ON THE FENG FAILURE CRITERION FOR COMPOSITES

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Abstract

The Feng failure criterion for composite materials, based upon the strain invariants (I_1, I_2, I_3, I_4, and I_5) of finite elasticity, has two parts. The failure criterion for matrix modes is

$$A_1(I_1-3)+A_{11}(I_1-3)^2+A_2(I_2-3)-1=0.$$

The failure criterion for fiber modes is

$$A_5(I_5-1)+A_{55}(I_5-1)^2+A_4(I_4-1)-1=0.$$

In this paper, the criterion is evaluated with experimental data for a boron/epoxy, symmetrically balanced, angle-ply laminate. The results prove that the failure states obtained by the criterion agree with the experimental data. Furthermore, the criterion also predicts whether the failure of a composite is due to matrix or fiber.

The advantages for Feng failure criterion over other criteria are summarized, and the criterion is implemented into LS-DYNA for composite materials.

Introduction

The finite-strain-invariant failure criterion developed by Feng [1] consists of two parts: the matrix- and fiber-dominated failure modes. Experimental data are used to evaluate the failure criterion. Since the strains to be evaluated are in the realm of infinitesimal strain, the Feng failure criterion has been reduced for the infinitesimal strain theory also. Five material properties are needed to determine the five constants in the failure criterion. These five material properties are, usually, longitudinal compressive and tensile strengths, transverse compressive and tensile strengths, and shear strength. In formulating the failure criterion, the strain criterion was developed. However, in most cases, experimental data are presented in stresses rather than in strains. For plane-stress problems, four more elastic constants are needed to convert stresses into strains. These four elastic constants are longitudinal and transverse elastic moduli, shear modulus, and the major Poisson's ratio.

In this paper, the elastic and failure properties of an unidirectional boron/epoxy composite, obtained by Hahn [2] are used. Besides these fundamental properties, Hahn also obtained the failure strengths of boron/epoxy, symmetrically balanced, angle-ply laminates with fiber angles ranging from 0 to 90 degrees. These experimental data for the off-angle, symmetrically balanced, angle-ply laminates are used to evaluate the validity of the Feng failure criterion of composites.

Failure criterion

The Feng failure criterion is in the domain of finite elasticity and consists of two modes. The matrix- and fiber-dominated failure modes are, respectively, functions of the strain invariants I_1, I_2, I_4, and I_5,

$$A_1(I_1-3)+A_{11}(I_1-3)^2+A_2(I_2-3)-1=0 \quad \text{(1a).}$$

$$A_5(I_5-1)+A_{55}(I_5-1)^2+A_4(I_4-1)-1=0. \tag{1b}$$

The strain invariants are functions of the Cauchy strains, $C_{\alpha\beta}$,

$$\begin{aligned}
I_1 &= C_{\alpha\alpha},\\
I_2 &= \frac{1}{2}\left[(C_{\alpha\alpha})^2 - C_{\alpha\beta}C_{\alpha\beta}\right],\\
I_3 &= \det(C_{\alpha\beta}),\\
I_4 &= V_\alpha C_{\alpha\gamma}C_{\gamma\beta}V_\beta\\
I_5 &= V_\alpha C_{\alpha\beta}V_\beta,
\end{aligned} \tag{2}$$

where V_α are components of the unit vector along the fiber direction before deformation.

The failure of a composite will be initiated when one of Equations (1) is satisfied.

The Feng failure criterion can be reduced to the infinitesimal strain-invariant criterion. The matrix- and fiber-dominated failure modes in terms of infinitesimal strain invariants are, respectively,

$$A_1J_1 + A_{11}J_1^{\ 2} + A_2J_2 - 1 = 0 \tag{3a}$$

$$A_5J_5 + A_{55}J_5^{\ 2} + A_4J_4 - 1 = 0. \tag{3b}$$

where J_1, J_2, J_4, and J_5 are related to the infinitesimal strain components by

$$\begin{aligned}
J_1 &= \varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33},\\
J_2 &= \frac{1}{6}\left[(\varepsilon_{11}-\varepsilon_{22})^2+(\varepsilon_{22}-\varepsilon_{33})^2+(\varepsilon_{33}-\varepsilon_{11})^2\right]+\varepsilon_{12}^2+\varepsilon_{23}^2+\varepsilon_{13}^2,\\
J_4 &= \varepsilon_{12}^2+\varepsilon_{13}^2,\\
J_5 &= \varepsilon_{11}.
\end{aligned} \tag{4}$$

Equations (1) in finite elasticity and Equations (3) in infinitesimal elasticity comprise the failure criterion for a composite structure.

