Global Sensitivity Analysis in Structural Optimization

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Summary:

The main purpose of global sensitivity analysis is to identify the most significant model parameters affecting a specific model response. This helps engineers to improve the model understanding and provides valueable information to reduce the computational effort in structural optimization. A structural optimization is characterized by a set of design parameters, constraints and objective functions formulated on basis of model responses. The computational effort of a structural optimization depends besides the complexity of the computational model heavily on the number of design parameters. However in many cases an objective function is dominated only by a few design parameters. Results of global sensitivity analysis may be used to select the most significant design parameters from a number of potential candidates and thereby reduce the optimization problem by the insignificant ones. In this paper different sensitivity measures and associated algorithms are evaluated with respect to their capabilities and computational costs. In particular the variance-based approach after Sobol is compared with the correlation analysis, the linear and quadratic ANOVA approaches, and the FAST approach. This is done using a comprehensible academic example as well as an optimization problem from engineering practice.

Keywords:

optimization, global sensitivity analysis, Sobol' indices

1 Introduction

Goal of the engineering design process is an optimal structural design. Each engineering design process is thus an optimization problem. Optimization problems are characterized by a set of design parameters, constraints and objective functions formulated on basis of model responses.Structural optimization of moderate to complex computational models is costly and often requires additionally effort due to the high number of design parameters that might be considered in the design process. Responses can depend on all design parameters, but often the significance onto the variation of the structural responses differs greatly. In many cases a structural response is dominated by only a few significant design parameters. For improvement in efficiency of solving the optimization problem and the engineering design process respectively, the optimization problem may be reduced by insignificant input variables, i.e. design parameters which do not have a part in the variation of the regarded structural responses. The identification of significant and insignificant input variables succeeds by means of global sensitivity analysis. Sensitivity analysis is the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variation [4]. Objective of a global sensitivity analysis ist thus the identification of significant and insignificant design parameters and the reduction of the input dimension (number of design variables) of the optimization problem. Furthermore global sensitivity analysis helps thus to find mistakes in the model and to improve the model understanding.

An adequate global sensitivity analysis approach for computation of sensitivities for general (nonlinear) computational models are the Sobol' indices. The Sobol's variance-based approach, the Sobol' indices, has received much attention in literature in the last years since they provide accurate sensitivity information for most computational models [6]. In contrast to that, well-established global sensitivity analysis approaches like the correlation analysis or the linear ANOVA require linear or weak nonlinear monotonic models. Also the quadratic ANOVA is based on a quadratic approximation of the structural responses. However the FAST approach is independent from the model behavior.

In this paper the global sensitivity analysis approaches mentioned above are evaluated and compared with respect to their capabilities and computational costs.

2 Global sensitivity analysis

2.1 Regression-based correlation analysis

Correlation, (often measured as a correlation coefficient), indicates the strength and direction of a linear relationship between two random variables. A number of different coefficients are used for different situations. The best known is the Pearson product-moment correlation coefficient, which is obtained by dividing the covariance of the two variables by the product of their standard deviations. The correlation coefficient $\rho_{X,Y}$ between two random variables X and Y is defined as the Pearson correlation coefficient

$$\rho_{X,Y} = \frac{E(XY) - E(X)E(Y)}{\sqrt{E(X^2) - E^2(X)}\sqrt{E(Y^2) - E^2(Y)}} \tag{1}$$

where $E(\cdot)$ denotes the expected value. For a result of a numerical experiment e.g. obtained through a Monte Carlo Simulation, the Pearson product-moment correlation coefficient can be used to estimate the correlation between X and Y. The Pearson correlation coefficient is then the best estimate of the correlation of X and Y. The Pearson correlation coefficient is written:

$$r_{x,y} = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{N}(x_i - \bar{x})^2 \sum_{i=1}^{N}(y_i - \bar{y})^2}} \tag{2}$$

where $\bar{x}$ is the mean (average) of the x_i values and $\bar{y}$ is the mean of the y_i values. The values of the correlation coefficients from Eqs. (1) and (2) are located in the interval $[-1, 1]$, where values close to 0 indicate a weak (linear) relationship and values close to -1/1 a strong (linear) relationship between the investigated random variables X and Y.

Consider a single response variable Y dependent upon a number of variables $\underline{X} = (X_1, X_2, ..., X_n)$, then the correlation coefficient may thus be used as sensitivity measure

$$S_i = \rho_{X_i, Y} \tag{3}$$

The correlation is a very popular and powerful measure to summarize linear relationships between variables but in the case of outliers or (weak) nonlinearity it may lead to wrong conclusions. Therefore a correlation analysis does not replace the individual examination of the data (e.g. through anthill plots).

