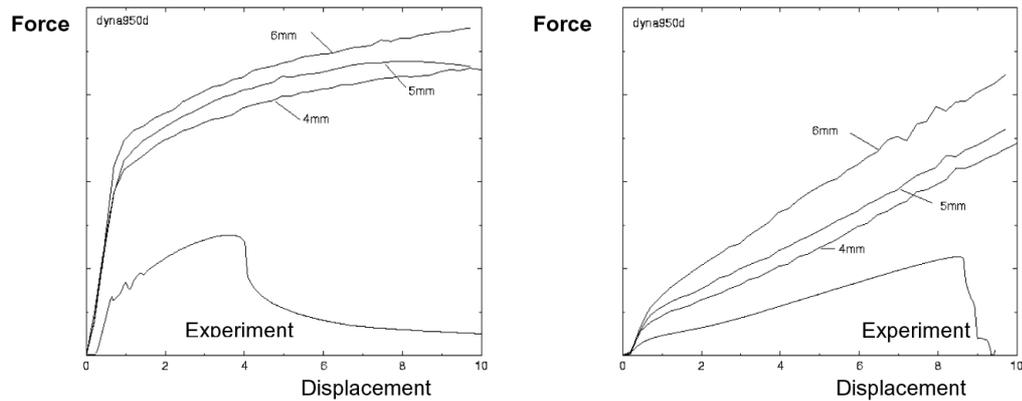


Figure 8 Measured spotweld forces for varied mesh density (element size 4/5/6 mm) of the spotweld nodes for contact shell sets; 0° (left, pure shear) and 90° (right, pure tension) loading

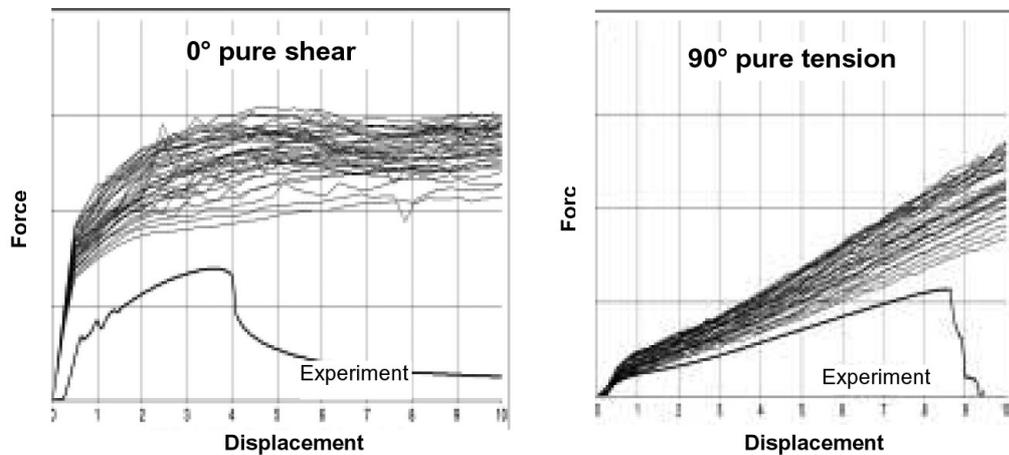
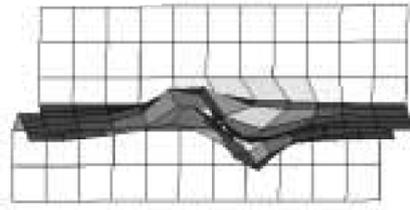


Figure 9 Measured spotweld forces vs. displacement varying the relative location of the continuum spotwelds; 0° (left, pure shear) and 90° (right, pure tension) loading

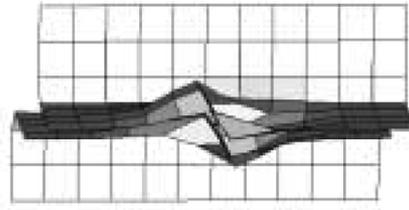
If continuum elements are used for the spotweld, then the behavior is similar to the contact shell set, if no torsional action is involved, see figure 9. There only the variation of the location is studied. The advantage of the continuum element besides torsional resistance is the higher efficiency compared to the contact shell set which contains two shell plus one beam element. The peaks in some load deformation curves indicate some problems of the contact search and release conditions for some counter deformation cases as mentioned above. If the contact is kept for all situations, then the curves are smooth. Further work on the implementation is under-way to guarantee this for such situations.

