

Finite element modeling of the ITER superconducting cables mechanical behaviour using LS-DYNA code

A.S. Nemov¹, A.I. Borovkov¹, B.A. Schrefler²

¹Computational Mechanics Laboratory (CompMechLab), St.Petersburg State Polytechnical University, St.Petersburg, Russia

²Department of Structural and Transportation Engineering, University of Padua, Via Marzolo 9, 35131 Padua, Italy

Summary:

Superconducting cables are one of the key technical solutions used for generation of strong magnetic field in modern tokamaks. It is very important for engineers to be able to predict the mechanical deformations of superconducting cables because superconductivity depends on strains, temperature and magnetic field. Superconducting cables for ITER the International Thermonuclear Experimental Reactor [1] currently under construction, have a complex structure that makes any analytical estimations hardly applicable. This paper presents the application of LS-DYNA [2] finite element code to the solution of different mechanical problems for ITER superconducts. Stretching, twisting and transverse compression are considered and results are compared with analytical estimations where possible.

The general view of the superconducting ITER cable is presented in Figure 1.



Figure 1: ITER Toroidal Field Model Coil TFMC Nb₃Sn superconducting cable, courtesy of ENEA-FRASCATI

Keywords:

Superconducting cables, stretching, twisting, transverse compression

1 Introduction

The cable superconductor is made up of six petals wound around a cooling channel. Each petal is made up of 237 strands arranged as $3 \times 3 \times 5 \times 5 + 3 \times 4$. This means that the petal has 4-level hierarchical structure. The first level (or, first macrostrand) is a triplet - 3 strands twisted around each other. Then three triplets are twisted around each other building the second order macrostrand 3×3 . The third level of the petal structure is the macrostrand $3 \times 3 \times 5$ - five second order macrostrands twisted around each other. Finally five macrostrands $3 \times 3 \times 5$ are twisted together and with an added core 3×4 form a petal. The successive stages of the developed macrostrands CAD models are presented in Figure 2. Pro\Engineer software was used for these models development.

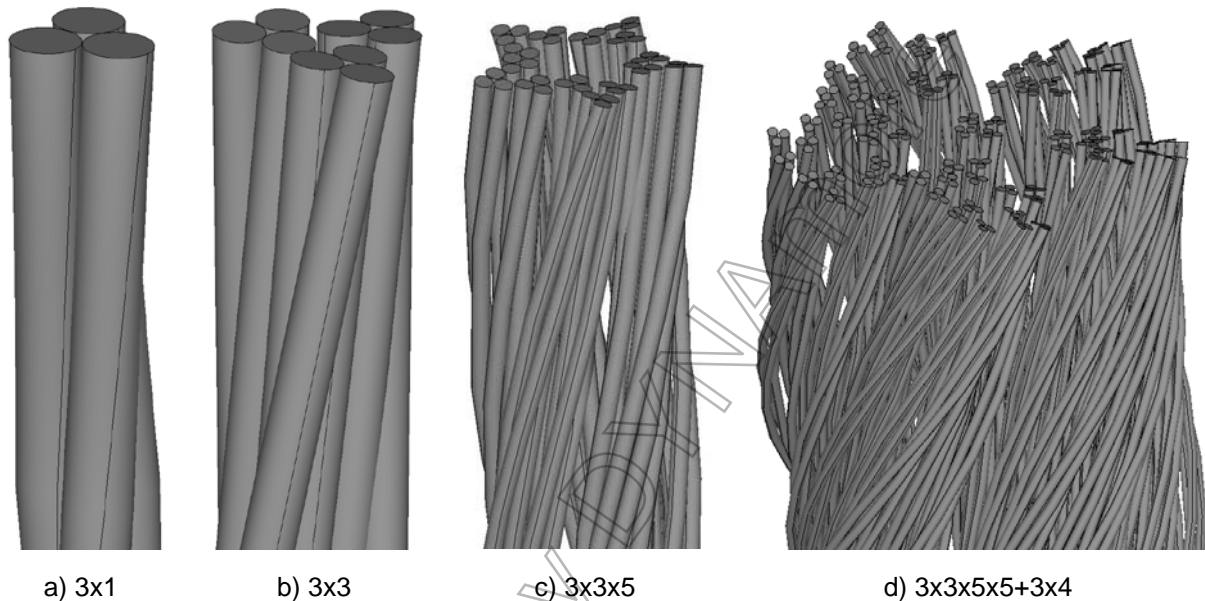


Figure 2: CAD models of macrostrands

From the mechanical point of view possible deformations of cable can be divided in four types: stretching, twisting, bending and transverse compression. The first two types of deformation for cables of relatively simple structure can be described with analytical estimations, that are shown below. However for cables with such complex structure as $3 \times 3 \times 5 \times 5 + 3 \times 4$ even for stretching and twisting analytical estimations become hardly applicable. For bending and transverse pressing analytical estimations can provide only rather rough ones and numerical computations become the only available way for performing such research. Finite element (FE) analysis [3] is the leading numerical method for the analysis of such complex structures as multi-strand cables subjected to complex loadings.