The Pearson correlation coefficient is associated with the regression coefficient obtained by linear regression analysis. Regression analysis also indicates the strength and direction of a relationship between two random variables X and Y. At this one random variable is defined to be dependent and modeled as a function of the independent variable, corresponding parameters, and a random error term. In linear regression for modeling n data points there is one independent variable x_i, two parameters a and b, and one error term ε_i.

$$y_i = a + bx_i + \varepsilon_i$$

The parameters a and b are estimated by using the least squares method.

$$\hat{b} = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{N}(x_i - \bar{x})^2} \tag{4}$$

$$\hat{a} = \bar{y} - \hat{b}\bar{x} \tag{5}$$

The interrelation between linear regression and the Pearson correlation coefficient is given by

$$\hat{b} = r_{x,y} \frac{s_y}{s_x} \tag{6}$$

where s_x and s_y are the standard deviations of the respective data.

The proportion of variability in the investigated data accounted for by the linear regression is defined by the coefficient of determination $R_{x,y}^2$. The variability of the data is measured through the residuals

$$\hat{u}_i = y_i - (\hat{a} + \hat{b}x_i) \tag{7}$$

The coefficient of determination $R_{x,y}^2$ is thus defined by

$$R_{x,y}^2 = 1 - \frac{\sum_{i=1}^{N} \hat{u}_i^2}{\sum_{i=1}^{N}(y_i - \bar{y})^2} \tag{8}$$

In the case of linear regression, $R_{x,y}^2$ is the square of the Pearson correlation coefficient $r_{x,y}$.

2.2 Variance-based approaches

The models under investigation are described by a function $\underline{Y} = f(\underline{X})$, where $\underline{X} = (X_1, X_2, ..., X_n)$ is the random input vector consisting of n random variables (i.e. structural parameters) and where $\underline{Y} = (Y_1, Y_2, ..., Y_m)$ denotes the random output vector consisting of random variables (i.e. structural responses). It is possible to decompose $\underline{Y} = f(\underline{X})$ into summands of increasing dimension.

$$f(\underline{X}) = f_0 + \sum_{i=1}^{n} f_i(X_i) + \sum_{1 \le i < j \le n} f_{ij}(X_i, X_j) + ... + f_{1,2,...,n}(X_1, ..., X_n) \tag{9}$$

Each random model response Y_j $(j = 1, 2, ..., m)$ is characterized by its variance V^j. According to Eq. (9) it is possible to decompose each variance V^j into partial variances associated with the single random input variables $X_1, X_2, ..., X_n$ as follows:

$$V^j \;=\; \sum_{i=1}^{n} V_i^j + \sum_{1 \leq i < k \leq n} V_{i,k}^j + ... + V_{1,2,...,n}^j \tag{10}$$

and to relate each partial variance to one sensitivity measure

$$S_{i_1,...,i_s}^j \;=\; \frac{V_{i_1,...,i_s}^j}{V^j} \quad \text{with} \quad 1 \leq i_1 < ...i_s \leq n, \quad s = 1, 2, ..., n \tag{11}$$

Each of the sensitivity measures $S_{i_1,...,i_s}^j$ describes which amount of each variance V^j is caused due to the randomness of the associated random input variables and their mapping onto the output variables. As special case the sensitivity measures S_i^j describing the sole influence of a single input variable X_i are called main effects. On the other hand the sensitivity measures $S_{i_1,...,i_s}^j$ describing the influence of combinations of input variables are called interaction effects.

In practice, all partial sensitivity measures involving the single input variable X_i $(i = 1, 2, ..., n)$ are summed up to the total sensitivity measure $S_{T_i}^j$ in order to evaluate the total effect of X_i. The total sensitivity measures consider thus interaction among the input variables. In order to quantify which amount of each variance V^j is caused due to a single input variable X_i the corresponding total sensitivity measure $S_{T_i}^j$ may be normalized according to Eq. (12).

$$\text{norm} S_{T_i}^j \;=\; \frac{S_{T_i}^j}{\sum_{k=1}^{n} S_{T_k}^j} \tag{12}$$

That is, input variable X_i has a share in variance V^j of $\text{norm} S_{T_i}^j$.

