

# On Adaptive Finite Element Analysis in Structural Dynamics of Shell-Like Structures – A Specific View on Practical Engineering Applications and Engineering Modelling

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## Summary:

Adaptive Finite element analysis in structural analysis has reached a fairly mature status; however, in practice only fairly little is visible in engineering applications. This contribution gives an overview over some major aspects with a specific focus on structural dynamics of shell-like structures.

Adaptive analysis in structural mechanics is mainly focusing on static problems where a large number of linear and nonlinear tasks in 3D continuum problems as well as in shell problems have been tackled. The developed procedures show really considerable improvements for the executed numerical problems. In statics on one side the competition among the methods is between low and high order approximation – the h- and p- or hp-enhancement and among error estimation between global and local estimation. For a good overview over the subject for a large number of problems it is referred to [11], for some mathematical background see [2]. For some simple benchmark problems the differences between the various approaches as presented in [5].

More recent developments are concerned with time dependent problems thus we are focusing here on structural dynamics [10]. In statics - linear or nonlinear – the spatial error distribution is the dominating quantity whereas in dynamics the consideration of the spatial error distribution over the complete considered time range and as well the consideration of the error due to time integration is needed. Here the consideration of dual problems – known in statics from the Betti-Maxwell principle and extended here with the according reciprocity idea, the Graffi-theorem [1] - allows checking the error in specific quantities at certain points in time, the so-called goal-oriented error computation or local error computation [3] [4]. On the basis of such error estimations, in principle, the adaptive modification of the finite element mesh as well as the time step is possible.

However, while in a semi-discretization approach the time step could be fairly easily adjusted – which is frequently done in the so-called explicit FE programs using the central difference scheme - the modification of the finite element mesh introduces major problems. First the data have to be properly mapped between meshes avoiding non-physical artifacts and second the dual error estimation scheme has to take into account different time steps and meshes. Both actions introduce further errors into the analysis which can hardly be judged. In addition the effort for the numerical analysis concerning the computation as well as the required storage becomes overly large [6] leading to the conclusion that adaptive analysis of real world problems based on dual error estimation cannot be handled - at least with the current computer environment.

Thus the focus of this contribution is on a discussion first on the importance of different parts of the error estimation and on the adaptive procedure and second how the major ingredients of the adaptive duality based analysis for practical engineering problems - restricting to shell problems - can still be used, regaining efficiency [7]. For some classes of shell type problems some simplifications can be suggested while still improving the quality of the analysis considerably by adaptive procedures [8].

In structural dynamics also eigenmodes and eigenvalues are important, thus improvements concerning these are also briefly discussed [9] [8]. Obviously the dominating quantity for achieving good results applying finite element methods in structural mechanics is a consistently refined mesh;

not unexpected for high frequency excitations and interest of the engineers in these almost uniformly refined meshes with high mesh densities are required.

It is shown, how the developed schemes can be applied to homogeneous problems and the limits concerning real world engineering models which include a large number of violations concerning standard continuum mechanics are presented.

Also the procedures implemented in LS-DYNA [12], [13] for adaptive analysis are discussed with the background set above. Further some model adaptivity for large structural computations where some parts are – at least in some early states of the analysis – hardly deforming. This effect can be used to introduce rigid bodies in the analysis; the question then arises, how this can be handled.

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# On Adaptive Finite Element Analyses in Structural Dynamics

A specific view on practical engineering applications and engineering modeling

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## Outline

- ▶ Introduction
  - Adaptivity as supportive tool for practical applications of FEM
  - Requirements for practical applications of adaptive methods
- ▶ Spatially adaptive methods in structural dynamics
  - Goal-oriented h-adaptive schemes for semidiscrete FEM
  - Goal-oriented h-adaptivity for dynamic eigenvalue problem
- ▶ A look on industrial applications of adaptive schemes
  - Crash simulations, Metal forming
  - Open problems in industrial applications
- ▶ Using adaptivity to create/verify engineering models
- ▶ Conclusions

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## Motivation

Adaptive schemes are:

- ▶ well-known in the scientific community
- ▶ highly sophisticated

But:

- ▶ “rarely” used in practical applications **WHY ?**

Adaptive schemes should be (engineering point of view):

- ▶ supportive tools for modeling and discretization purposes

Demands:

- ▶ minimal effort
- ▶ good indication of error in the quantity of interest
- ▶ hints for improvements of the model and discretization
  - e.g. use of different element modifications should be possible
    - assisting to create/verify simplified models
- ▶ provide further insight into the problem

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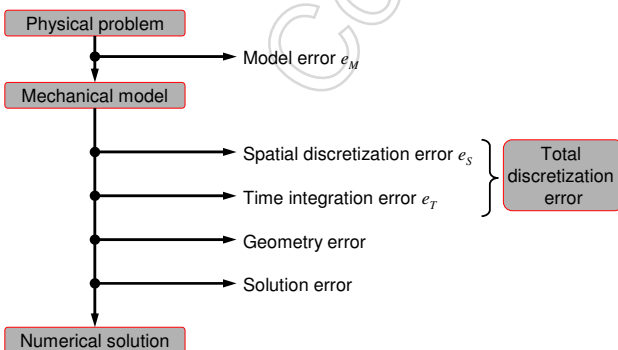
## Steps of simulation process

- ▶ **Modeling**
  - Beams – Shells – 3D-Continua
    - Details: e.g. which beam- or shell-formulation?
  - Constitutive model
  - Joints – w./wo. Friction (*Model adaptivity ?*)
- ▶ **Finite element discretization**
  - Element formulation – Time stepping scheme
  - Mesh density → **Adaptivity**
  - Time step size → **Adaptivity**
- ▶ **Solution**
- ▶ **Verification of solution**
  - Hints from **Adaptivity**

Considered here: Semidiscretization in structural dynamics

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## Errors in semidiscretization in structural dynamics



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## Splitting of total discretization error

$$e = e_S + e_T = \frac{\mathbf{u} - \mathbf{u}_h}{e_S} + \frac{\mathbf{u}_h - \mathbf{u}_{h,k}}{e_T}$$

With:

- $\mathbf{u}$  : exact solution
- $\mathbf{u}_{h,k}$  : numerical solution – applying spatial and temporal discretization
- $\mathbf{u}_h$  : exact solution of semidiscrete equation of motion (no time integration error → not computable)

All other errors (model, geometry, solution) are neglected

Goal:

- ▶ **Estimation and control of spatial discretization error** in an arbitrary quantity of interest

Main focus here:

- ▶ Practical application of adaptive schemes. Possible simplifications ?
- ▶ Provide interpretations of adaptive schemes for engineering purposes

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Adaptive schemes in dynamics

### Modeling and Spatial FE Discretization

**3D continuum**

No specific assumptions for kinematics

Shell theory  $\Rightarrow$  kinematics  $\epsilon_{xx}, \epsilon_{yy} = a \cdot z + b$  (linear in thickness direction)

$\sigma_{zz}=0$   
 $\sigma_{xz}; \sigma_{yz}$  constant  $\Rightarrow$  Reissner / Mindlin theory

Ansatz functions in all directions identical

Ansatz functions only in shell surface directions in z-direction only loading! (no z-stress)

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Adaptive schemes in statics

### FE Discretization and Spatial Error Estimation

Standard:  $C^0$  - elements  $\Rightarrow$  continuous in displacements/rotations  
not continuous in strains / stresses } between elements

$\Rightarrow$  jumps in boundary terms  $\Rightarrow$  boundary error

approximation in domain / in element  $\Rightarrow$  domain error

**Classical error estimation:** ("a posteriori" estimation)

$$\|e_r\| \leq \left[ \sum_{\text{elem}} \|C_{ik} \cdot \gamma_k \cdot R_i\|_{L_2, \Omega}^2 \right] + \left[ \sum_{\text{elem}} \|C_{2k} \cdot \sqrt{h_k} \cdot R_k\|_{L_2, \partial \Omega}^2 \right]$$

**Rem.:** independent of order of approximation

$R_i$  = error of residuum (= satisfaction of DE in element)  
 $R_k$  = stress jump at element borders  
 $h_k$  = characteristic element size  
 $C_k$  = interpolation constant (can be computed from eigenvalue analysis for elements)

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Adaptive schemes in statics

### FE Discretization and Error estimation (contd.)

vectors  $R_i$  can be further subdivided e.g. for shells in normal-, membrane- or transversal shear error

**Ex.:** boundary term stress jump:

Strains at boundary 3-4 e.g. in element n as:

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \left( u = \sum_{i=1}^4 \phi_i(\xi, \eta) u_i \right)$$

in element n+1 as:

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \left( u = \sum_{j=1}^4 \phi_j(\xi, \eta) v_j \right)$$

are **not identical**.