Stretching, twisting and transverse compression of cables on the example of ITER superconducting cable elements are analyzed in the current research. Analytical approach based on Glushko's formulae [4] and numerical approach based on LS-DYNA computations are presented. Despite of the fact that analyzed problems are quasi-static (slow enough) in their nature, they are solved by a dynamic approach and explicit LS-DYNA code is used for this. The reason for this is that using explicit time integration is one of the ways to overcome the convergence difficulties that are usual for solution of problems with complex contact interaction (like multiple contact interaction between strands in a cable) when using implicit time integration which is natural for quasi-static problems.

Every strand of ITER superconducting cable is a complex two level composite structure, where Nb_3Sn filaments are arranged in groups and surrounded by bronze matrix [5] (Figure 3). Since direct modeling of such composite structure in a cable analysis is not feasible, homogenization procedures should be used to obtain effective strand properties and strand effective material should be used in cable analysis. The appropriate two-level homogenization procedure for this problem is described in [5]. As a first approximation in the frame of current research the strand material is assumed to be homogeneous and isotropic.

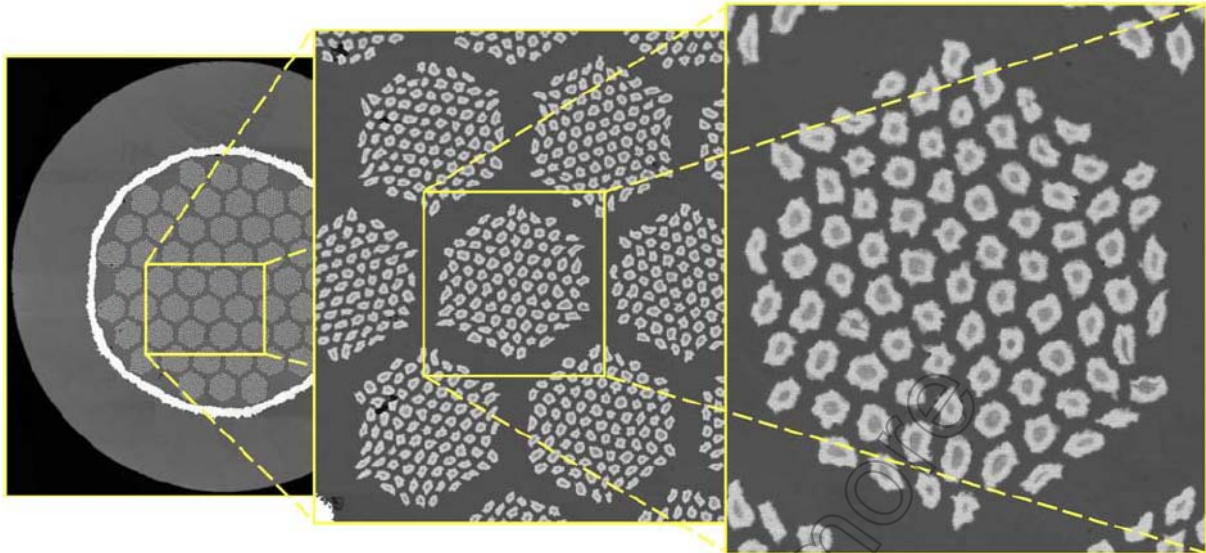


Figure 3: Structure of superconducting cable strand [5]

2 Stretching and twisting of a triplet

According to the principle "from simple to complex" this research starts with triplet analysis. Triplet is the simplest element of superconducting cable after a strand and corresponds to three strands twisted around each other. The considered triplet parameters are: strand diameter $d = 0.81$ mm (all strands are assumed to be identical and have a circular cross section), twisting pitch 45 mm, Young's modulus of strand material $E = 117.7$ GPa, Poisson's ratio $\nu = 0.3$.