For computationally demanding models, e.g. finite element models in engineering mechanics, efficient meta models should be applied. Besides polynomial response surface approximations, Neural and Radial Basis Function networks and Krigging approximations may also be used as response surfaces [5]. In this case, an additional amount of variance due to the approximation in dependency of the quality of the response surface appears. To take this additional variance into consideration the variance of the discrepancies between the exact functional response and the response surface at the experimental points is computed and denoted by V_{unex}^j. The normalized total sensitivity measure $S_{T_i}^j$ is thus modified according to Eq. (13), where V^j and $S_{T_i}^j$ are computed only using the response surface.

$$\text{norm} S_{T_i}^j \;=\; \frac{\frac{S_{T_i}^j}{\sum_{k=1}^{n} S_{T_k}^j} V^j}{V^j + V_{\text{unex}}^j} \tag{13}$$

In the following three different algorithms for the numerical computation of the partial variances $V_{i_1,...,i_s}^j$ and thus the sensitivity measures $S_{i_1,...,i_s}^j$ are presented and compared.

2.2.1 Linear and quadratic ANOVA

The linear and quadratic ANOVA (ANalysis Of VAriance) approach according to [5] is explained in the following. The approach is based on response surface methodology, that is, a design surface is fitted to predetermined response values using regression analysis. Least squares approximation is used for this purpose. Consider a single response variable Y dependent upon a number of variables $\underline{X} = (X_1, X_2, ..., X_n)$. The exact functional relationship between these quantities is

$$Y \;=\; f(\underline{X}) \tag{14}$$

The exact functional relationship is now approximated (e.g. polynomial approximation) as

$$f(\underline{X}) \quad \approx \quad f^*(\underline{X}) \tag{15}$$

The approximating function $f^*(\underline{X})$ is assumed to be a summation of basis functions

$$f^*(\underline{X}) \quad = \quad \sum_{i=1}^{L} a_i \phi_i(\underline{X}) \tag{16}$$

where L is the number of basis functions $\phi_i(\cdot)$ used to approximate the model. The constants $\underline{a} = (a_1, a_2, ..., a_L)^T$ have to be determined in order to minimize the sum of square error

$$\sum_{p=1}^{P} \left\{ [f(\underline{X}_p) - f^*(\underline{X}_p)]^2 \right\} \quad = \quad \sum_{p=1}^{P} \left\{ [f(\underline{X}_p) - \sum_{i=1}^{L} a_i \phi_i(\underline{X}_p)]^2 \right\} \tag{17}$$

P is the number of experimental points and $f(\underline{X}_p)$ is the exact functional response at the experimental points $\underline{X}_p$. The numerical computation ist presented in the following.

The P experimental points are arrayed in the matrix

$$\mathbf{X} = \begin{bmatrix} X_{11} & \ldots & X_{1i} & \ldots & X_{1N} \\ \vdots & & \vdots & & \vdots \\ X_{j1} & \ldots & X_{ji} & \ldots & X_{jN} \\ \vdots & & \vdots & & \vdots \\ X_{P1} & \ldots & X_{Pi} & \ldots & X_{PN} \end{bmatrix} \quad \text{with} \quad i = 1 \ldots n \quad \text{and} \quad j = 1 \ldots P \tag{18}$$

and the mean values

$$E(\mathbf{X}_{\langle i \rangle}) = \sum_{j=1}^{P} \mathbf{X}_{\langle i \rangle, j} \tag{19}$$

and variances

$$V(\mathbf{X}_{\langle i \rangle}) = \sum_{j=1}^{P} \left[(\mathbf{X}_{\langle i \rangle, j} - E(\mathbf{X}_{\langle i \rangle}))^2 \cdot \frac{1}{P} \right] \tag{20}$$

are computed. With

$$\widehat{\mathbf{X}} = \begin{bmatrix} 1 & X_{11} & \ldots & X_{1i} & \ldots & X_{1N} \\ \vdots & \vdots & & \vdots & & \vdots \\ 1 & X_{j1} & \ldots & X_{ji} & \ldots & X_{jN} \\ \vdots & \vdots & & \vdots & & \vdots \\ 1 & X_{P1} & \ldots & X_{Pi} & \ldots & X_{PN} \end{bmatrix} \quad \text{and} \quad \mathbf{Y} = \begin{bmatrix} f(\underline{X}_1) \\ \vdots \\ f(\underline{X}_j) \\ \vdots \\ f(\underline{X}_P) \end{bmatrix} \tag{21}$$

the constants $\underline{a} = (a_1, a_2, ..., a_L)^T$ of a linear approximation (that is linear ANOVA) may be computed by

$$\underline{a} = (\widehat{\mathbf{X}}^\mathbf{T} \widehat{\mathbf{X}})^{-1} \widehat{\mathbf{X}}^\mathbf{T} \mathbf{Y} \tag{22}$$