**Reason:** stress boundary terms are also dependent on quantities defined outside of common boundary

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Adaptive schemes in statics

### FE Discretization and Error Estimation (contd.)

**Error estimation according to Zienkiewicz and Zhu:** „Superconvergent Patch Recovery“ 1992 IJNME

$$\|u_h - u_{ex}\|_{L_2, \Omega} = \int_{\Omega} (\sigma_h - \sigma_{ex})^T C^{-1} (\sigma_h - \sigma_{ex}) d\Omega \approx \int_{\Omega} (\sigma_h - \sigma')^T C^{-1} (\sigma_h - \sigma') d\Omega$$

with:  $\sigma' = \sum_{i=1}^n N_i \sigma'_i$  ... improved Finite-Element stresses  
 $\sigma'_i$  ... improved Finite-Element nodal stresses

Approximation of stress components in patch

$$\sigma'_i(x, y) = P(x, y) a_i$$

$P(x, y) = (1, x, y)$  ... smoothing functions  
 $a_i = (a_{i1}, a_{i2}, a_{i3})^T$  ... coefficients

**Rem.:** optimally used for low order test functions!

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Adaptive schemes in statics

### FE discretization and Error Estimation (contd.)

**Error estimation according to Zienkiewicz and Zhu:** „Superconvergent Patch Recovery“ 1992 IJNME

Least square fit to determine  $a_i$ :  $F(a_i) = \sum_{j=1}^n (\sigma'_j(x_j, y_j) - \sigma'_i(x_j, y_j))^2 \rightarrow \min$

with:  $x_j, y_j$  ...coordinates of superconvergent stress points  
 $\sigma'_j$  ...stress components of FE-solution  
 $n$  ...number of stress points in patch

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Adaptive schemes in statics

### FE Discretization and Error Estimation (contd.)

**Linear Statics:**

**Global estimators of strain energy**

**Residual Concept** (including stress jumps/residual terms) = classical concept  
*Babuska/Miller 1987; Johnson/Hansbo 1992*

**Patch - oriented estimators:** *Zienkiewicz/Zhu 1992*

**Nonlinear Statics:** Error estimation at end of converged step

**Rheinboldt 1985:** Working with linearized problem  $\rightarrow$  same error estimator as for linear problems  
**Restriction:** Linearized problem must be close to nonlinear problem  $\rightarrow$  only small stepsize in nonlinear analyses allowed

**Stein et. al. 1993:** Special treatment of stability problems e.g. vicinity of bifurcation points

**Suttmeier/Rannacher 1996/97:** Plasticity - linearized dual problem  $\rightarrow$  very good results and efficient meshes for local quantities

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Adaptive schemes in statics

**Morley Plate**

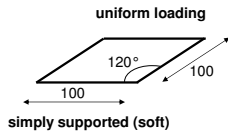
- A-posteriori error estimation
- Starting mesh: 1 finite element

$$E = 10^7$$

$$\nu = 0.3$$

$$p = 1$$

$$t = 1$$

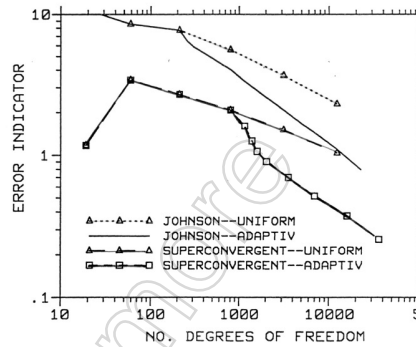


- The following results are obtained:
  - An adaptive analysis is more efficient than an uniform mesh
  - Comparison of error indicators:
    - The stress-jump indicator (Johnson / Hansbo) leads to an upper bound of the error
    - For both indicators the generated meshes are nearly identical

⇒ **Model problem: In boundary layer error is hardly decreasing.**

Adaptive schemes in statics

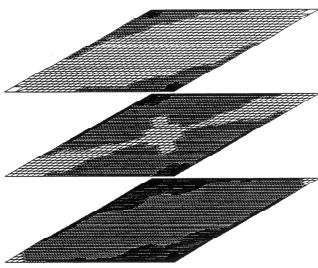
**Morley Plate (contd.) : Convergence Diagram**



- Remarks:
- Global error decreases
  - Local error may be still larger (e.g. boundary layer)

Adaptive schemes in statics

**Morley Plate (contd.)**



10. iteration  
2322 elements

11. iteration  
5316 elements

12. iteration  
11796 elements

Meshes for different iteration states - superconvergent patch error indicator Zienkiewicz / Zhu

Adaptive schemes in dynamics

Estimation of error in a quantity of interest

Main steps:

- ▶ Choose quantity of interest  $Q(u)$
- ▶ Suitable error representation for the error  $E(u, u_h) = Q(u) - Q(u_h)$  applying **Dual-Weighted-Residual- approach (DWR)** see e.g. :
  - Eriksson & Johnson 1991
  - Eriksson, Estep, Hansbo & Johnson 1995
  - Rannacher & Becker 1996
  - Rannacher & Bangerth 2003
  - and others

Basic idea:

- ▶ Residual of equation of motion is weighted with dual solution (influence function)
- ▶ Choose initial conditions of dual problem w.r.t. the quantity of interest
- ▶ Numerical evaluation of error representation

Adaptive schemes in statics

**Global vs. Local Error Estimation**

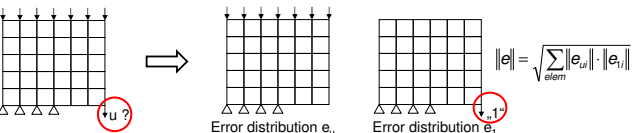
**Global error**

Sum of energy error over all elements / total energy  
⇒ error can differ considerably dependent on location / but global behavior well captured

$$\|e\|_{Glob} = \left( \sum_{elem} \|e_{ui}\|^2 \right)^{1/2}$$

**Local error**

influence of corresponding error on a local quantity e.g. displacement/ force  
Principle : Betti - Maxwell – actuating variable ⇒ **Dual approach**



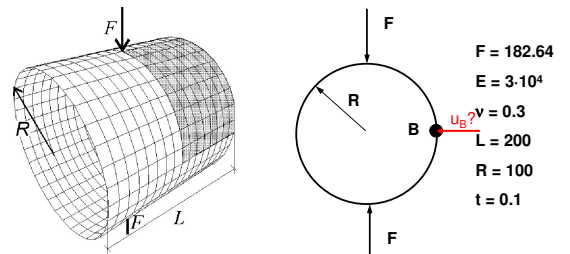
Rem.: Weighting of "global" error with local quantity

Rannacher / Becker / Suttmeier , Heidelberg 1998/99 ; Ramm / Cirak, Stuttgart 1998/99

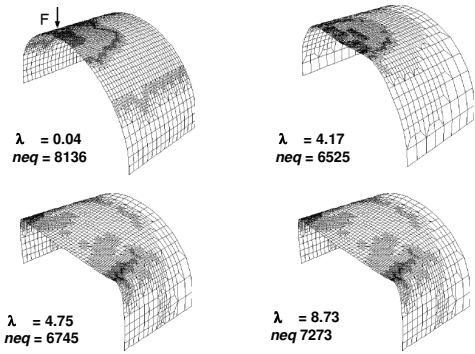
Adaptive schemes in nonlinear statics

**Mesh development for different error estimation**

**Example: Pinched Cylinder**

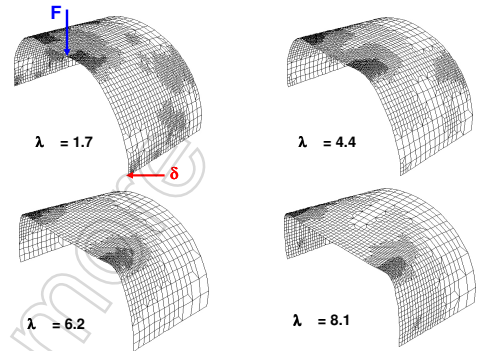


Adaptive schemes in nonlinear statics  
**Mesh development for global error estimation**



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Adaptive schemes in nonlinear statics  
**Mesh adaptation with dual error estimation**

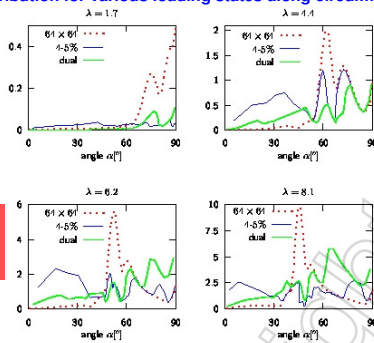


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Adaptive schemes in nonlinear statics  
**Mesh adaptation with dual error estimation (contd.)**

Absolute error distribution for various loading states along circumference

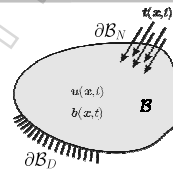
- Comparison of
- uniform mesh 61 x 61
  - global error est. 4-5 %
  - dual error estimation



Dual error estimation  
→ efficient mesh adaptation

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Adaptive schemes in dynamics



Equation of motion

$$\begin{aligned} \operatorname{div} \sigma + \rho b - \rho \dot{u} &= 0 & \text{on } B & 0 \leq t \leq T \\ \sigma n &= t & \text{on } \partial B_N & 0 \leq t \leq T \\ u &= u_D & \text{on } \partial B_D & 0 \leq t \leq T \\ u(t=0) &= u_0 & & t=0 \\ \dot{u}(t=0) &= \dot{u}_0 & & t=0 \end{aligned}$$

Weak form of equation of motion (with damping)

$$\rho(\ddot{u}, w) + c_M \rho(\dot{u}, w) + c_K a(\dot{u}, w) + a(u, w) = \mathcal{F}_u(w) \quad \forall w \in W, 0 \leq t \leq T$$

With:  $(\ddot{u}, w) = \int_B \ddot{u} \cdot w \, dV$

$$a(u, w) = \int_B \sigma : \operatorname{grad} w \, dV = \int_B \sigma : \partial \varepsilon(w) \, dV$$

$$\mathcal{F}_u(w) = \int_B \rho b \cdot w \, dV + \int_{\partial B_N} t \cdot w \, dA$$

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Adaptive schemes in dynamics  
**Physical interpretation of weak formulation**

Interpretation of the testfunction  $w$

as

virtual displacement

$$w = \delta u$$

virtual velocity

$$w = \delta \dot{u}$$

yields

Principle of virtual work

$$\delta \Pi = 0$$

Principle of virtual power

$$\delta P = 0$$

Both principles yield different reciprocity theorems

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Adaptive schemes in dynamics

**Spatial discretization**

Assumption: Only spatial discretization → no time integration error

$$\rho(\ddot{u}_h, w_h) + c_M \rho(\dot{u}_h, w_h) + c_K a(\dot{u}_h, w_h) + a(u_h, w_h) = \mathcal{F}_u(w_h) \quad \forall w_h \in W^h, 0 \leq t \leq T$$