In many practical situations, the experimental failure state is measured in terms of stresses rather than strains. Therefore, it is necessary to convert the strains to stresses. For plane-stress problems and for infinitesimal strains, the constitutive relationships that relate stresses and strains are

$$\{\tilde{\varepsilon}\} = [\tilde{S}]\{\tilde{\sigma}\} \tag{5}$$

where

$$\begin{aligned}
\{\tilde{\varepsilon}\}^t &= \{\varepsilon_{11} \quad \varepsilon_{22} \quad \varepsilon_{33} \quad \varepsilon_{12}\}\\
\{\tilde{\sigma}\}^t &= \{\sigma_{11} \quad \sigma_{22} \quad 0 \quad \sigma_{12}\}
\end{aligned} \tag{6}$$

and the compliance matrix is

$$\left[\tilde{S}\right]=\begin{bmatrix} \frac{1}{E_{11}} & -\frac{\nu_{12}}{E_{11}} & -\frac{\nu_{12}}{E_{11}} & 0 \\ -\frac{\nu_{12}}{E_{11}} & \frac{1}{E_{22}} & -\frac{\nu_{23}}{E_{22}} & 0 \\ -\frac{\nu_{12}}{E_{11}} & -\frac{\nu_{23}}{E_{22}} & \frac{1}{E_{22}} & 0 \\ 0 & 0 & 0 & \frac{1}{2G_{12}} \end{bmatrix} \tag{7}$$

In equation (7), E_{11} and E_{22} are longitudinal and transverse elastic moduli, G_{12} is the shear modulus, and ν_{12} is the major Poisson's ratio.

It must be noted that

$$\frac{\nu_{12}}{E_{11}}=\frac{\nu_{21}}{E_{22}} \tag{8}$$

has been applied in equation (7)

Another Poisson's ratio, ν_{23}, in a plane perpendicular to the axis of isotropy, can be obtained from the approximate formula:

$$\frac{1}{1+\nu_{23}}=\frac{1-\nu_{12}}{1-\nu_{12}^2 E_{22}/E_{11}} \tag{9}$$

Uniaxial Extension of a Symmetrically Balanced, Angle-ply Laminate

Before comparing the failure criterion with the experimental data, the strain state in a loaded composite should be obtained. The coordinate system for a symmetrically balanced, angle-ply laminate is shown in Figure 1. The fiber axis and the direction transverse to the fiber axis are denoted by 1 and 2, respectively. A uniaxial load is applied along the x-axis. The lay-up of the composite is $\left[\pm\alpha\right]_{ns}$, where α is the angle between the loading axis and the fiber axis.

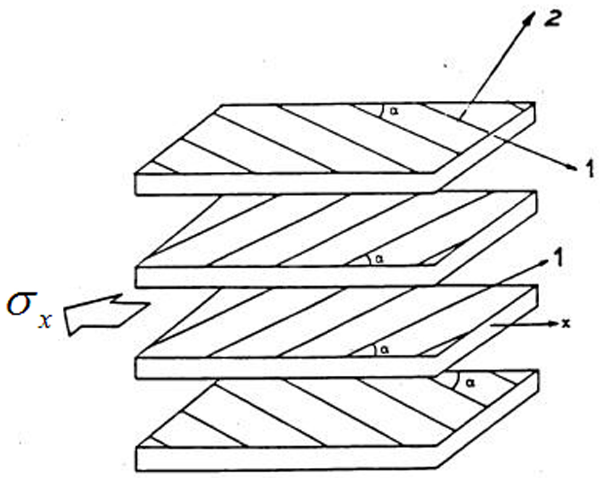


Figure 1. Coordinate system for a symmetrically balanced, angle-ply laminate.