The partial variances associated with each basis function of the linear ANOVA are then given by

$$V_i \quad = \quad a_i^2 \cdot V(\mathbf{X}_{\langle i \rangle}) \quad \text{with} \quad i = 1 \ldots L \tag{23}$$

The quadratic ANOVA is based on a quadratic approximation of the response variable Y and quadratic basis functions respectively. The parameter matrix given by Eq. (21) changes thus to

$$\widehat{\mathbf{X}} = \begin{bmatrix} 1 & X_{11} & \dots & X_{1N} & \dots & X_{1l} \cdot X_{1k} & \dots & (X_{11})^2 & \dots & (X_{1N})^2 \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ 1 & X_{j1} & \dots & X_{jN} & \dots & X_{jl} \cdot X_{jk} & \dots & (X_{j1})^2 & \dots & (X_{jN})^2 \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ 1 & X_{P1} & \dots & X_{PN} & \dots & X_{Pl} \cdot X_{Pk} & \dots & (X_{P1})^2 & \dots & (X_{PN})^2 \end{bmatrix} \tag{24}$$

The computation of the partial variances differs slightly in that case and is explained in detail in [5].

2.2.2 Fourier Amplitude Sensitivity Test (FAST)

The Fourier Amplitude Sensitivity Test according to [1] is based on the transformation of the response variable Y dependent upon a number of variables $\underline{X} = (X_1, X_2, ..., X_n)$ into a onedimensional space which is described by the variable s. This section just provides a short introduction. In the first step each input variable X_i is associated with a characteristic integer frequency ω_i. None of the chosen frequencies may be represented by a linear combination with integer coefficients r_i of the other frequencies.

$$\sum_i r_i \omega_i \neq 0 \quad \text{with} \quad -\infty \leq r_i \leq +\infty \tag{25}$$

Furthermore the number M of considered harmonics has to be chosen and P discrete points s_j ($j = 1, ..., P$), which have to be analysed in the onedimensional space, must be defined.

$$s_j = -\pi + \frac{\pi}{P} + \frac{2\pi}{P}(j-1) \tag{26}$$

In the next step realizations of each input variable X_i are computed according to their inverse probability distribution functions F_i^{-1} at the discrete points s_j in the onedimensional space and arrayed in the matrix $\widehat{\mathbf{X}}$.

$$\widehat{\mathbf{X}}_{ji} = F_i^{-1}\left[\frac{1}{2} + \frac{1}{\pi}\arcsin(\sin(\omega_i s_j))\right] \tag{27}$$

The partial variances associated with each input variable are then given by

$$V_i = \sum_{m=1}^{M} \Lambda_{m\omega_i} \tag{28}$$

with

$$\Lambda_{m\omega_i} = \frac{1}{2} \cdot \left[(\mathbf{A}_{m\omega_i})^2 + (\mathbf{B}_{m\omega_i})^2\right] \tag{29}$$

the Fourier coeficients

$$\mathbf{A}_{m\omega_i} = \frac{2}{P} \cdot \sum_{j=1}^{P} \mathbf{Y}_j \cos(m \cdot \omega_i \cdot s_j) \tag{30}$$

$$\mathbf{B}_{m\omega_i} = \frac{2}{P} \cdot \sum_{j=1}^{P} \mathbf{Y}_j \sin(m \cdot \omega_i \cdot s_j) \tag{31}$$

and

$$\mathbf{Y}_j = f(\widehat{\mathbf{X}}_{\langle i \rangle})_j \tag{32}$$

2.2.3 Sobol' indices

The Sobol' indices are practically computed using Monte Carlo simulation [6]. Using Monte Carlo simulation, the partial variances associated with each input variable X_i are computed as

$$V_i = \frac{1}{N_{\text{sim}}} \sum_{m=1}^{N_{\text{sim}}} f(\underline{x}_m^{(1)}) f(\underline{x}_{(\sim i)m}^{(2)}, \underline{x}_{im}^{(1)}) - \hat{f}_0^2 \tag{33}$$

with the mean value

$$\hat{f}_0 = \frac{1}{N_{\text{sim}}} \sum_{m=1}^{N_{\text{sim}}} f(\underline{x}_m) \tag{34}$$

and N_{sim} the number of simulations. The superscripts (1) and (2) in Eq. 33 indicate that two different simulations are generated and mixed in such a way that the model response $f(\underline{x}_{(\sim i)m}^{(2)}, \underline{x}_{im}^{(1)})$ is computed using the input vector $(\underline{x}_{1m}^{(2)}, \underline{x}_{2m}^{(2)}, ..., \underline{x}_{(i-1)m}^{(2)}, \underline{x}_{im}^{(1)}, \underline{x}_{(i+1)m}^{(2)}, ..., \underline{x}_{nm}^{(2)})$.