Residual

$$\mathcal{R}_u(w) = \mathcal{F}_u(w) - (\rho_0(\ddot{e}_s, w) + c_M \rho(\dot{e}_s, w) + c_K a(\dot{e}_s, w) + a(e_s, w))$$

$$\mathcal{R}_u(w) = \rho_0(\ddot{e}_s, w) + c_M \rho(\dot{e}_s, w) + c_K a(\dot{e}_s, w) + a(e_s, w) \quad \forall w \in W, 0 \leq t \leq T$$

Differential equation of discretization error

Galerkin orthogonality:  $\mathcal{R}_u(w_h) = 0 \quad \forall w_h \in W^h, 0 \leq t \leq T$

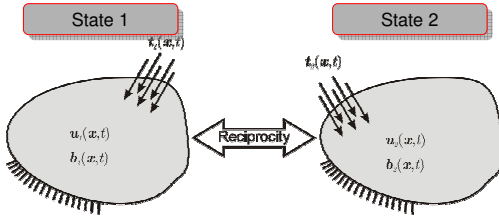
Summary:

- ▶ Spatial discretization error is a solution of the equation of motion with residual as loading function
- ▶ Spatial discretization error is an elastodynamic state for which we can apply elastodynamic reciprocity theorems

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Reciprocity in elastodynamics

„Reciprocity is a good thing. Something is given and something else, equally or more valuable, is returned.“ [Achenbach, 2003]



- ▶ Reciprocity theorems relate two admissible elastodynamic states of an elastic body
- ▶ Two reciprocity theorems formulated in the time domain based on:
  - Principle of virtual work (Graffi-Theorem)
  - Principle of virtual power

Reciprocity in elastodynamics (2)

Reciprocity in the time domain based on principle of virtual work

**Graffi (1946):** Given two elastodynamic states  $u_1(x,t)$  and  $u_2(x,t)$  of an elastic body, for each time  $t_n$  the following reciprocity theorem h

$$\int_{\partial S} \mathbf{t}_1(\mathbf{x}, t_n) * \mathbf{u}_2(\mathbf{x}, t_n) dA + \int_S \rho \mathbf{b}_1(\mathbf{x}, t_n) * \mathbf{u}_2(\mathbf{x}, t_n) dV + \int_S \rho \mathbf{u}_1(\mathbf{x}, 0) \dot{\mathbf{u}}_2(\mathbf{x}, t_n) + \rho \dot{\mathbf{u}}_1(\mathbf{x}, 0) \mathbf{u}_2(\mathbf{x}, t_n) dV = \int_{\partial S} \mathbf{t}_2(\mathbf{x}, t_n) * \mathbf{u}_1(\mathbf{x}, t_n) dA + \int_S \rho \mathbf{b}_2(\mathbf{x}, t_n) * \mathbf{u}_1(\mathbf{x}, t_n) dV + \int_S \rho \mathbf{u}_2(\mathbf{x}, 0) \dot{\mathbf{u}}_1(\mathbf{x}, t_n) + \rho \dot{\mathbf{u}}_2(\mathbf{x}, 0) \mathbf{u}_1(\mathbf{x}, t_n) dV \quad \forall t_n > 0$$

With the convolution:  $\mathbf{a}(t_n) * \mathbf{b}(t_n) = \int_0^{t_n} \mathbf{a}(\tau) \cdot \mathbf{b}(t_n - \tau) d\tau$

**Note:** No damping for brevity reasons – damping can be added fairly simply

Reciprocity in elastodynamics (3)

Reciprocity in the time domain based on principle of virtual power

Given two elastodynamic states  $u_1(x,t), v_1(x,t)$  and  $u_2(x,t), v_2(x,t)$  of an elastic body, for each time  $t_n$  the following reciprocity theorem holds:

$$\int_{\partial S} \mathbf{t}_1(\mathbf{x}, t_n) * \mathbf{v}_2(\mathbf{x}, t_n) dA + \int_S \rho \mathbf{b}_1(\mathbf{x}, t_n) * \mathbf{v}_2(\mathbf{x}, t_n) dV + \int_S \rho \mathbf{v}_1(\mathbf{x}, 0) \mathbf{v}_2(\mathbf{x}, t_n) + \sigma_1(\mathbf{x}, 0) : \epsilon_2(\mathbf{x}, t_n) dV = \int_{\partial S} \mathbf{t}_2(\mathbf{x}, t_n) * \mathbf{v}_1(\mathbf{x}, t_n) dA + \int_S \rho \mathbf{b}_2(\mathbf{x}, t_n) * \mathbf{v}_1(\mathbf{x}, t_n) dV + \int_S \rho \mathbf{v}_2(\mathbf{x}, 0) \mathbf{v}_1(\mathbf{x}, t_n) + \sigma_2(\mathbf{x}, 0) : \epsilon_1(\mathbf{x}, t_n) dV \quad \forall t_n > 0$$

**Note:** Theorem can be derived in same fashion as Graffi-Theorem

Reciprocity theorems are independent of the discretization

Reciprocity and error estimation

General idea:

- ▶ State 1: spatial discretization error  $e_s \rightarrow$  Residual is loading function
- ▶ State 2: auxiliary (dual) problem with solution  $z \rightarrow$  homogeneous problem

Two identities:

1. Based on principle of **virtual displacement** – displacement  $z$  of dual problem as weighting function of residual – via Graffi-theorem:

$$[\rho(\dot{e}_s, z) - \rho(e_s, \dot{z}) + c_M \rho(e_s, z) + c_K a(e_s, z)]_0 + \int_0^{t_n} \mathcal{R}_u(z) dt = [\rho(z, \dot{e}_s) - \rho(\dot{z}, e_s) + c_M \rho(z, e_s) + c_K a(z, e_s)]^{t_n}$$

Used for definition of initial conditions of dual problem

2. Based on principle of **virtual velocities** – velocities  $\dot{z}$  of dual problem as weighting function of residual:

$$[\rho(\dot{e}_s, \dot{z}) + a(e_s, z)]_0 + \int_0^{t_n} \mathcal{R}_u(\dot{z}) dt = [\rho(\dot{z}, \dot{e}_s) + a(z, e_s)]^{t_n}$$

Used for definition of initial conditions of dual problem

Weak formulation of dual problem

From definition of convolution:

- ▶ **Backward problem in time**

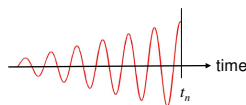
$$\rho(\dot{z}, w) - c_M \rho(\dot{z}, w) - c_K a(\dot{z}, w) + a(z, w) = 0 \quad \forall w \in W, 0 \leq t \leq t_n$$

- ▶ **Initial conditions at “end” time  $t_n$**

Features of dual problem:

- ▶ Dual problem describes the spatial and temporal transport
- ▶ Negative damping terms: Only limited temporal transport for damped problems („loss of memory“):

Temporal evolution of dual solution for damped problem:



Quantity of interest

Target functional:  $Q(\mathbf{u})^{t_n}$

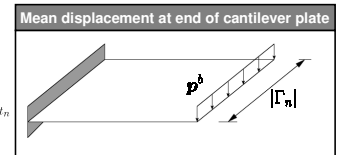
Error measure:  $E(\mathbf{u}, \mathbf{u}_h)^{t_n} = Q(\mathbf{u})^{t_n} - Q(\mathbf{u}_h)^{t_n}$

Example: Mean error of the displacements in subdomain at time  $t_n$

$$Q(\mathbf{u})^{t_n} = \frac{1}{|\Gamma_n|} \int_{\Gamma_n} \mathbf{p}^b \cdot \mathbf{u} ds^{t_n}$$

with  $|\mathbf{p}^b| = 1$

$$E(\mathbf{u}, \mathbf{u}_h)^{t_n} = \frac{1}{|\Gamma_n|} \int_{\Gamma_n} \mathbf{p}^b \cdot \frac{(\mathbf{u} - \mathbf{u}_h)}{e_s} ds^{t_n}$$



Initial conditions of dual problem define the quantity of interest

Adaptive schemes in dynamics  
Error representations for the quantity of interest

**Error representation based on principle of virtual work**

Initial conditions of dual problem are chosen such that:

$$E(\mathbf{u}, \mathbf{u}_h)|^{t_n} = \rho(\mathbf{z}, \dot{\mathbf{e}}_S)|^{t_n} - \rho(\dot{\mathbf{z}}, \mathbf{e}_S)|^{t_n} + c_M \rho(\mathbf{z}, \mathbf{e}_S)|^{t_n} + c_K a(\mathbf{z}, \mathbf{e}_S)|^{t_n}$$

Example: Mean error of the displacements in subdomain at time  $t_n$

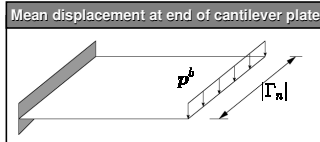
Error measure: 
$$E(\mathbf{u}, \mathbf{u}_h)|^{t_n} = \frac{1}{|\Gamma_n|} \int_{\Gamma_n} \mathbf{p}^b \cdot \mathbf{e}_S d\mathbf{s}|^{t_n}$$

Initial conditions

$\mathbf{z}(t_n) = \mathbf{0}$  (chosen)  
$$-\rho_0(\dot{\mathbf{z}}, \mathbf{e}_S)|^{t_n} = \frac{1}{|\Gamma_n|} \int_{\Gamma_n} \mathbf{p}^b \cdot \mathbf{e}_S d\mathbf{s}|^{t_n}$$

Impulse load at time  $t_n \rightarrow \dot{\mathbf{z}}(t_n)$

→ Backward analysis



Adaptive schemes in dynamics  
Error representations for the quantity of interest (2)

**Error representation based on principle of virtual power**

Initial conditions of dual problem are chosen such that:

$$E(\mathbf{u}, \mathbf{u}_h)|^{t_n} = \rho(\dot{\mathbf{e}}_S, \dot{\mathbf{z}})|^{t_n} + a(\mathbf{e}_S, \mathbf{z})|^{t_n}$$

Example: Mean error of the displacements in subdomain at time  $t_n$

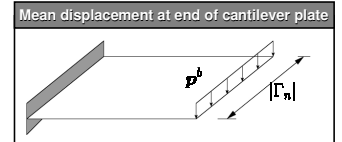
Error measure: 
$$E(\mathbf{u}, \mathbf{u}_h)|^{t_n} = \frac{1}{|\Gamma_n|} \int_{\Gamma_n} \mathbf{p}^b \cdot \mathbf{e}_S d\mathbf{s}|^{t_n}$$

Initial conditions

$\dot{\mathbf{z}}(t_n) = \mathbf{0}$  (chosen)  
$$a(\mathbf{z}, \mathbf{e}_S)|^{t_n} = \frac{1}{|\Gamma_n|} \int_{\Gamma_n} \mathbf{p}^b \cdot \mathbf{e}_S d\mathbf{s}|^{t_n}$$

Quasi-static problem at time  $t_n \rightarrow \mathbf{z}(t_n)$

→ Backward analysis



Adaptive schemes in dynamics  
Classification of solutions of equation of motion

Equation of motion

Depending on: Loading, boundary conditions etc.