The deformation pattern for the contact shell sets is depicted in figure 10 for the case of 0° (pure shear) and 90° (pure tension) loading and two different locations of the connection. It becomes obvious, that the contact shell sets located such, that the center of the set is directly at an element center of the specimen, lead to a less stiff behavior than any other connection. The deformation patterns for the continuum element connection are almost identical and thus not depicted.

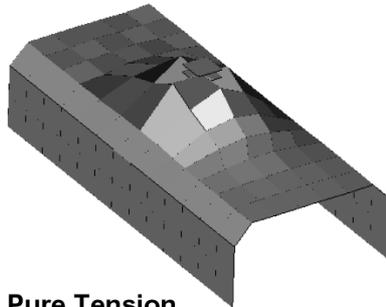
0° Pure Shear



Center of set at mesh node



Center of set at element center



90° Pure Tension

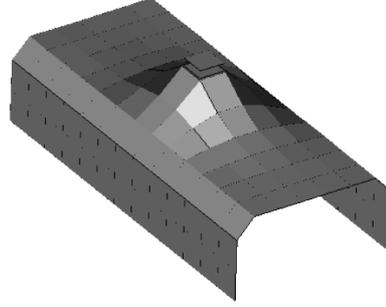


Figure 10 Deformation of specimen for connection via contact-shell-set; 0° (pure shear, side view) and 90° (pure tension, top view) loading. Location of contact shell set varied.

Multiple Spotweld Connections

At a first look it is expected that for parts connected by many spotwelds the difference in the results will not depend so much on the location of the spotwelds with reference to the mesh of the specimen. Thus a study is performed comparing nodal spotweld and continuum spotwelds for the so-called H-test example with two rows of five evenly spaced spotwelds under pure shear loading. The loading is applied in the same fashion as for the single spotweld example and increased from rest to $v = 1$ m/s within 0.1 ms. In addition the influence of the element type on the results was investigated. The latter did not show any significant difference in the results. However, the location of the nodal spotwelds still has a major influence, as is visible in figure 11.

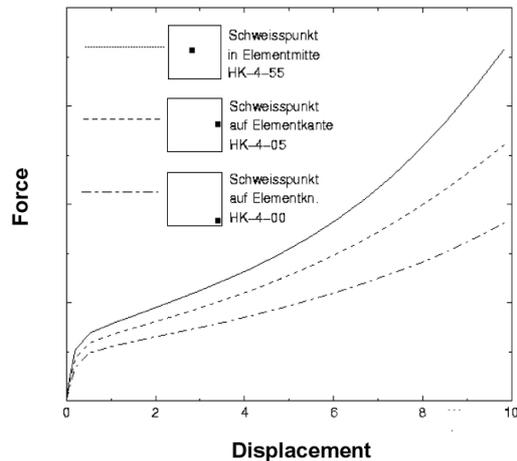
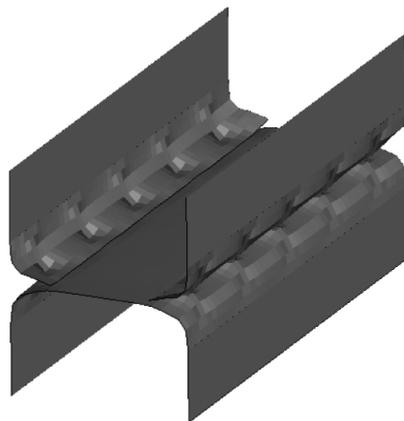


Figure 11 Multiple spotweld example; deformation pattern (left); measured spotweld forces vs. deformation for varied location of the spotweld nodes for nodal connections (right); 0° (pure shear) loading