The higher-order partial variances are computed analog as

$$V_{ij} = \frac{1}{N_{\text{sim}}} \sum_{m=1}^{N_{\text{sim}}} f(\underline{x}_m^{(1)}) f(\underline{x}_{(\sim ij)m}^{(2)}, \underline{x}_{ijm}^{(1)}) - \hat{f}_0^2 \tag{35}$$

In the same way the partial variances $V_{ijk}, ...$ are computed.

The partial variance considering the total effect of an input variable X_i may be computed as

$$V_{\sim i} = \frac{1}{N_{\text{sim}}} \sum_{m=1}^{N_{\text{sim}}} f(\underline{x}_m^{(1)}) f(\underline{x}_{\sim im}^{(1)}, x_{im}^{(2)}) - \hat{f}_0^2 \tag{36}$$

2.3 Comparison of the approaches

In the following the Ishigami-function [3]

$$Y_3 = f_3(X_1, X_2, X_3) = \sin X_1 + a \sin^2 X_2 + b X_3^4 \sin X_1 \tag{37}$$

is investigated. The input variables are uniformly distributed in the interval $[-\pi, \pi]$. The example is carried out using the numerical values $a = 7$ and $b = 0.1$. The plot ($X_3 = 1.0$) of this nonlinear and non-monotonic function is shown in Fig. 1.
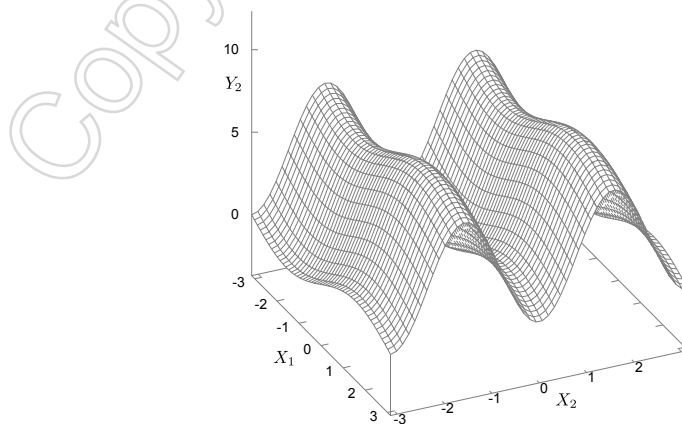

Fig. 1: Function plot of the Ishigami function ($X_3 = 1.0$)

The results of the computation of the regression-based correlation coefficient, the variance-based linear and quadratic ANOVA, and the Sobol' indices are gathered in Tab. 1.

Tab. 1: Results of the global sensitivity analysis

	correlation	linear ANOVA	quadratic ANOVA[1]	FAST	Sobol	analytic[2]
S_1	0.44	0.19	0.11	0.314	0.314	0.314
S_2	0	0	0.42	0.444	0.442	0.442
S_3	0	0	0.19	0	0	0
S_{12}	–	–	0.01	–	0	0
S_{13}	–	–	0.25	–	0.244	0.244
S_{23}	–	–	0.03	–	0	0
S_{123}	–	–	–	–	0	0
normS_{T_1}	–	–	0.28	–	0.45	0.45
normS_{T_2}	–	–	0.35	–	0.35	0.35
normS_{T_3}	–	–	0.36	–	0.2	0.2

[1]LS–OPT results, normalized

[2]according to Eq. (10)

It is observed that the influences of X_2 and X_3 are negated by the correlation analysis as well as by the linear ANOVA. The results of the quadratic ANOVA depend on the quality of the polynomial approximation. Here the contributions of the input variable X_2 and the interacting input variables (X_1, X_3) are detected, however the significance of X_1 is negated. The total effect of X_1 is thus underestimated by the quadratic ANOVA. The FAST may compute the main effects correctly but it yields no information about interaction effects. Furthermore it may lead to differing results in higher input dimensions because of the frequencies and number of harmonics to be chosen. Only the Sobol's approach corresponds with the analytic solution and yields information about the main effects, the interaction effects, and the total effects.