When the composite laminate is subjected to an uniaxial loading in the x-direction, the stress in the fiber direction (σ_{11}), the stress transverse to the fiber direction (σ_{22}) and the in-plane shear stress (σ_{12}) are given by Tsai [3]:

$$\begin{aligned}
\sigma_{11} &= \frac{\sigma_x}{2}\left[1+\frac{A\sin^2 2\alpha + B\cos 2\alpha}{\sin^2 2\alpha + B\cos^2 2\alpha}\right] \\
\sigma_{22} &= \frac{\sigma_x}{2}\left[1-\frac{A\sin^2 2\alpha + B\cos 2\alpha}{\sin^2 2\alpha + B\cos^2 2\alpha}\right] \\
\sigma_{12} &= \frac{\sigma_x}{2}\left[\frac{1-A\cos 2\alpha}{\sin^2 2\alpha + B\cos^2 2\alpha}\sin 2\alpha\right],
\end{aligned} \tag{10}$$

where A and B are functions of the material constants

$$A = \frac{(E_{11}/E_{22})-1}{1+2\nu_{12}+(E_{11}/E_{22})}$$

$$B = \frac{(E_{11}/G_{12})}{1+2\nu_{12}+(E_{11}/E_{22})} \tag{11}$$

Hence, the stresses in each ply are determined for a given loading (σ_x).

Comparison with the Experimental Data

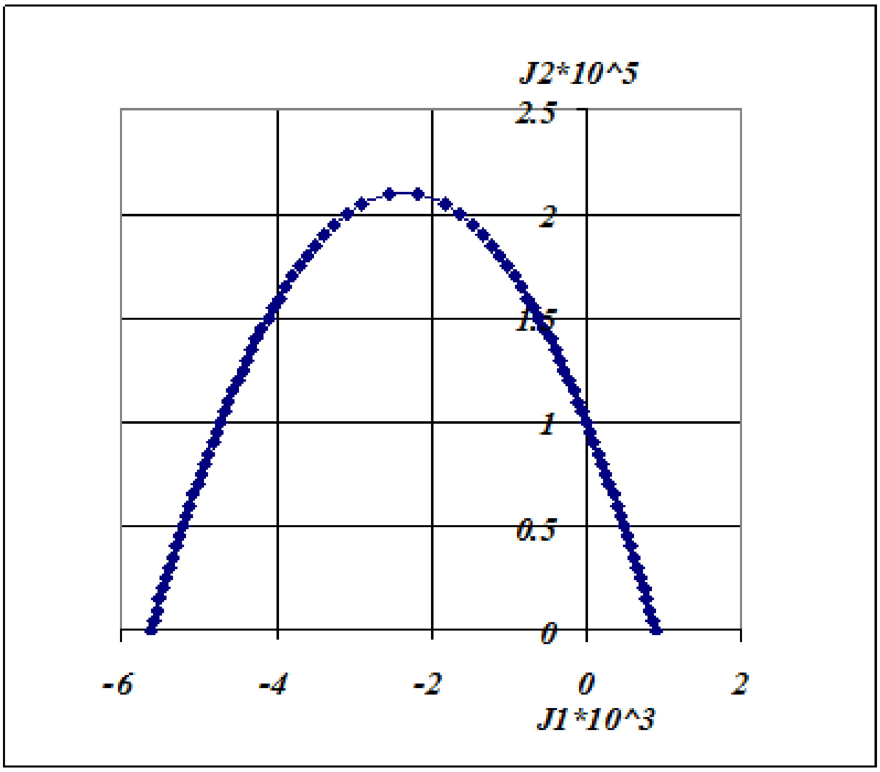
The experimental data for the elastic properties and strengths of an unidirectional boron/epoxy composite were obtained by Hahn[2].