If continuum spotwelds are used, then this influence is considerably reduced for the two cases, if the element is either in contact with a full element or with two elements over an edge, see figure 12. However, some differences are still present, as the geometry of the spotweld introduces a different rigidity with each variation relative to the mesh. This, of course, would disappear with very fine meshes for the spotweld itself and the contacting surfaces, but for crashworthiness models coarse meshes with one element per spotweld are a more realistic model. The mesh influence is particularly large for the case, if the element is in contact with four elements, thus a node of the

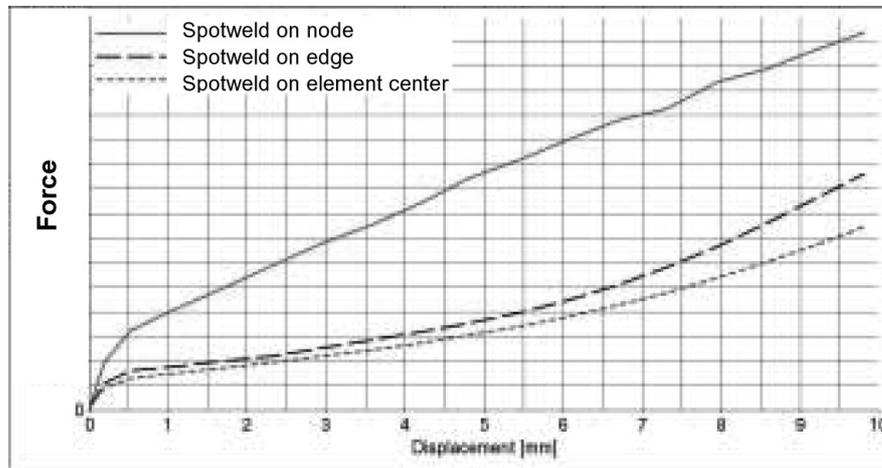


Figure 12 Measured spotweld forces vs. displacement for varied location of the spot-weld nodes for continuum spotwelds; 0° (pure shear) loading

mesh of the specimen is at the center of the surface of the continuum element. Then a rather large rigidity is introduced into the structure for this particular, fairly coarse mesh.

The latter effect becomes more visible, if the deformation pattern is examined in more detail. In figure 13 a partial view of the deformed structure is given; the stiffen-ing effect of the particular position, if four elements on each side are in contact with the spotweld continuum element, becomes obvious. A cure to this effect would be a refinement of the mesh of the specimen such that the edges of the shell elements would be at the edges of the continuum element.

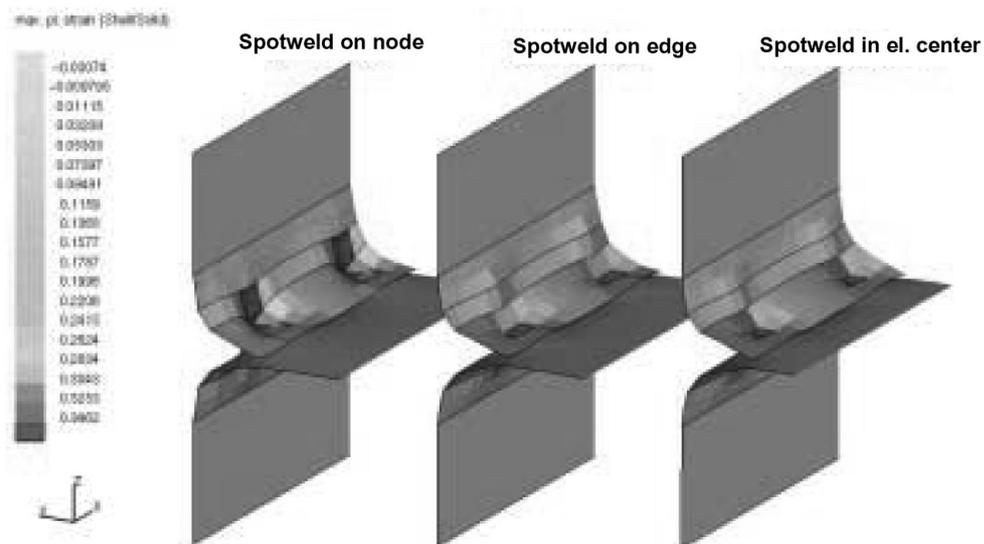


Figure 13 Deformation pattern of multiple spotweld example under pure shear load-ing; continuum spotweld; partial view