As a conclusion, this academic example shows the potential of the global sensitivity analysis by the Sobol' indices to investigate the dependencies of nonlinear functions with respect to the input parameters. Indeed all three input variables are important, which the common correlation analysis, the linear ANOVA and the FAST are not able to detect. Due to the limited capacity of the polynomial approximation used by the quadratic ANOVA the associated ranking of the input variables is differing to the analytic solution.

3 Application

Structural optimization within the engineering design process is demonstrated by means of the optimization of the crash behavior of a car model at front crash against a stonewall [2].
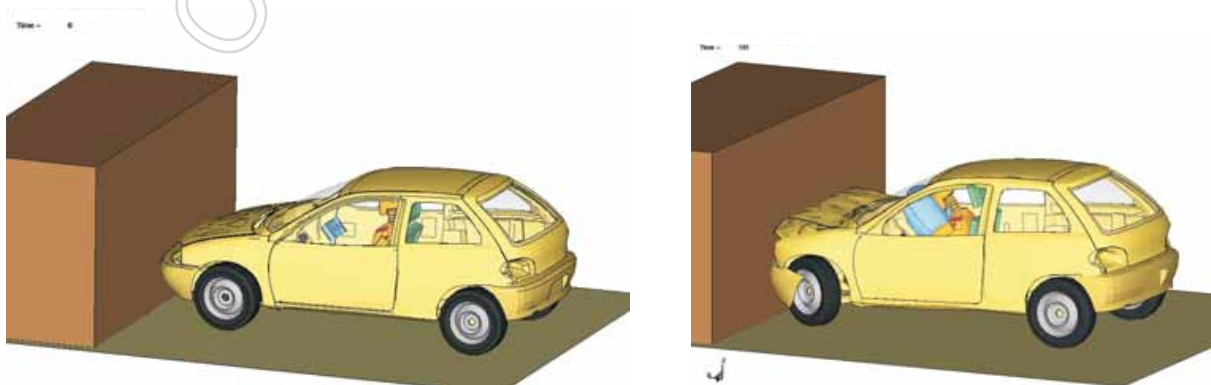

Fig. 2: Front crash of a car model against a stonewall

The design parameters are `lb1`, `lb2`, `lb3`, `c1`, `c2`, and `c3`, that is, the sheet thickness of the longitudinal and crossmembers according to Fig. 3.
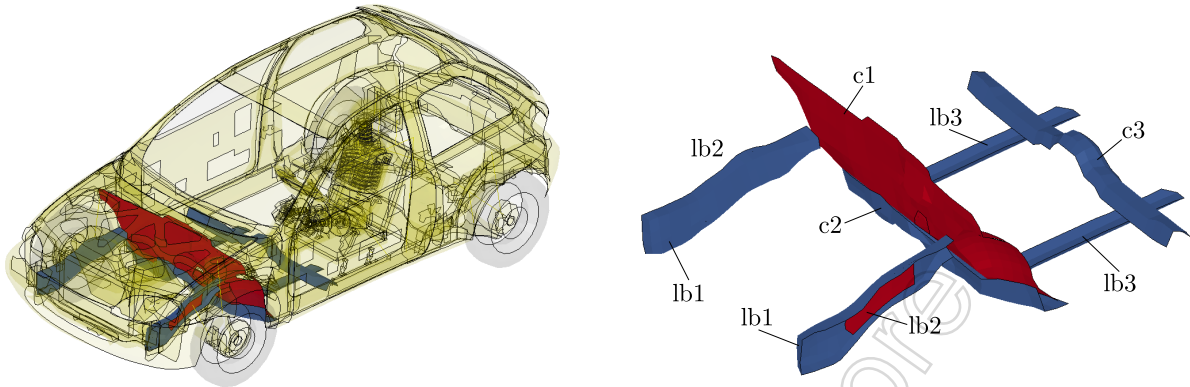

Fig. 3: Design parameters

The possible ranges of the design parameters are given in Tab. 2.

Tab. 2: Design parameters

Parameter	Name	Minimum [mm]	Maximum [mm]
longitudinal 1	lb1	1.0	1.5
longitudinal 2	lb2	1.3	1.8
longitudinal 3	lb3	0.8	1.3
crossmember 1	c1	0.8	1.3
crossmember 2	c2	0.8	1.3
crossmember 3	c3	1.0	1.5

As constraint of the optimization problem the total mass of the car is limited to $m_{max} = 923.92\,\mathrm{kg}$.