Wave propagation

- ▶ Examples: Impact, Shockwaves
- ▶ Short time dynamics
- ▶ High frequencies
- ▶ Transport of energy is dominant
- ▶ Discretization must capture the transport

Vibration

- ▶ Example: harmonic oscillation
- ▶ Long time dynamics
- ▶ Lower frequencies
- ▶ Transport of energy is of minor interest
- ▶ Classic solution: Modal superposition

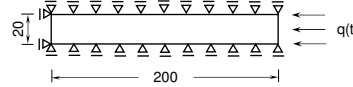
Remark: Adaptive scheme should take into account

physical nature of the considered problem → **problem oriented simplification ?**

Adaptive schemes in dynamics  
Example: Adaptive computation of wave propagation

**Thin rectangular slab subjected to half-sinusoidal impulse**

Problem



Adaptive scheme

- ▶ Error estimation with standard energy norm estimators (global)
- ▶ Hierarchical mesh refinement and coarsening

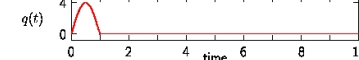
Material:  $\rho = 10.0, E = 10000.0, \nu = 0.3, t = 1.0$

Error bounds:  $\epsilon_{local} = 0.0, \epsilon_{upper} = 3\%$

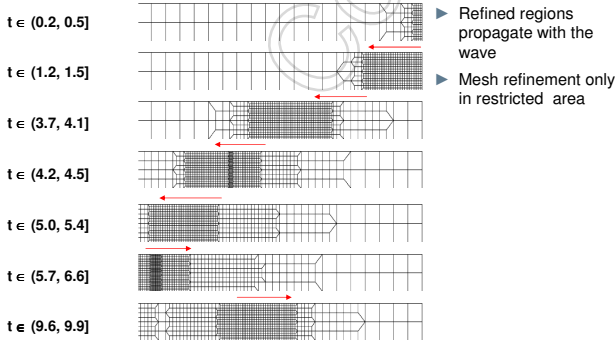
$\epsilon_{prcns} = 2.5\%$

From pre-simulation:  $\|\mathbf{u}\|_{max, prcns} = 0.89$

Load - time history:



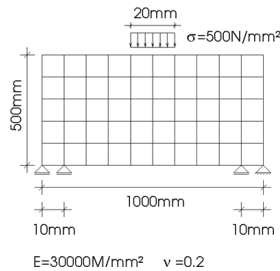
Adaptive schemes in dynamics  
Wave propagation: FE-meshes at different time intervals



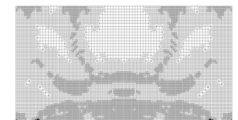
Adaptive schemes in dynamics  
Example: Plane pressure wave in concrete specimen

**Simulation of an explosive charge on a concrete specimen**

Problem



Mesh evolution



Time span: 0 sec. - 0.002 sec



Practical problems of wave propagation simulations

- ▶ Dispersion due to spatial discretization
  - ▶ Reflections at borders of fine to coarse meshes
  - ▶ Error due to transfer between different meshes
  - ▶ Demands for transfer of data:
    - Global energy preservation?
    - Preservation of local behavior of wave
  - ▶ Complex geometries → often fairly uniform meshes needed
- But: reduction of effort with DRW approach (Bangerth & Rannacher 2001)

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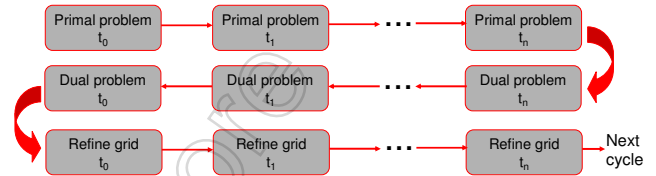
Adaptive schemes in dynamics

Goal-oriented adaptivity for wave propagation problems

See: Rannacher/Bangerth (1999), Fuentes, Oden, Littlefield, Prudhomme(2005)

- ▶ Temporal evolution of error is of high practical interest
  - Full dual problem is needed

▶ Adaptive scheme [from Rannacher/Bangerth (1999)]:



Advantage:

- ▶ Complete information of dual problem used for adaptation
- Full error information

Drawbacks:

- ▶ Very high numerical effort
- ▶ Large memory requirements
- ▶ Transfer/estimation with different refined meshes ? 38

Adaptive schemes in dynamics

Goal oriented error estimation for vibration problems

Modal decomposition of spatial discretization error

Exact solution:  $u(x, t) = \sum_{i=1}^{\infty} U_i(x) \cdot f_i(t)$

Discrete solution:  $u_n(x, t) = \sum_{i=1}^{n_{dof}} U_i^h(x) \cdot f_i^h(t)$

Discretization error:  $e_S(x, t) = \sum_{i=1}^{n_{dof}} \left( \frac{(U_i - U_i^h) \cdot f_i^h(t) + U_i \cdot (f_i(t) - f_i^h(t))}{E_i(x)} \right) + h.o.t.$

Spatial discretization error consists of:

- ▶ Error in spatial approximation of natural modes  $\sum_{i=0}^{n_{dof}} (E_i(x) \cdot f_i^h(t))$
- ▶ Error in approximation of natural frequencies - phase error  $\sum_{i=0}^{n_{dof}} (U_i \cdot e_i^p(t))$
- ▶ Truncation of higher modes (h.o.t.) → usually negligible

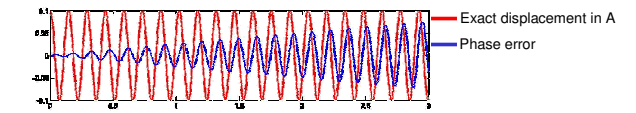
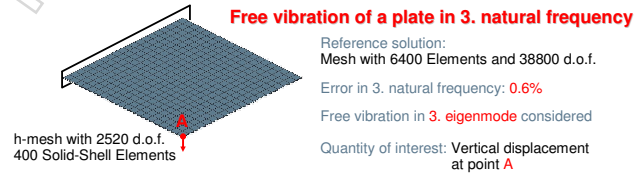
Question: Which part of the error could be neglected for practical purposes?

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Adaptive schemes in dynamics

Example for phase error due to spatial discretization

Free vibration of a plate in 3. natural frequency



Note: Phase error can have order of magnitude of the exact displacement  
 For practical purposes: Exact temporal solution of minor interest  
 More important: Maximum displacements, stresses etc.

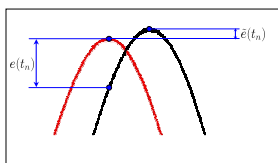
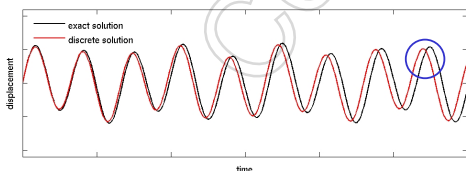
Proposition: Phase error should be neglected

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Adaptive schemes in dynamics

Error measure without phase error

Characteristic evolution of exact and discrete solution for vibration problem



$e(t_n)$ : total error at time  $t_n$   
 comparison of different states in discrete and exact solution ( $\dot{u}_h = 0$  but  $\dot{u} \neq 0$ )  
 → estimation with full backward problem

$\tilde{e}(t_n)$ : comparison of same states in discrete and exact solution ( $\dot{u}_h = 0$  and  $\dot{u} = 0$ )  
 → estimation with state variables at  $t_n$

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Adaptive schemes in dynamics

Consequences of neglecting phase error

Modified error measure:  $\tilde{e}_S(x, t) = \sum_{i=1}^{n_{dof}} E_i(x) \cdot f_i^h(t)$

- ▶ No phase shift between exact and numerical solution  $f_i^h(t) = f_i(t)$
- ▶ Temporal evolution of spatial discretization error assumed to be known
- ▶ Current state is „accepted“ → only state variables at current time necessary for error estimation

▶ Proposed approach: Use terms at end time  $t_n$  for error estimation

- Only „spatial information“ of dual problem is necessary
- Error representation based on velocities of dual problem is used

Error representation:  $E(u, u_h) = [\rho_0(e_S, \dot{z}) + a(e_S, z)]^{t_n}$

▶ Goal: Deriving simple error indicator to judge the current mesh w.r.t. quantity  $Q(u)$

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A simple error indicator

Subtracting Galerkin orthogonality & Application of Cauchy-Schwarz inequality yields:

$$|E(\mathbf{u}, \mathbf{u}_h)|^n \leq [\rho] \|\dot{\mathbf{e}}_S\|_{L_2} \cdot \|\dot{\mathbf{e}}_Z\|_{L_2} + \|\mathbf{e}_S\|_a \cdot \|\mathbf{e}_Z\|_a$$

Estimation of energynorm  $\|\cdot\|_a$  by Zienkiewicz-Zhu error indicator (1992)