Girder Type Structure under Crashworthiness Loading

In order to show the influence of the spotweld modeling on the global behavior of a structure a girder similar to a longitudinal girder of an automotive structure is used for comparison purposes of all three spotweld models discussed above. In the numerical model the girder is fixed on one side and is impacted on the other side by a rigid wall of about 600 kg moving with 40 km/h. In all models the connected parts of the girder are meshed without considering the location of the spotwelds. The latter were added after the meshing with constraining nodes to the contact surfaces independent of their meshing. Though the difference is only in the spotweld modeling the difference in the deformation behavior is substantial, see figures 14 to 16. All show initially the same behavior with some local folding. Then the nodal spotweld leads to a global buckling of the girder, whereas both other models show a continuation of the local folding process. The time history curve for the longitudinal force, however, does not reflect the differences so much, see figure 17. The forces are rather close up to 0.018 sec, then the nodal spotweld (beam model) model deviates stronger from the other two curves. In general the deviations occur at a fairly low force level compared to the level up to 0.017 sec. As a general statement we can conclude from this model that further investigations are necessary for different locations of the spotwelds, in order to find out whether a similar behavior could have been found also for the con-tact-shell-set resp. the continuum model considering other variations.

This appears to



Figure 14 Longitudinal girder model under crash loading; deformation states; nodal spotweld model



Figure 15 Longitudinal girder model under crash loading; deformation states; contact shell set spotweld model



Figure 16 Longitudinal girder model under crash loading; deformation states; continuum spotweld model

be a typical example for a structure under stability loading which may be sensitive against less significant structural modifications compared to the spotweld modeling.

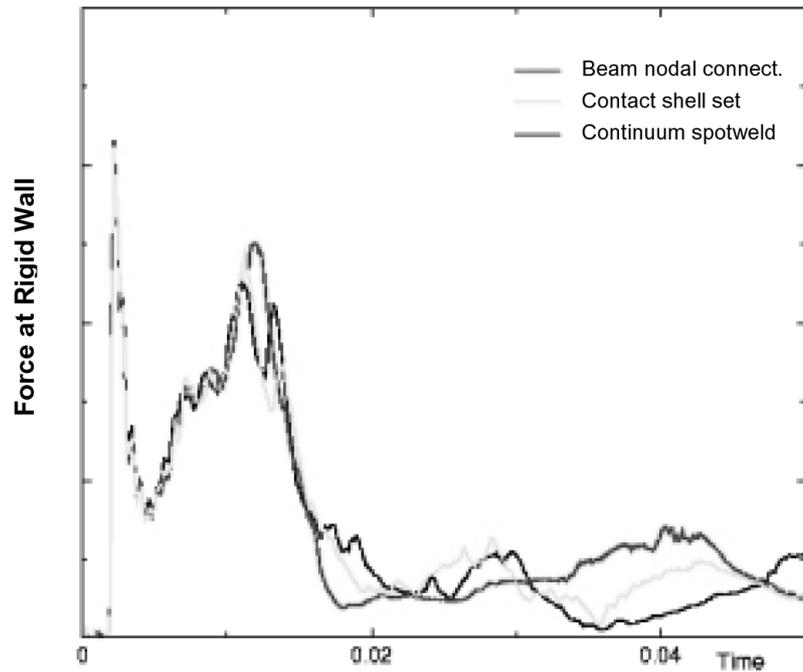


Figure 17 Longitudinal girder model under crash loading; comparison of time history for longitudinal force of three spotweld models