The considered response is the maximum intrusion of the chassis (`MaxIntru2`) into the passenger compartment as highlighted in Fig. 4.
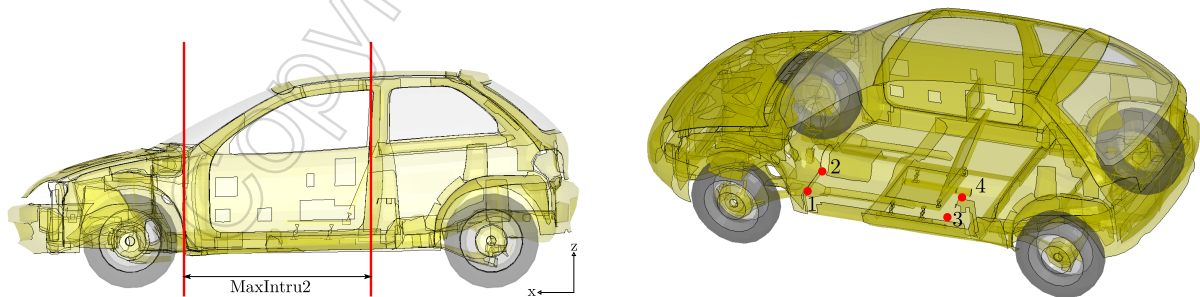

Fig. 4: Evaluated response of the car model

The objective function z of the optimization problem may thus be formulated as

$$z = \max_t \left[\left(\frac{x_1(0) + x_2(0)}{2} - \frac{x_3(0) + x_4(0)}{2} \right) - \left(\frac{x_1(t) + x_2(t)}{2} - \frac{x_3(t) + x_4(t)}{2} \right) \right] \rightarrow \min$$

with $x_i(t)$ are the displacement functions in x direction in of the regarded nodes $i = 1, 2, 3, 4$ as highlighted in Fig. 4. Because of the time dependence of the displacement functions the maximum of the relative displacement is evaluated and minimized.

There is no information about the probability distribution functions (pdf's) of the design parameters. In that case, a straightforward approach of a uniform probability distribution of the design parameters is assumed. It has to be kept in mind that the choice of the probability distribution functions influence the results of the sensitivity analysis significantly.

The global sensitivity analysis after Sobol was carried out with the software tool D-SPEX [7] using 250 experimental points and a radial basis function neural network as metamodel provided by LS-OPT. The Sobol' indices are given in Tab. 3.

Tab. 3: Results of the global sensitivity analysis after Sobol

Parameter	S_i = main effect	$\mathrm{norm}S_{T_i}$ = total effect
lb1	0.16	0.45
lb2	0	0.09
lb3	0	0.02
c1	0.10	0.25
c2	0.24	0.13
c3	0	0.04
residuals	0.07	0.02

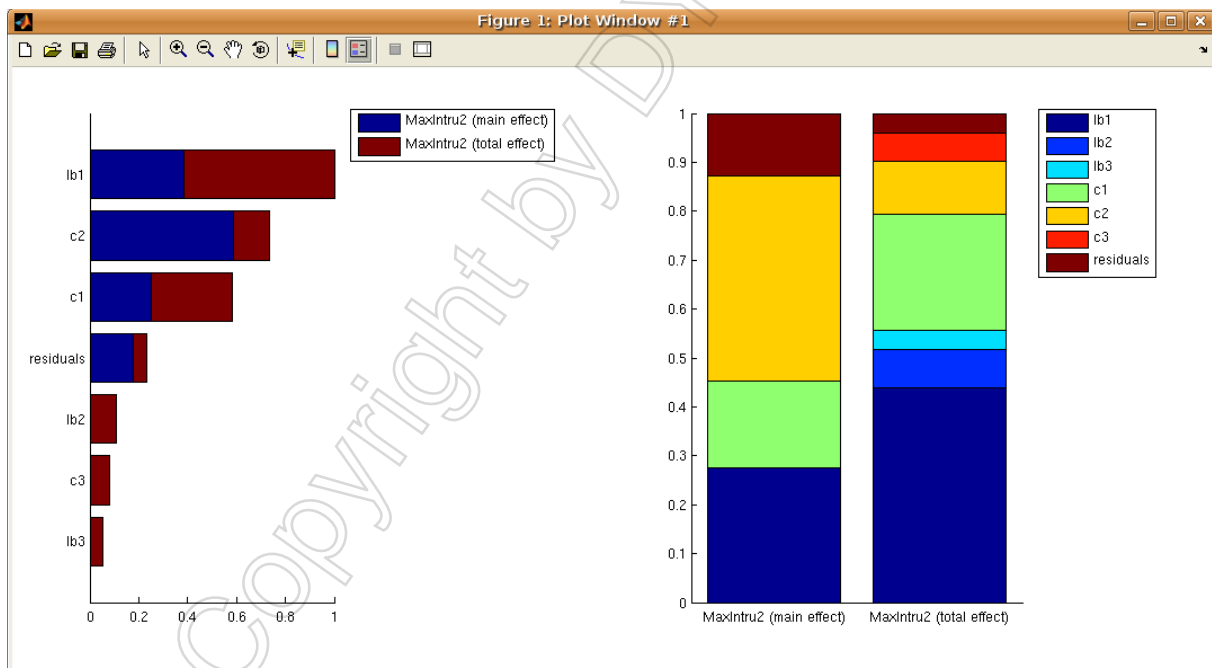The results of the sensitivity analysis may also be visualized with D-SPEX, Fig. 5.