The first approach is based on the well-known theory of rope stretching and twisting [4]. According to this theory, linear equations, governing the stretching and twisting of the rope can be written as:

$$\begin{cases} A \cdot \varepsilon + C \cdot \Theta = T \\ C \cdot \varepsilon + B \cdot \Theta = M \end{cases}, \quad (1)$$

where T and M are the applied tensile force and twisting moment correspondingly, ε and Θ the longitudinal and angular rope deformations. A , B and C are the generalized rope stiffness coefficients. A represents the stretching stiffness of the rope, B the twisting stiffness, and C is called the mutual influence coefficient.

Glushko [4] presents formulae for analytical estimation of stiffness coefficients for single and double lay ropes taking into account contact forces between strands. For the triplets these generalized rigidity coefficients can be obtained by the following equations:

$$A = m(EF \cos^3 \alpha + EI \frac{\sin^4 \alpha}{r^2} \cos^3 \alpha + GJ \frac{\sin^6 \alpha}{r^2} \cos^2 \alpha) \quad (2)$$

$$B = m(EF r \cos \alpha \sin^2 \alpha + GJ \cos^2 \alpha + E(1 + \cos^2 \alpha)^2 \sin^2 \alpha \cos \alpha) \quad (3)$$

$$C = m(EFr \cos^2 \alpha \sin \alpha + GJ \frac{\cos^4 \alpha}{r} \sin^3 \alpha - EI(1 + \cos^2 \alpha) \frac{\cos^2 \alpha}{r} \sin^3 \alpha). \quad (4)$$

In the above expressions m is the number of strands, F is the strand cross section area, I is the strand cross section moment of inertia, J the strand cross section polar moment of inertia, α is the twisting angle, r is the helix radius.

For the triplet with parameters listed above eqn. (2) - (4) lead to the following values of the triplet stiffness coefficients: $A = 180.8$ kN, $B = 5.87$ mN·m², $C = 5.5$ N·m.

The described approach allows to make a very quick estimation of the rope "macro" stress-strain state, but has rather strong limitations concerning the range of problems for which it can be

applied (only elastic strand material, only cables with relatively simple structure, only small displacements and deformations e.t.c.). It also does not allow to obtain the strain distribution across the strand cross section and does not take into account the transverse deformation of strands.

The other approach to the cable investigation is the direct finite element analysis. This approach requires significant preparatory work (creation of geometrical and FE models), but has a lot of benefits in comparison with the first one. For example, it allows to take into account elasto-plastic behavior of strand material, contact interaction with friction between the strands, large deformations, any arbitrary loadings and can be applied to cables with any structure. The other significant benefit of this approach is that as a results it provides the detailed stress-strain state of the each strand in the cable. FE model of triplet developed for LS-DYNA FE is presented in Figure 4.

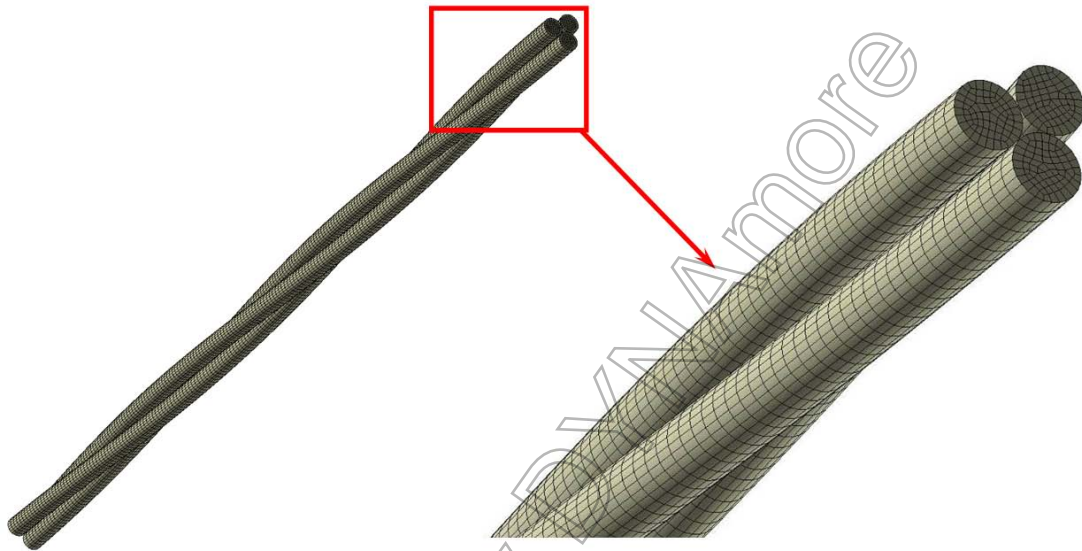


Figure 4: LS-DYNA FE model of a triplet

The presented model consists of 43 014 8-node Solid164 elements. For the purpose of comparison of the two described approaches, three special problems presented in Figure 5 were solved for the triplet. These are the three problems which results allow to obtain the generalized rigidity coefficients of the triplet. These problems are referenced as "three test problems" further in the text.