Summary and Conclusions

The investigations of the spotweld models available in LS-DYNA have been extended to the application of so-called contact-shell-set models and a continuum spotweld model. From an efficiency point of view simple nodal spotweld models are most efficient, however, they may lead to higher mesh dependencies and to a softer response, in particular, as no torsional resistance is provided. Furthermore hour-glassing is often excited by the large local point loading introduced by this model. The alternative contact-shell-set model also has no torsional stiffness, but reflects the real geometry better due to the size of the contact shells. The major disadvantage is the high modeling and computational effort needed for this model. If a simple continuum element is used stiffness and geometry is represented for all types of loading, though a close adjustment of one element to the real behavior e.g. for all load cases is certainly difficult. Nevertheless, it provides a fairly robust and efficient model which can be defined in a simple fashion. The results concerning the deformation behavior as well as the force deformation resp. time history of the forces show that it is rather important to resolve the deformation of the shell structures in such a fashion, that the bending is not restricted by the geometry of the spotweld. Otherwise a considerable mesh resp. location dependency may be obtained. If, however, a reasonably fine mesh is used with elements such that they can be easily adjusted to fit the edges of the continuum spotweld element, then the correct deformation behavior of the numerical model - compared to experimental results - can be expected. It must be noted that this “mesh alignment” requires a fairly high modeling effort. Further investigations currently underway are dedicated towards the dynamics of the spotweld connections with particular focus on failure situations.

References

1. BELYTSCHKO, T., TSAY, C.S. (1983) "A stabilization procedure for the quadri-lateral plate element with one-point Quadrature", Int.J.Num.Meth.Engng. Vol. 19, pp. 405-419.
2. HAHN, O., WENDER, B. (1983) "Finite-Element-Analyse einer auf Zug bean-spruchten einschnittig überlappten Widerstandspunktschweißverbindung mit Hilfe unterschiedlicher Strukturmodelle", Schweißen & Schneiden 35.
3. HAHN, O., GIESKE, D. (1994) "Neue Eielementprobe zum Prüfen von Punktschweißverbindungen unter kombinierter Belastung", Schweißen & Schneiden 46.
4. HAHN, O.; KURZOK, J.R.; ROHDE, A.; THESING, T. (1999) "Rechnergestützte Dimensionierung widerstandspunktgeschweißter und mechanisch gefügter Bau-teile", Schweißen & Schneiden, 51 , pp. 17_23.
5. HAHN, O., KURZOK, J., OETER, M.; (2000) "On Experimental Failure Investiga-tions of Single Spotweld Connections under Highly Transient Loading" , (int. Re-port for Forschungsvereinigung Automobiltechnik e.V working group 27, FAT-AK 27) Laboratorium für Werkstoff- und Füge-technik, University - Gesamt-hoch-schule Paderborn.
6. HALLQUIST, J.O., LIN, T., TSAY, C.S. (1993) "LS-DYNA Theoretical Manual", "Nonlinear dynamic analysis of solids in three dimensions", Livermore Software Technology Corp., Livermore.
7. HALLQUIST, J.O., TSAY, C.S. (2000) "LS-DYNA vs. 950 Users Manual", "Non-linear dynamic analysis of solids in three dimensions", Livermore Software Tech-nology Corp., Livermore.
8. MAYER, A., PREIS, N. (1994) "Berechnungsverfahren zur Spannungsbeurtei-lung von Punktschweißverbindungen", Konstruktion 46.
9. RADAJ, D., GIERING, A. (1995) "Spot-Welded Double-Cup Specimen", Materi-alprüfung 37.
10. SCHMID, W. (1999) "Schweißpunktverbindungen im Automobilbau; Rechen-modelle für transient belastete Strukturen", Diplomarbeit, Institut für Mechanik, Universität Karlsruhe, Germany (sup.visor Schweizerhof/Klamser).
11. SCHRÖDER, R., MACHERAUCH, E. (1983) "Berechnung der Eigenspannung bei Widerstandspunktschweißverbindungen unter Zugrundelegung unterschiedli-cher mechanisch-thermischer Werkstoffdaten, Schweißen und Schneiden 35.
12. VOPEL, W.; HILLMANN, W. (1996) "Berücksichtigung der Schweißpunkte bei der FEM_Modellierung von Karosserien", VDI Bericht 1283, VDI Düsseldorf, 1996, pp. 171_181.