Fig. 5: Plot of normalized total Sobol' indices $_{\mathrm{norm}}S_{T_i}$

In the Sobol' indices plots the length of each section in one stacked bar represents the contribution of the specific parameter (`lb1`, `lb2`, `lb3`, `c1`, `c2`, `c3`) to the total variance of the respective result (right chart). The left chart is an alternative depiction and illustrates the main effect and the total effect of the specific parameter to the respective result.

The Sobol' indices show that the sheet thicknesses of `lb1`, `c1`, and `c2` are significant and the sheet thicknesses of `lb2`, `lb3`, and `c3` are insignificant with respect to the objective function and the intrusion of the chassis respectively. The objective function is thus reduced by the insignificant parameters, that is deterministic values are assigned a priori, and the optimization problem is solved considering only the significant ones.

The optimization was carried out with LS-OPT. The results of the reduced optimization are given in Tab. 4 and compared with the results of the entire optimization. Compared with the results of the entire optimization a proper approximation of the global optimum has been found while reducing the computational effort substantially.

Tab. 4: Comparison – reduced and entire optimization

	Reduced [mm]	Entire [mm]
lb1	1.0	1.0
lb2	1.8[1]	1.3
lb3	0.8[1]	0.8
c1	1.3	1.3
c2	1.1	1.2
c3	1.0[1]	1.4
maximum intrusion	14.7	12.7

[1]assigned deterministic values

4 Conclusion

In this paper the Sobol' indices as a suitable global sensitivity analysis approach within structural optimization problems are presented and demonstrated by means of analytical and practical examples. The Sobol' indices are sensitivity measures for arbitrary (nonlinear) computational models. They quantify which amount of the variance of result variables is caused due to a single input variable or combinations thereof and may thus identify significant and insignificant input variables of structural optimization problems. Three further global sensitivity analysis approaches are presented with particular consideration of nonlinearity of the underlying computational model. The correlation analysis reflects the strength and direction of a linear relationship, but not any aspects of nonlinear relationships. The same applies to the linear ANOVA. The quadratic ANOVA may take nonlinearities into account, but restricted in terms of a quadratic approximation. The FAST considers the main effects of nonlinear non-monotonic problems correctly but does not yield information about interaction effects and may lead to differing results in higher input dimensions.

Acknowledgment

References

[1] CUKIER, R.: *Nonlinear Sensitivity Analysis of Multiparameter Model Systems*. Journal of Computational Physics, 26, 1978.

[2] National Crash Analysis Center, George Washington University: `http://www.ncac.gwu.edu`.

[3] SALTELLI, A., K. CHAN, and E. M. SCOTT (EDS.): *Sensitivity analysis*. John Wiley and Sons, New York, 2000.

[4] SALTELLI, A., M. RATTO, T. ANDRES, F. CAMPOLONGO, J. CARIBONI, D. GATELLI, M. SAISANA, and S. TARANTOLA: *Global Sensitivity Analysis – The Primer*. John Wiley and Sons, 2008.

[5] STANDER, N., W. ROUX, T. EGGLESTON, T. GOEL, and K. CRAIG: *LS-OPT User's Manual – A Design Optimization and Probabilistic Analysis Tool for the Engineering Analyst*. Livermore Software Technology Corporation, Livermore, 2007.

[6] SUDRET, B.: *Global sensitivity analysis using polynomial chaos expansions*. Reliability Engineering and System Safety, 93(7):964–979, 2008.

[7] WITOWSKI, K., M. THIELE, G. SEER, W. MÜHLHUBER, and M. VAN DEN HOVE: *New features in d-spex with applications*. In *Proceedings of the 6th German LS-DYNA Forum*, Frankenthal, 2007.