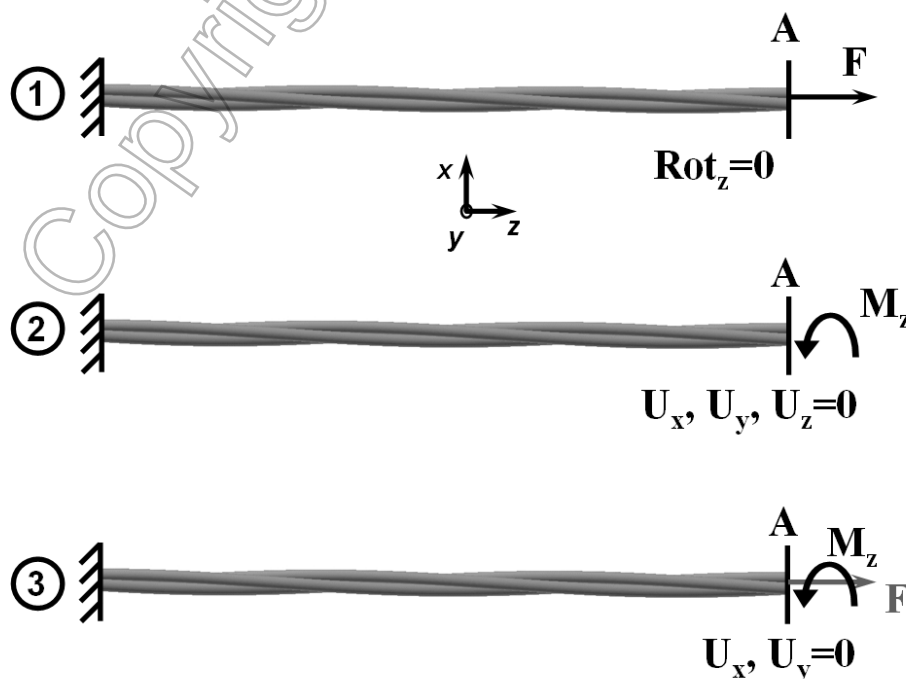


Figure 5: Three problems for obtaining cable stiffness coefficients

The solutions obtained for the above described three problems allow to plot the dependencies of the cable end displacement (twisting angle) on applied force (moment) curves. These dependencies are almost linear for the triplet and the generalized stiffness coefficients can be calculated from them. The stretching stiffness A^{FE} is calculated by two points of the displacement-force curve extracted from the results of the first problem as:

$$A^{FE} = \frac{\Delta F}{\Delta U_z} L = \frac{F^{(2)} - F^{(1)}}{U_z^{(2)} - U_z^{(1)}} L \approx 179.5 \text{ kN} \quad (5)$$

On the basis of the results of the second problem the torsional stiffness B^{FE} is calculated by two points from torsion angle-moment curve:

$$B^{FE} = \frac{\Delta M}{\Delta \theta_z(A)} L = \frac{M^{(2)} - M^{(1)}}{\theta_z^{(2)} - \theta_z^{(1)}} L \approx 5.7 \text{ mN} \cdot \text{m}^2 \quad (6)$$

The influence coefficient C is calculated based on results of third problem using already calculated stiffness coefficients A and B . There are two ways to obtain C : either based on twisting angle-moment curve (7) or based on displacement-force curve (8).

$$C_1^{FE} = \frac{\Delta M - B \Delta \theta_z(A)}{\Delta U_z(A)} L \approx 5.24 \text{ N} \cdot \text{m} \quad (7)$$

$$C_2^{FE} = \frac{\Delta F - A \Delta U_z(A)}{\Delta \theta_z(A)} L \approx 5.4 \text{ N} \cdot \text{m} \quad (8)$$

It is natural to take the average of these two values as a final value of the stiffness coefficient C which is $C^{FE} = 5.32 \text{ N} \cdot \text{m}$.

It can be seen that in spite of the difference in two applied approaches (analytical estimations and FE modeling), the respective triplet stiffness coefficients are rather close: the difference does not exceed 4%.