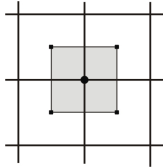
$$\|\mathbf{e}_S\|_a = \int_B (\boldsymbol{\sigma}^* - \boldsymbol{\sigma}_h) : \mathbb{C}^{-1} : (\boldsymbol{\sigma}^* - \boldsymbol{\sigma}_h) dV$$

Basic idea:

- ▶ Postprocessing of stresses with
- ▶ Superconvergent patch recovery (SPR)

Advantages:

- ▶ Implementation simple
- ▶ Simple to use with different element formulations (only stresses are needed for error estimation)



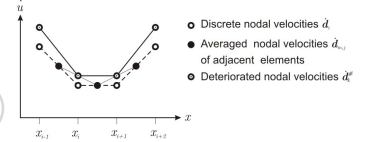
A simple error indicator (2)

Estimation of L2-norm  $\|\cdot\|_{L_2}$ : Estimator with deteriorated velocities (Riccius/Schweizerhof [1998])

$$\|\dot{\mathbf{e}}_S\|_{L_2}^2 \approx c^2 \|\dot{\mathbf{u}}^{\#} - \dot{\mathbf{u}}_h\|_{L_2}^2 = c^2 \int_B (\dot{\mathbf{u}}^{\#} - \dot{\mathbf{u}}_h)^2 dV$$

with  $c = 0.365$

Illustration for 1-D-problem:



Basic ideas:

- ▶ No superconvergent points for velocities
- ▶ Generate field of deteriorated velocities (= velocities on coarser mesh → factor c from similar problems)

Advantage:

- ▶ Numerical effort fairly small

A simple error indicator (3)

Example for error indicator: Estimation of vertical displacement at particular point A

Quasi-static dual problem

1. Error estimation for primal problem:  $\|\mathbf{e}_S\|_a$
2. Solve discrete dual problem:  $\mathbf{K} d_Z = \mathbf{F}_Z \rightarrow z_h$
3. Error estimation for dual problem:  $\|\mathbf{e}_Z\|_a$
4. Multiplication:  $|E_{est}|^n = \|\mathbf{e}_S\|_a \cdot \|\mathbf{e}_Z\|_a$

Advantages of error estimation by simplified approach:

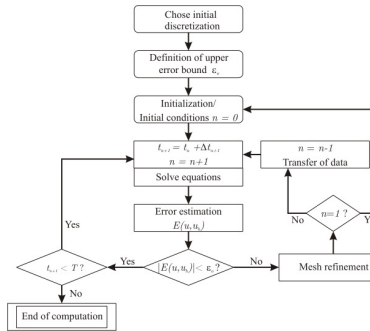
- ▶ No backward integration of dual problem
- ▶ Dual problem must only be computed once and if mesh is changed
- ▶ Suitable basis for mesh adaptation at current time  $t_n$
- ▶ Efficient use of important features of the dual problem

Main disadvantage:

- ▶ No control of total discretization error

Goal-oriented h-adaptive schemes

Strategy I: Single pass with sequential mesh adaptation and transfer of data



Advantages:

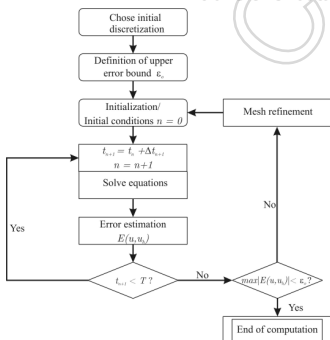
- ▶ Only one computation of whole problem necessary

Shortcomings:

- ▶ Additional error due to transfer of data
- ▶ Resulting mesh only reflects the states at times of mesh modifications

Goal-oriented h-adaptive schemes (2)

Strategy II: Multi pass with mesh adaptation at final step without transfer of data



Advantages:

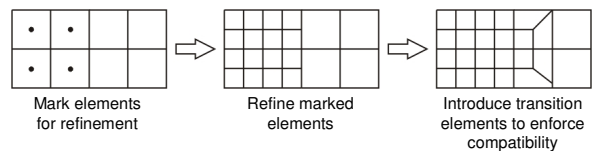
- ▶ Error at states of all times captured for mesh adaptation
- ▶ No transfer error
- ▶ More robust than strategy I
- ▶ Very attractive for highly nonlinear analysis and robust

Shortcoming:

- ▶ Several re-computations of whole problem necessary

Mesh refinement scheme for bilinear shell elements

Hierarchical mesh refinement with transition elements



Advantage:

- ▶ Only small difference in subsequent meshes
- ▶ Clear hierarchy of meshes → transfer of data very simple (coarse to fine)

Disadvantage:

- ▶ Transition elements → badly shaped elements

Adaptive schemes in dynamics

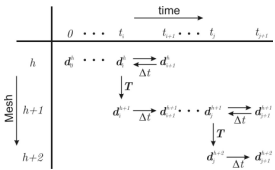
Consistent transfer procedure (Ortiz/Radovitzky 1999)

For Newmark scheme:  $\mathbf{d}_{n+1} = \mathbf{T}\mathbf{d}_n + \Delta t \mathbf{T}\dot{\mathbf{d}}_n + \Delta t^2 \left[ \left(\frac{1}{2} - \beta\right)\mathbf{T}\ddot{\mathbf{d}}_n + \beta\ddot{\mathbf{d}}_{n+1} \right]$   
 (Note: CG version)  $\dot{\mathbf{d}}_{n+1} = \mathbf{T}\dot{\mathbf{d}}_n + \Delta t \left[ (1 - \gamma)\mathbf{T}\ddot{\mathbf{d}}_n + \gamma\ddot{\mathbf{d}}_{n+1} \right]$

Transfer operator:  $\mathbf{T} = \mathbf{M}_{n+1}^{-1} \cdot \int_{\mathcal{B}} \rho_0 \mathbf{N}_{n+1}^T \cdot \mathbf{N}_n dV$

For hierarchical refinement:  $\mathbf{N}_n = \mathbf{N}_{n+1} \cdot \mathbf{C}$   $\mathbf{C} = \text{Constraint matrix}$

$\mathbf{T} = \mathbf{M}_{n+1}^{-1} \cdot \int_{\mathcal{B}_0} \rho_0 \mathbf{N}_{n+1}^T \cdot \mathbf{N}_{n+1} dV \cdot \mathbf{C} = \mathbf{C}$



Steps of transfer procedure:

- Geom. interpolation of solution at  $t_n$
- Compute solution at  $t_{n+1}$  with time integration scheme

Adaptive schemes in dynamics

Consistent transfer procedure (2)

Some demands for general transfer operators (Rashid 2002):

- Transfer should generate sufficiently smooth solution
- Transfer should pertain local character of solution (e.g. wave propagation)
- Self consistency
- Global energy preservation vs. Local momentum preservation?
- Application possible also to modified element formulations:
  - Assumed strain formulations
  - Enhanced strains
  - Underintegrated elements

Practical questions:

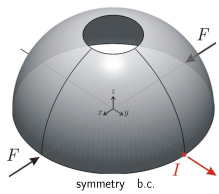
- Error due to transfer?
- Which features of transfer operator are important for the problem at hand ?

Remark: For practical purposes the transfer operator should be designed w.r.t. the physical nature of the computed problem

Adaptive schemes in dynamics

Numerical example

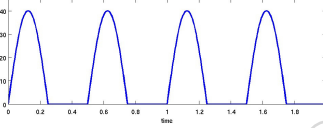
Hemisphere: Mesh adaptation due to displacement error at point I



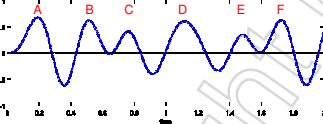
radius of hemisphere:  $R = 10$   
 thickness:  $t = 0.04$   
 radius of hole:  $r = 3$   
 modulus of elasticity:  $E = 6.8 \cdot 10^7$   
 Poisson ratio:  $\nu = 0.3$   
 density:  $\rho_0 = 5$   
 Rayleigh damping:  $C_{in} = 0.0003$ ,  $C_k = 0.0001$

Time stepping: Newmark scheme with  $\Delta t = 0.005$   
 CG version

Loading function:



Displacement at point I:



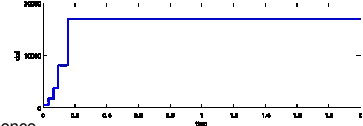
Spatial discretization: Bilinear Solid-Shell-Elements ANS3DEAS

Adaptive schemes in dynamics

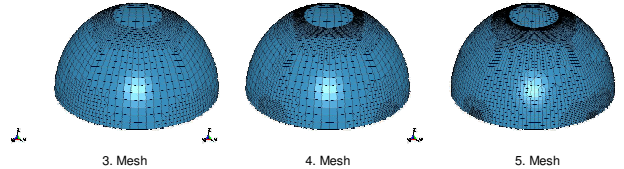
Numerical example (2)

Hemisphere: Strategy 1 – mesh adaptation with transfer of data

Evolution of number of degrees of freedom (for the discretized quarter)



Mesh sequence



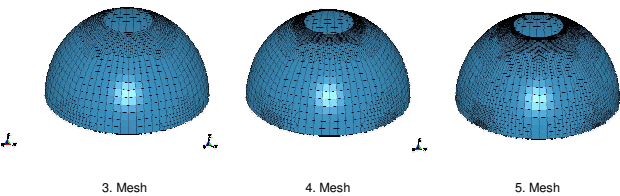
Adaptive schemes in dynamics

Numerical example (3)

Hemisphere: Strategy 2 – mesh adaptation without transfer of data

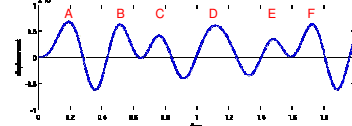
- 5 computations of whole problem necessary to reach desired accuracy

Mesh sequence



Adaptive schemes in dynamics

Numerical example (4)