3 Stretching and twisting of a 3x3 macrostrand

The next level structure in the superconducting cable after triplet is the second order macrostrand 3x3 (Figure 2, b). Analytical estimations of stiffness coefficients for the double lay rope according to [4] are:

$$A = m(A_0 \cos^3 \beta + 2 \frac{C_0}{r} \sin^3 \beta \cos^3 \beta + \frac{B_0 \sin^2 \beta + g_0 \cos^2 \beta}{r^2} \sin^4 \beta \cos \beta) \quad (9)$$

$$B = m(A_0 r_0^2 \sin^2 \beta \cos \beta + 2C_0 r_0 \sin \beta \cos^4 \beta + B_0 \cos^7 \beta + g_0 (1 + \cos^2 \beta)^2 \sin^2 \beta \cos \beta) \quad (10)$$

$$C = m \cdot (A_0 r \cos^2 \beta \sin \beta + B_0 \frac{\cos^4 \beta}{r} \sin^3 \beta - g_0 (1 + \cos^2 \beta) \frac{\cos^2 \beta}{r} \sin^3 \beta + C_0 (1 + tg^4 \beta) \cos^5 \beta) \quad (11),$$

where A_0 , B_0 and C_0 are the generalized stiffness coefficients of the single lay rope (the triplet that was analyzed in the previous section), and g_0 is its bending stiffness. Bending stiffness of triplet g_0 is assumed to be equal to $3EI$ in the framework of this research. For the considered macrostrand 3x3 the stiffness coefficients calculated according to eqn. (9) – (11) have the following values: $A = 538 \text{ kN}$, $B = 21 \text{ mN} \cdot \text{m}^2$, $C = 51 \text{ N} \cdot \text{m}$.

According to the second approach the 3D FE model of macrostrand 3x3 was developed and three test problems were solved for it. The developed FE model is presented in Figure 6.

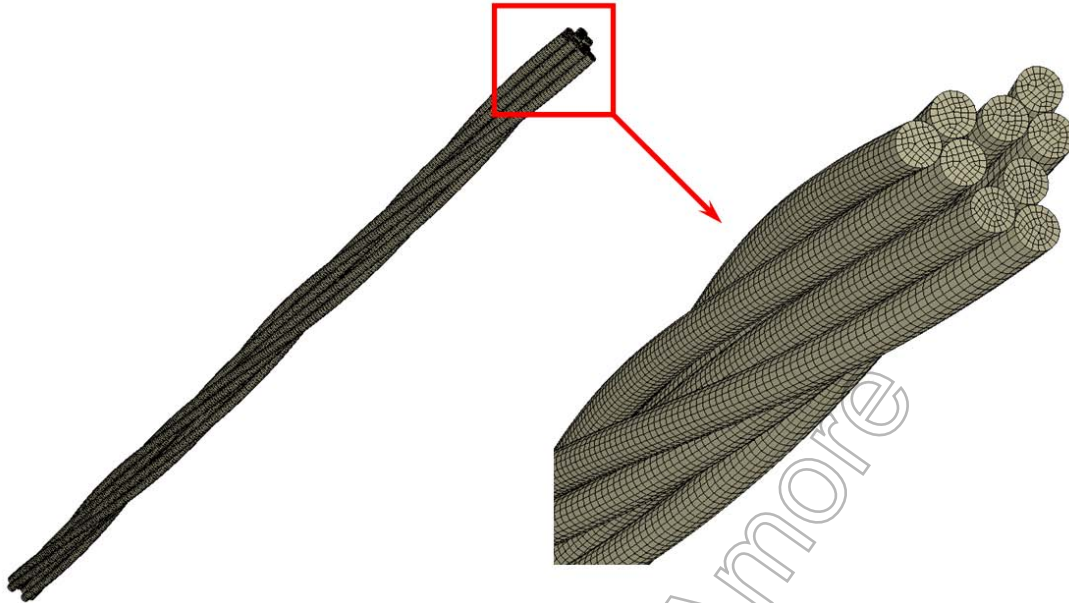


Figure 6: LS-DYNA FE model of 3x3 macrostrand

The model consists of 126 900 8-node Solid164 elements. The following values of stiffness coefficients were calculated: $A^{FE} = 513 \text{ kN}$, $B^{FE} = 20 \text{ mN}\cdot\text{m}^2$, $C^{FE} = 48 \text{ N}\cdot\text{m}$. It can be seen that the difference between FE results and analytical estimations has increased due to the more complex geometry of macrostrand 3x3 in comparison with that of the triplet but does not exceed 6%.

4 Stretching and twisting of higher order macrostrands

The analytical estimations of generalized stiffness coefficients were developed in [4] for single and double lay ropes, and though if necessary they can be also applied to higher order macrostrands, the result may be rather far from reality. For higher order macrostrands (3x3x5 and 3x3x5x5+3x4) direct FE modeling becomes the most appropriate research method. For cables of such complex structure the question of initial strands configuration becomes very important. The geometrical models of 3x3x5 and 3x3x5x5+3x4 macrostrands of Figure 1, c,d represent one of the possible initial configurations. This geometry was built with CAD system ProEngineer. The center lines of all strands are described in it mathematically by considering the cable as a multi-level helix. It can be seen that such algorithm of model creation leads to significant voids inside the macrostrand which strongly affects the mechanical behavior of the cable. The importance of this is shown on the example of a petal (3x3x5x5+3x4 macrostrand).

The developed FE model of 3x3x5x5+3x4 cable is presented in Figure 7.

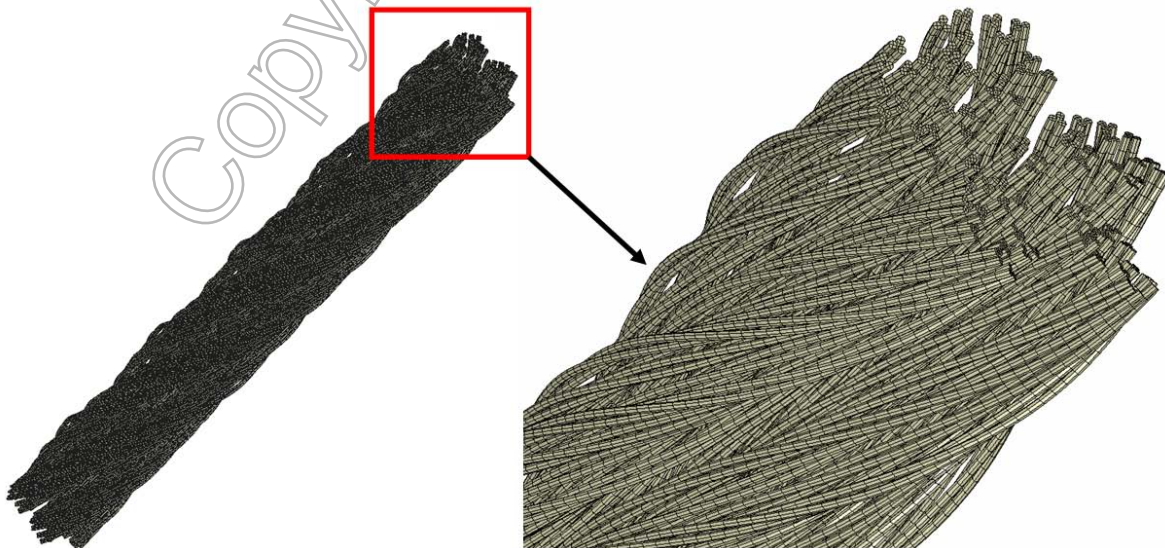


Figure 7: LS-DYNA FE model of a petal

This model consists of 189 600 8-node Solid164 elements and includes totally 237 strands that are in contact interaction with each other.

The evolution of the petal deformed shape during the increase of applied tensile load (Problem #1 from Figure 5) is presented in Figure 8.