Relative errors at maximum amplitudes – comparison with uniform mesh

	Uniform mesh ndof = 24960	Strategy I ndof <sub>max</sub> = 16972	Strategy II ndof = 15516
Uniform mesh as reference solution with ndof = 99500			
A	0.3 %	0.8 %	0.8 %
B	0.4 %	0.2 %	0.8 %
C	2.1 %	0.4 %	1.0 %
D	0.4 %	0.2 %	0.7 %
E	4.2 %	0.2 %	0.4 %
F	0.9 %	0.3 %	0.9 %

Adaptive schemes in semidiscretization - Conclusions

Simplification of error indicator

- ▶ Simplification mandatory for large size, practical applications
- ▶ Use of standard energynorm indicator possible

Adaptive schemes

- ▶ Adaptation yields efficient spatial discretizations compared to uniform refinement
- ▶ Scheme without transfer of data more robust (in complex cases)

Limitations

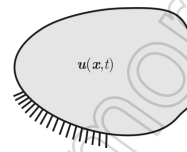
- ▶ Restriction to vibration problems
- ▶ No control of total error

Alternative: Goal-oriented adaptivity for eigenfunctions for Modal analysis

Basic idea:

- ▶ Split quantity of interest into modal parts:  $Q(u(x, t)) = \sum_{i=1}^{\infty} Q(u_i(x)) \cdot f_i(t)$
- ▶ Mesh adaptation for quantity of interest in single eigenmodes
  - Rannacher & Heuveline 2001
  - Oden, Prudhomme, Westerman & Bass 2003

Dynamic eigenvalue problem



Homogeneous equation of motion

$$\text{div} \sigma(u(x, t)) - \rho \ddot{u}(x, t) = 0$$

Continuous eigenvalue problem

$$\text{div} \sigma(\bar{u}) + \rho \lambda \bar{u} = 0$$

Natural frequencies

$$\lambda = \omega^2 = 2\pi f$$

Dynamic eigenvalue problem – weak formulation

Generalized eigenvalue problem and normalization of modes:

$$a(\bar{u}, w) = \lambda \rho(\bar{u}, w) \quad \forall w \in W$$

$$\rho(\bar{u}, \bar{u}) = 1$$

Normalization necessary to define unique solution

Compact notation

Eigenpair:  $U = \{\bar{u}, \lambda\} \in \mathcal{V} = W \times \mathbb{R}$

Testpair:  $W = \{w, \pi\} \in \mathcal{V} = W \times \mathbb{R}$

With:  $A(U; W) = \lambda \rho(\bar{u}, w) - a(\bar{u}, w) + \pi[\rho(\bar{u}, \bar{u}) - 1]$

Eigenvalue problem & normalization:

$$A(U; W) = 0 \quad \forall W \in \mathcal{V}$$

Dynamic eigenvalue problem – discretization

Discretization:

$$A(U_h; W_h) = 0 \quad \forall W_h \in \mathcal{V}^h = W^h \times \mathbb{R}$$

Matrix representation:  $(K - \lambda_h M) d_u = 0$

$$|d_u^T M d_u| = 1$$

Dual problem for linear target functionals

Minimization problem (Rannacher & Becker 1996; Oden, Prudhomme et al. 2003):

with  $Q(U)$  as an arbitrary functional „optimal control“ leads to → find  $U$  such that:

$$Q(U) = \inf_{V \in M} (Q(V)) \quad \text{with } M = \{V \in \mathcal{V}; A(V; W) = 0 \quad \forall W \in \mathcal{V}\}$$

For single eigenvalues

Find saddle point  $(U, Z) \in \mathcal{V} \times \mathcal{V}$  with  $Z = \{z, \mu\} \in \mathcal{V}$  of the Lagrangian:  $\mathcal{L}((U, Z)) = Q(U) - A(U; Z)$

Dual problem for linear target functionals (2)

The saddle point problem yields:

$$A(U; W) = 0 \quad \forall W \in \mathcal{V} \quad \leftarrow \text{Eigenvalue problem}$$

$$A'(U; V, Z) = Q(V) \quad \forall V \in \mathcal{V} \quad \leftarrow \text{dual problem}$$

Final dual problem for error estimation for linear target functionals:

$$-a(v, z) + \lambda \rho(v, z) = Q(V) - Q(U) \rho(\bar{u}, v) \quad \forall V \in \mathcal{V}$$

orthogonality condition:  $\rho(\bar{u}, z) = 0$

Error representation and error indicator

Error of primal problem:  $E_u = U - U_h = \{\bar{u}, \lambda\} - \{\bar{u}_h, \lambda_h\} = \{e_u, e_\lambda\}$

Error of dual problem:  $E_z = Z - Z_h = \{z, \mu\} - \{z_h, \mu_h\} = \{e_z, e_\mu\}$

Error representation (see e.g. Oden, Prudhomme et al. 2003):

$$E(U, U_h) = \mathcal{R}_u(E_z) + \underbrace{e_\lambda(e_u, z) + \frac{1}{2} \mu \rho(e_u, e_u)}_{\Delta_1} \quad \leftarrow \text{Small - higher order}$$

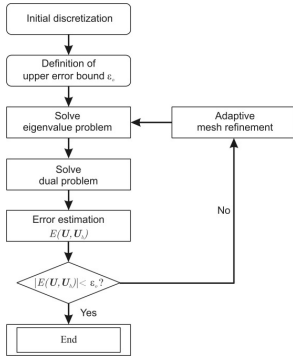
$E(U, U_h) \approx \mathcal{R}_u(E_z)$  { Error can be expressed by dual weighted residual

$$|E(U, U_h)| \approx a(e_u, e_u)^{\frac{1}{2}} \cdot a(e_z, e_z)^{\frac{1}{2}} = \|e_u\|_a \cdot \|e_z\|_a$$

Here: Estimation of energynorm error of primal and dual problem with e.g. Zienkiewicz-Zhu estimator

Adaptive schemes in dynamics

Dynamic eigenvalue problem – adaptive scheme



Refinement scheme:

- Hierarchical mesh refinement with transition elements

Numerical effort:

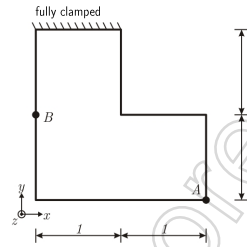
- Several computations of eigenvalue problem and dual problem necessary

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Adaptive schemes in dynamics

Dynamic eigenvalue problem – numerical example

Example: Plate vibration; L-shaped domain



**Material data**  
 thickness  $t = 0.1$   
 modulus of elasticity:  $E = 3 \cdot 10^7$   
 Poisson ratio:  $\nu = 0.3$   
 density:  $\rho = 2.5$

**Spatial discretization:**  
 Bilinear Solid-Shell-Elements ANS3DEAS

Quantities of interest:

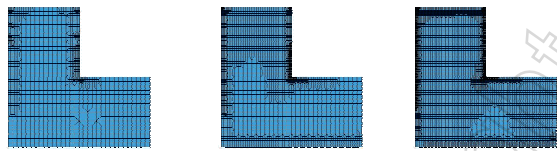
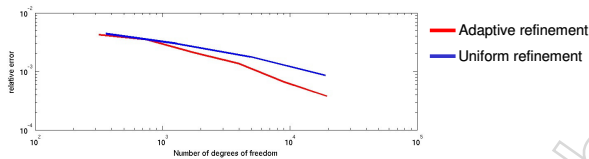
Displacements in z-direction at points A and B for 1. eigenmode e.g. for control of max. displacement due to design requirements

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Adaptive schemes in dynamics

Dynamic eigenvalue problem – numerical example (2)

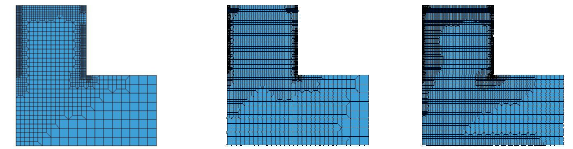
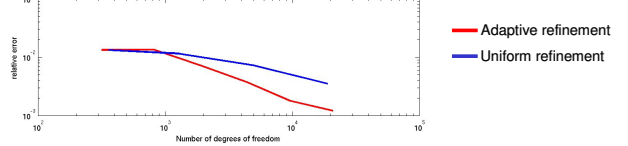
L-shaped domain: Error in vertical displacement at node A



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Dynamic eigenvalue problem – numerical example (3)

L-shaped domain: Error in vertical displacement at node B

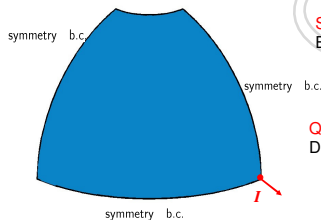


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Adaptive schemes in dynamics

Dynamic eigenvalue problem – numerical example (4)

Example: Curved shell



**Spatial discretization:**  
 Bilinear Solid-Shell-Elements ANS3DEAS

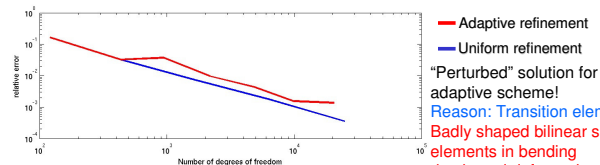
**Quantities of interest:**  
 Displacement at point for 1. eigenmode

radius of hemisphere:  $R = 10$   
 thickness  $t = 0.04$   
 radius of hole:  $r = 3$   
 modulus of elasticity:  $E = 6.8 \cdot 10^7$   
 Poisson ratio:  $\nu = 0.3$   
 density:  $\rho = 5$

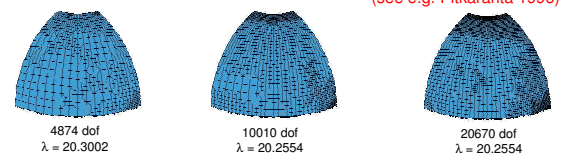
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Dynamic eigenvalue problem – numerical example (5)

Curved Shell: Error in displacement at node A – 1. eigenmode



Mesh evolution for adaptive scheme:



“Perturbed” solution for adaptive scheme!  
 Reason: Transition elements  
 Badly shaped bilinear shell elements in bending dominated deformation states lead to reduced convergence (see e.g. Pitkäranta 1996)

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Dynamic eigenvalue problem - Conclusions

**Error indicator**

- ▶ Focus on single eigenfrequencies
- ▶ Standard global error indicators can be used

**Mesh adaptation**

- ▶ Hierarchical mesh adaptation with bilinear elements superior to uniform refinement for plane geometry (as expected)
- ▶ Curved shells with bilinear elements **transition elements may yield perturbed solution** in bending dominated case  
→ lower eigenmodes are usually bending dominated  
(effect is considerably reduced with biquadratic elements)
- ▶ **Problems:** adaptive refinement with transition elements for highly imperfection sensitive problems  
→ e.g. for static buckling of shells perturbed eigenmodes  
→ BUT less sensitive for dynamic postbuckling analysis

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Adaptive finite element schemes and practical applications

Adaptive schemes are highly sophisticated but often suffer from limitations as:

- ▶ Only geometries with reduced complexity are considered
- ▶ Fairly uniform constitutive models and similar kinematic assumptions for whole domain are taken into account

But **practical (industrial) applications** consist of:

- ▶ Complex geometries such as e.g. shell intersections and many engineering model assumptions
- ▶ Combination of different element types
  - Assumed/enhanced strains
  - Selective/underintegration (→ hourglassing)
- ▶ Combination of rigid bodies and elastic parts
- ▶ Constraints, Spotwelds, Joints etc.

For **practical problems:** Mathematically strict treatise of the problem yields very high effort → **Simplifications necessary**

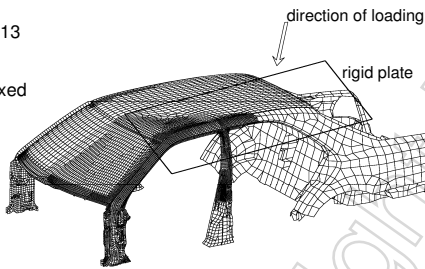
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Industrial adaptive computations in automotive industry

**Roof Crush Problem using adaptivity**

- Model data:
- ▶ 27712 shell elements
  - ▶ 16115 nodes
  - ▶ self contact type 13
  - ▶ rigid wall contact
  - ▶ lower boundary fixed
  - ▶ applied velocity

Fairly complex problems with multiple shell intersections

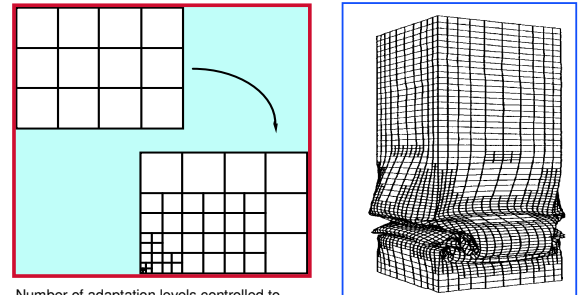


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Examples of adaptive computations in automotive industry

**Mesh refinement scheme in LS-DYNA**

Hierarchical mesh - One side two-neighbor rule – hanging node strategy



Number of adaptation levels controlled to keep certain time step size – explicit time integration

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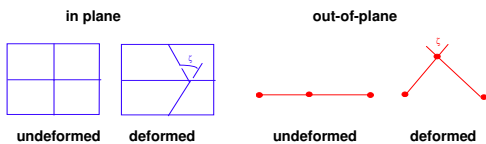
Adaptive computations in automotive industry

**Mesh refinement scheme e.g. in commercial program LS-DYNA**

**Goal:** Efficient, element independent, must work also for shell intersections

**Refinement indicators:** → dedicated to detect large deformations

- Angle based indicators
  - ⇒ Check deformation between elements



⇒ Total angle change - for duration of calculation  
⇒ Incremental angle change - between adaptive steps

**Remark:** Tolerance values from experience / trial runs

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Adaptive computations in automotive industry

**Mesh refinement scheme e.g. in commercial program LS-DYNA**

**Refinement indicators (cont.)**

- **Thickness**
  - ⇒ Refine based on thickness changes
- **Impending contact - action in advance**
  - ⇒ Refine before contact occurs
  - ⇒ One pass adaptivity can be used
  - ⇒ May create more elements than necessary.
- **Interpenetration in contact**
  - ⇒ Small geometric features on one side of a contact interface contacting "large" elements on the other.
    - Two pass adaptivity is necessary

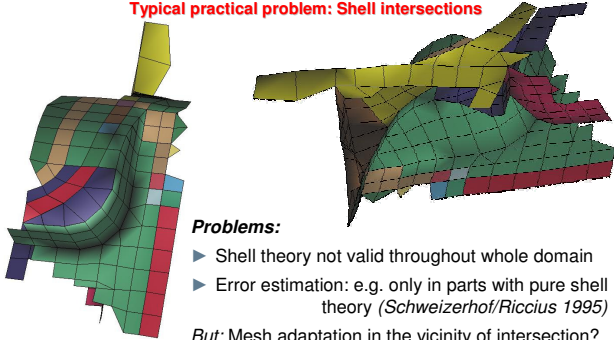
**Remark:** Tolerance values from experience / trial runs

- no patch information necessary (or available) – locality important for parallel execution

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Adaptive computations in automotive industry

Typical practical problem: Shell intersections



Problems:

- ▶ Shell theory not valid throughout whole domain
- ▶ Error estimation: e.g. only in parts with pure shell theory (Schweizerhof/Riccio 1995)

But: Mesh adaptation in the vicinity of intersection?

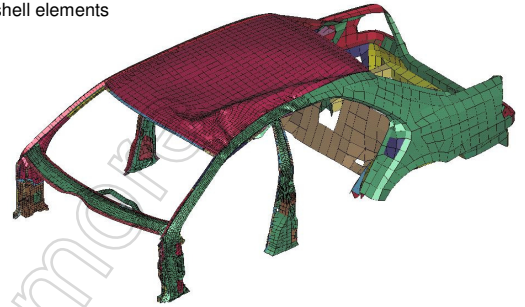
Use simple indicators !

Adaptive computations in automotive industry with LSDYNA

Roof Crush Problem using adaptivity

Run without adaptive refinement:

- ▶ 27712 shell elements

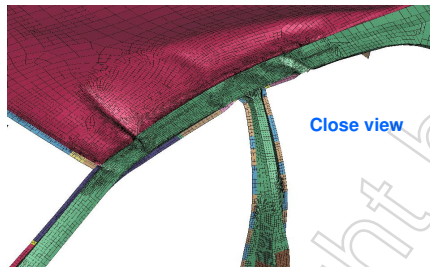


Adaptive computations in automotive industry with LSDYNA

Roof Crush Problem using adaptivity

Refinement level 3:

- ▶ 58926 shell elements



Close view

Side aspects of adaptivity with commercial program - explicit time integration

Memory

- ▶ Memory allocation and disk space usage must be determined dynamically as the problem size grows

Contact

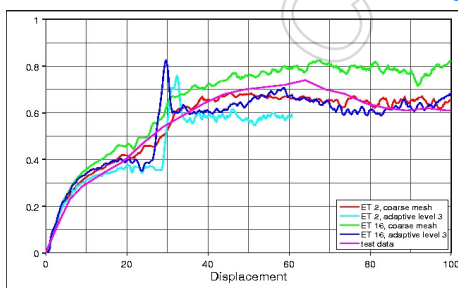
- ▶ Automatic contact definitions are required between parts which are adapted.
- ▶ Thickness offsets for shells are mandatory
- ▶ Adaptive constraints (due to hanging nodes) must be handled in contact searching

Mass scaling

- ▶ Adaptivity leads to subdivision of elements  
 → Courant – F.-L. – conditions yields **small timestep size** (stability)
- ▶ Efficiency: Keep timestep size constant by **mass scaling**

Adaptive computations in automotive industry

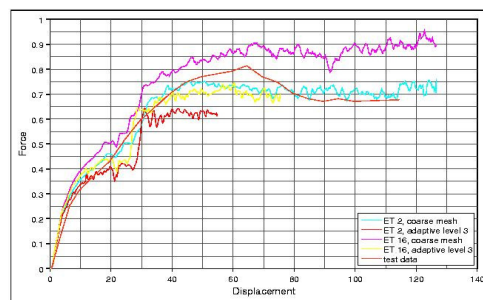
Roof Crush: Comparison with experimental data - **high** mass scaling



Artificial spike in response curve

Adaptive computations in automotive industry

Roof Crush: Comparison with experimental data - **reduced** mass scaling



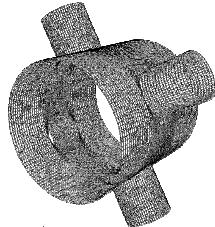
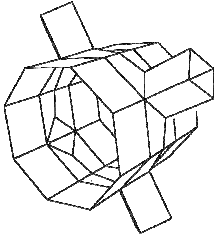
Artificial spike in response curve removed

Adaptive computations in automotive industry

Desirable tool: Adaptive remeshing with/on CAD data

Coarse initial mesh

Refined mesh

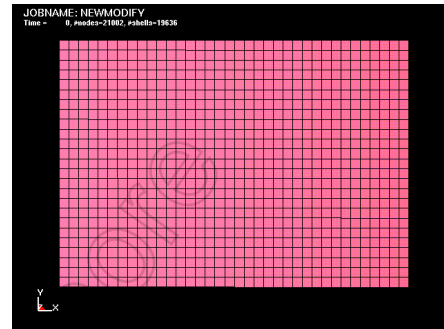


**Problem:**

Geometry for nonlinear problems with large deformations not known

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Adaptive computations in automotive industry – with LS-DYNA  
Sheet metal forming problem