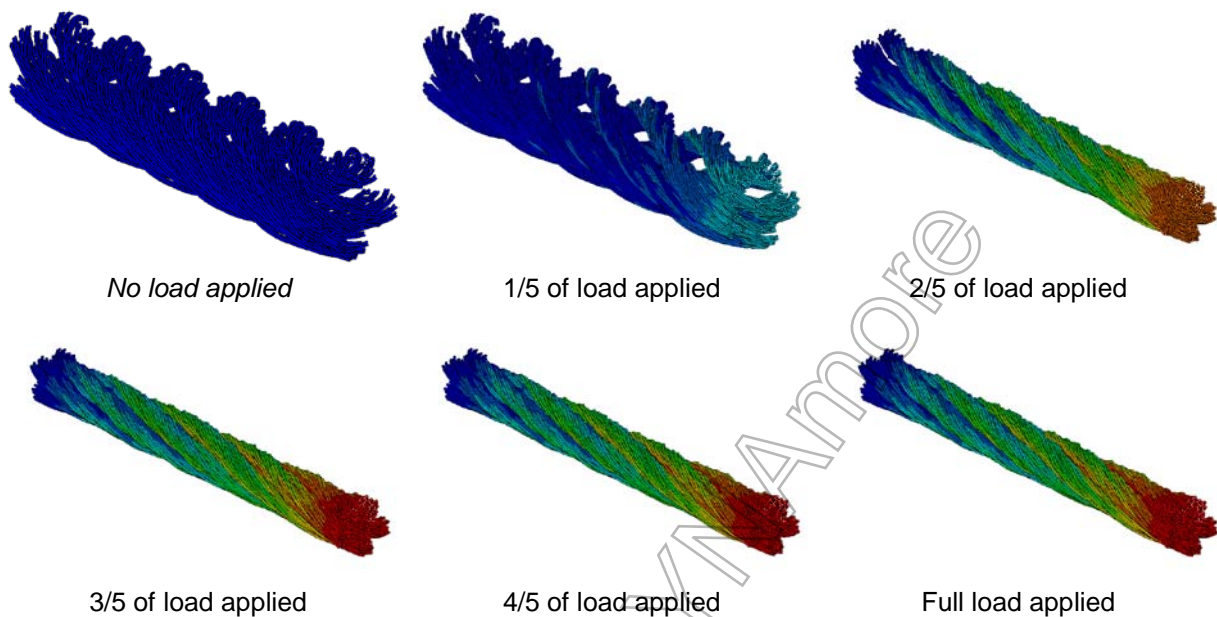


Figure 8: Successive stages of petal deformation under tensile load

The effect of transverse compression of the cable (also called cable reduction) during its stretching can be observed in Figure 8. Such a strong reduction as presented in Figure 8 is caused by the fact that the geometry of the petal created with CAD system has significant voids between strands inside the cable. For low order macrostrands (like triplet or 3x3 macrostrand) these voids are negligibly small due to the simplicity of their geometry. In case of the petal the effect of cable reduction leads to strongly non-linear dependences of the cable end displacement (twisting angle) on applied force (moment) curves. For example, the displacement versus force curve obtained for shown in Figure 7 cable model is presented in Figure 9.

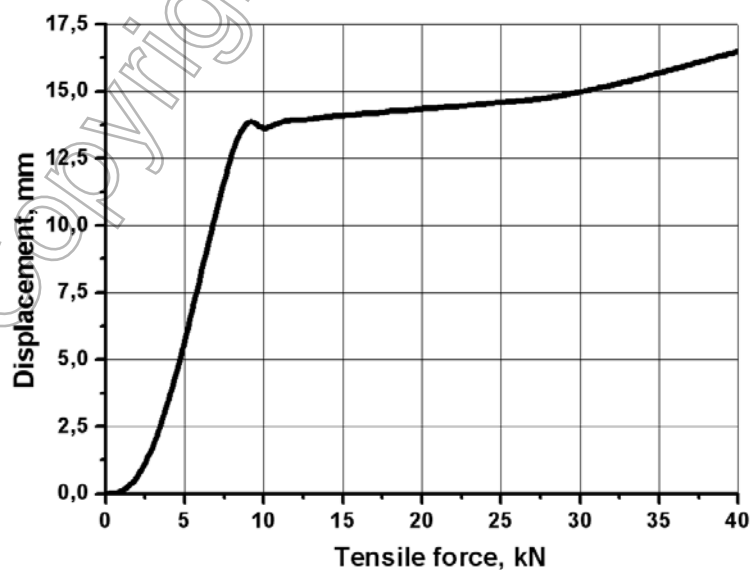


Figure 9: Cable end displacement versus the applied force for 3x3x5x5+3x4 cable

One of the possible ways to develop the FE model of the complex cable without voids inside it is assuming the obtained deformed shape under some tensile load (similar to presented in Figure 8) to be initial configuration.

5 Simulation of cable transverse compression experiment

Besides the problems of stretching and twisting the cables the problem of cable transverse compression is very important because it is the type of deformation where friction between strands plays a great role and heat generation can be significant. The photo of experimental device in the NHMFL (National High Magnetic Field Laboratory) used for transverse compression tests of the whole superconducting cable conducted under contract with US-ITER [6] is shown in Figure 10.



Figure 10: An NHMFL transverse pressure apparatus for mechanical characterization of a variety of ITER-relevant cable designs for the US ITER Magnet Team

In the presented device the cable is placed between two rigid blocks. The bottom block is fixed while the upper is able to move in vertical direction. A force is applied to it and the decreasing gap between two blocks is measured.

The statement of problem for the analogous numerical experiment for the 3x3 macrostrand is presented in Figure 11.