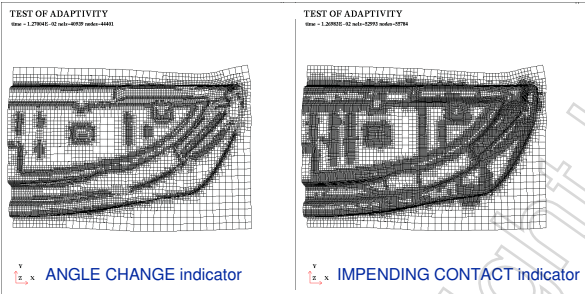
Refinement based on indicators for large deformation problems

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Adaptive computations in automotive industry with LS-DYNA

Sheet metal forming problem

Effect of ref.-indicators: ANGLE CHANGE vs. IMPENDING CONTACT

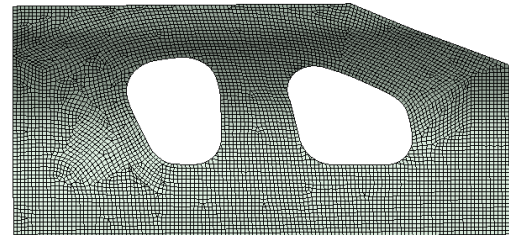


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Adaptive computations in automotive industry with LS-DYNA

Sheet metal forming problem

Combined use of refinement indicators  
ANGLE CHANGE and IMPENDING CONTACT



➤ Simple transfer of data due to hierarchical meshing and refinement only  
Second pass analysis with final refined mesh (for verification because of contact)

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Using adaptivity for model reduction  
– from flexible to rigid body analysis

**Goal:** For many engineering problems efficient e.g. rigid body models are mandatory

- for long time analyses
- for design studies of fairly rigid machine parts with “minor” modifications

**Current situation**

► Rigid body models are created simply on engineering assumptions

**Improvement**

► Verify resp. create rigid body models by adaptivity  
► Monitoring deformation indicators

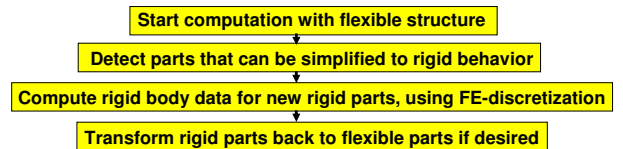
**Examples**

► Simple transient problem  
► Verifying/creation of rigid bodies for efficient demolition simulations in case of demolition using explosives

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Using adaptivity for model reduction  
– from flexible to rigid body analysis

General procedure



- **Coupling between rigid and flexible parts:** Transformation to minimal coordinates
- **Time Integration method:** implicit midpoint rule  
Specific variation Energy Momentum Method [Simo et al, 1992]
- **Solution Method for nonlinear equations:** Implicit Newton - Raphson

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Using adaptivity for model reduction - from flexible to rigid body analysis

**Criterion to switch status from flexible to rigid**

Check at each time step for every flexible element:

**Normalized strain rate**

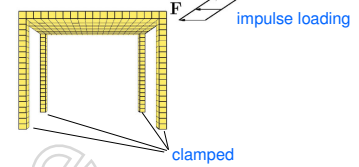
$$e^e = \frac{\sqrt{\int (E_{10} - E_{00}) : (E_{10} - E_{00}) dV_0^e}}{\Delta t V_0^e} \leq tol$$

$E$  Green-Lagrange strain tensor  
 $V_0^e$  Element volume (reference)  
 $\Delta t$  time step size  
00/10 beginning / end of time step

- strain rate based, geometrical and objective criterion.
- directly correlated to discretization.
- efficient ( $E$  also needed for stiffness matrix)

Using adaptivity for model reduction - from flexible to rigid body analysis

**Frame Structure**



Geometry:

Plate: 400 x 200 x 20 [cm]  
Column: 20 x 20 x 300 [cm]

Material: Youngs-Modulus  $E$ :  $2.1 \cdot 10^5 \frac{N}{mm^2}$   
Poisson ratio  $\nu$ : 0.2  
mass density:  $8 \frac{g}{cm^3}$   
Num. damping [Armero, Peiöcz 1998]

Loading:  $0.0 \leq t \leq 0.05s$  Forces  $\sum F = 11000N$

Time step size: 0.05 s (Energy-Momentum-Scheme)

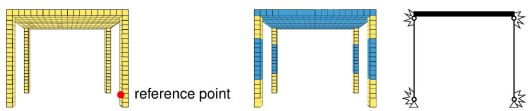
**Rigid body detection**

Check criterion in every time step and mark violating elements  
Check allocation rigid - flexible in time interval:  $\Delta t = 0.5 s$

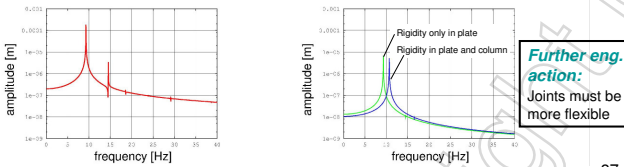
**Frame Structure: Results**

$T = 0.05s$

$T = 2.00s$



Amplitude-Frequency Spectra for horizontal displacement of reference point



Using adaptivity for model reduction - from flexible to rigid body analysis

**Lime works Borna, Germany**

**Application to building demolition by explosives**



Silo in Weimar, Germany  
Videos by courtesy of Dr.-Ing. Rainer Melzer  
Planungsbüro für Abbruchsprengen

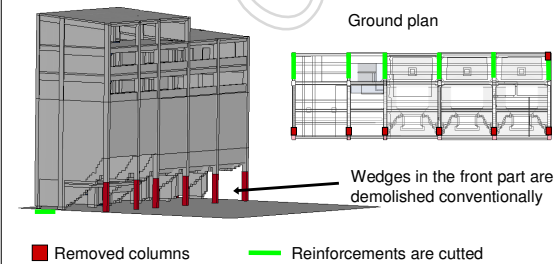
Construction data

- ▶ Height 25.2 m
- ▶ length 36.0 m
- ▶ width 12.0 m
- ▶ reinforced concrete construction with concrete and masonry walls
- ▶ Main part silo installations
- ▶ Mass of building is approximately 3630 t

Demolition companies:  
-Thüringer Sprenggesellschaft  
-Dr. Ing. R. Melzer

**Borna - Blasting strategy**

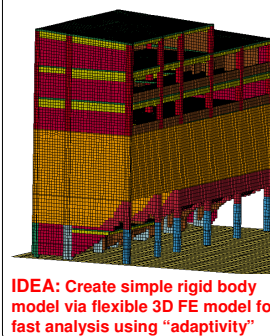
Structure should topple in direction of weak axis



Removed columns      Reinforcements are cutted

Using adaptivity for model reduction - from flexible to rigid body analysis

**Computational Model - Discretization**



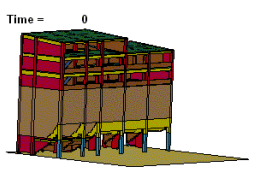
- ▶ Building discretized via 75570 hexahedral elements (reduced integrated, hourglass stabilization)
- ▶ Ground plate discretized via 1330 shell elements
- ▶ Elastic-plastic material model used
- ▶ Failure modeled via element erosion (criterion: plastic strains)
- ▶ Contact via segment-to-segment penalty formulation
- ▶ FE-Code LS-DYNA
- ▶ Blast process modeled by removing elements

**IDEA: Create simple rigid body model via flexible 3D FE model for fast analysis using "adaptivity"**

Using adaptivity for model reduction - from flexible to rigid body analysis

Results – numerical analysis (Controlled demolition)

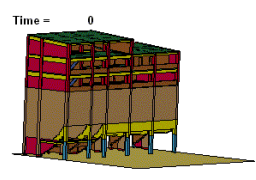
Time = 0



Model A  
(no fracture in silo assumed)  
analyzed with flexible parts

**Visual comparison shows good agreement**

Time = 0




Model B  
(no fracture in silo/columns assumed)  
analyzed with flexible parts

**Visual comparison shows good agreement**

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Using adaptivity for model reduction - from flexible to rigid body analysis

Validation – video source – Model B



**Common overlay of numerical analysis results and video detail screen**

**Result:**  
Location of fracture adequate / Kinematics are similar

**Problem (of validation):**  
Only a single video available  
For rendering process location and view angle not known

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Using adaptivity for model reduction – from flexible to rigid body analysis

Application to demolition by explosives:

Nice idea - BUT:

- Application to 3D structure not straightforward – **not automatic**
- For hinge evolution fairly detailed model necessary
- Hinge lines develop – resolution/discretization must capture form
- Fracture must be captured somehow
- Contact evolution must be captured – rigid body must contain appropriate joints/hinges
- Many more difficulties .....

Project in progress – FOR500 research group involving Universities Bochum-Dresden-Karlsruhe

Conclusions

- **Goal oriented error estimators** based on dual weighted residual very powerful method for mesh adaptation in structural dynamics
  - Applications and problems discussed - computational effort, transfer of data etc.
  - Adaptive schemes are usually designed and tested on rather simple "academic" problems, e.g. simple geometries, uniform problems
  - Simplifications are necessary for practical – large scale - applications of adaptive schemes (due to computational effort needed)
  - For **different problem classes simplifications** are possible, e.g.:
    - Vibration dominated problems
    - Large deformation problems involving contact
- Consequences:** Error estimation /refinement indication is limited, but shows fairly good results for eng. purposes

Conclusions/Outlook

- ▶ Practical problems – however - usually consist of
  - very complex geometries, shell intersections,
  - combinations of elements – beams, shells, solids
  - constraints with arbitrary conditions, point connections etc.
- Consequences:**
  - In simulation programs with adaptive algorithms many complex algorithmic details are needed beyond adaptivity
  - Simple (though rather limited) algorithms are necessary for robustness
- ▶ Adaptive schemes/error estimation further provide hints for modeling and discretization
  - Many useful hints for model limits can be obtained
  - Application for model reduction – creation of **engineering models**
  - BUT: Automatic creation via application of adaptive algorithm to find simple models not straightforward (if at all possible)

Thank you for your attention !