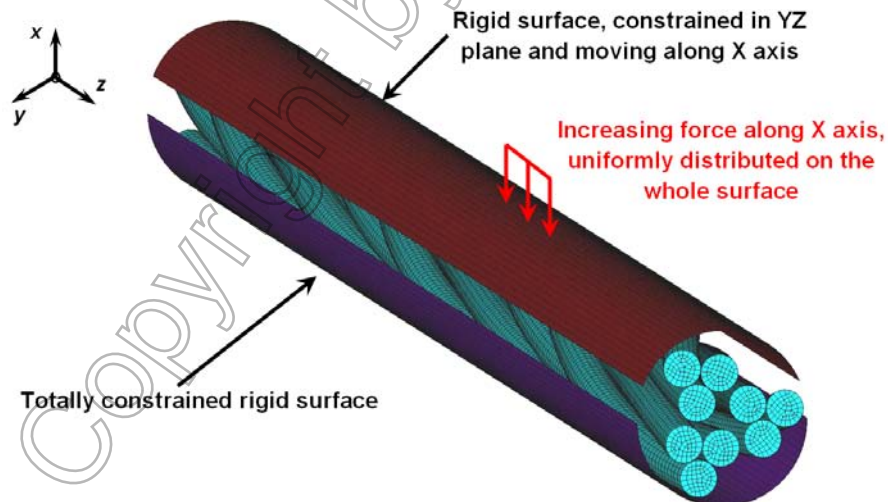


Figure 11: Photo of experimental device for cable transverse pressing

In a FE experiment the 3x3 cable is placed between two rigid surfaces, one of which is constrained and the other is pushed with distributed force. The obtained evolution of the macrostrand deformed shape during the increase of applied transverse load is presented in Figure 12. The color contours correspond to the displacement vector sum.

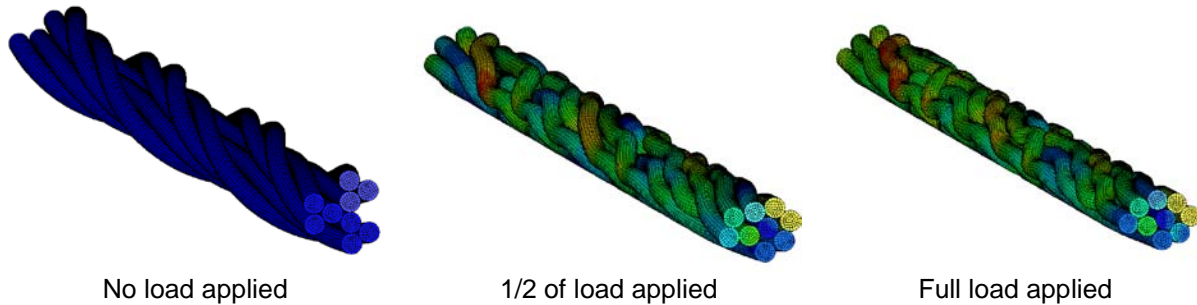


Figure 12: Successive stages of 3x3 macrostrand deformation under transverse load

The results demonstrate that direct FE modeling is can be used for such a complex multi-contact problem as cable transverse pressing.

6 Conclusions

Two approached to the research of superconducting cables were considered. The analytical approach can be useful for fast prediction of global cable behavior for the cables with not very complex structure subjected to relatively simple loads (like tensile load and twisting moment). The other approach - direct FE modeling - is applicable to any cables and any loadings. The only limitation is time required for model development, capabilities of used computers and time required for computations. Stretching, twisting and transverse compression of macrostrands starting from triplet and up to a petal (3x3x5x5+3x5 macrostrand) were investigated. Results of FE modeling are in a good comparison with analytical estimations in cases where the last ones are applicable. LS-DYNA is a suitable code for solving cable analysis problems with high number of strands and complex contact interaction between them.

7 Literature

- [1] Holtkamp N. and for the ITER Project Team: "An overview of the ITER project", Fusion Engineering and Design, 82 (2007), 427-434.
- [2] Hallquist J.O. LS-DYNA. Theoretical Manual. Livermore Software Technology Corporation, Livermore, California, 1998.
- [3] Zienkiewicz O.C., Taylor R.L., Zhu J.Z.: "The Finite Element Method: Its Basis and Fundamentals. Sixth edition", Elsevier Butterworth-Heinemann, Oxford, 2005.
- [4] Glushko M.F. Steel Hoisting Ropes [in Russian]. Technika, Kyiv, 1966.
- [5] Boso D.P., Lefik M., Schrefler B.A.: "A multilevel homogenised model for superconducting strands thermomechanics", Cryogenics, 45,4 (2005), 259-271
- [6] Miller J.R., Martovetski N.N., Hatfield D.R., US ITER Project Office; Walsh R.P., NHMFL: "Options for the design and manufacture of large scale, Nb3Sn-based, cable-in-conduit conductors", , , submitted for presentation at the 2009 CEC/ICMC Conference