For the elastic properties of the composite: longitudinal and transverse elastic moduli are 206.7 GPa and 20.7 GPa respectively, shear modulus is 6.9 GPa, and the major Poisson's ratio is 0.3. For the strengths of the composite: The tensile and compressive strengths in the fiber direction are 1300 MPa and -2490 MPa respectively, the tensile and compressive strengths transverse to fiber direction are 62 MPa and –310 MPa respectively, and the shear strength is 68.9 MPa.

With these properties the material constants for the failure criterion, Equation (3), are

$$\begin{aligned}
A_1 &= 939 \\
A_{11} &= 199{,}145 \\
A_2 &= A_4 = 10{,}000 \\
A_5 &= 76 \\
A_{55} &= 13{,}200
\end{aligned} \tag{12}$$

The matrix failure surface (J_1-J_2) and the fiber dominated failure surface (J_5-J_4), shown in Figure 2, are for three-dimensional boron-epoxy composite structures. It definitely applies to a boron/epoxy, symmetrically balanced, angle-ply laminate.



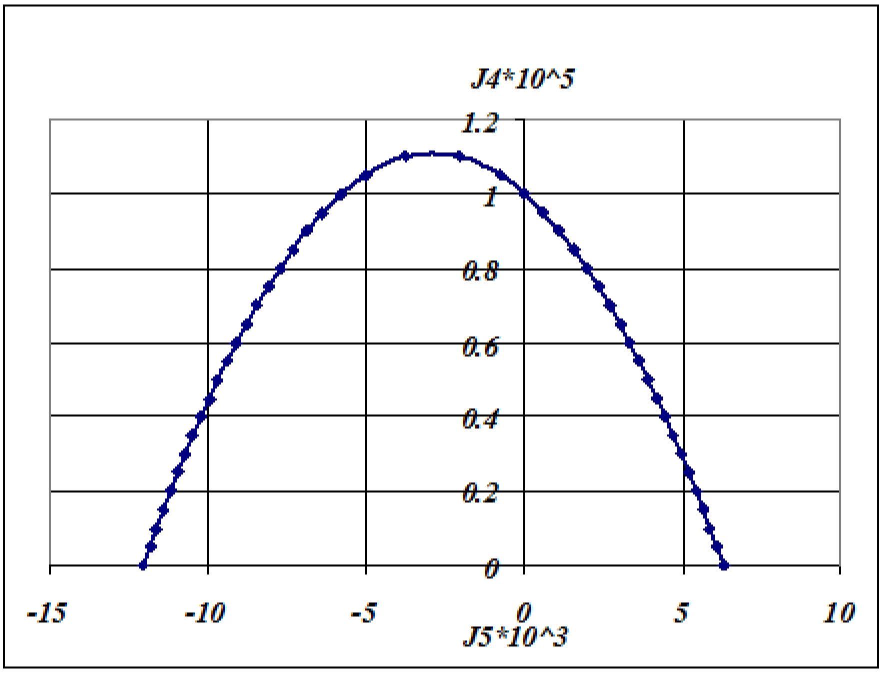


Figure 2. The failure surfaces for boron-epoxy composite

The maximum uniaxial stresses before failure for a boron/epoxy, symmetrically balanced, angle-ply laminate with various angles of orientation are shown in Figure 3. The basic strength data used in determining the failure criterion are shown as open dots.

As can be seen from the graph, the failure is divided into two parts: the fiber-dominated failure mode and the matrix-dominated failure mode. It is worth noting, that for unidirectional composite ($\alpha = 0$), the criterion predicts that the matrix failure will precede the fiber failure. The test data used assume total failure of the composite; therefore the fiber-dominated failure mode is assumed for determining the failure criterion. However, the failure criterion indicates that the matrix-dominated failure occurs first for a unidirectional boron/epoxy composite subjected to uniaxial loading in the fiber direction.

The solid dots shown in Figure 3 are the test data of the failure stress of boron/epoxy symmetrically balanced angle-ply composite laminates. These values are not used in determining the failure material constants A_1, A_{11}, A_2 (A_4), A_5 and A_{55}. It can be seen that the failure predicted by the failure criterion agrees with the experimental data.

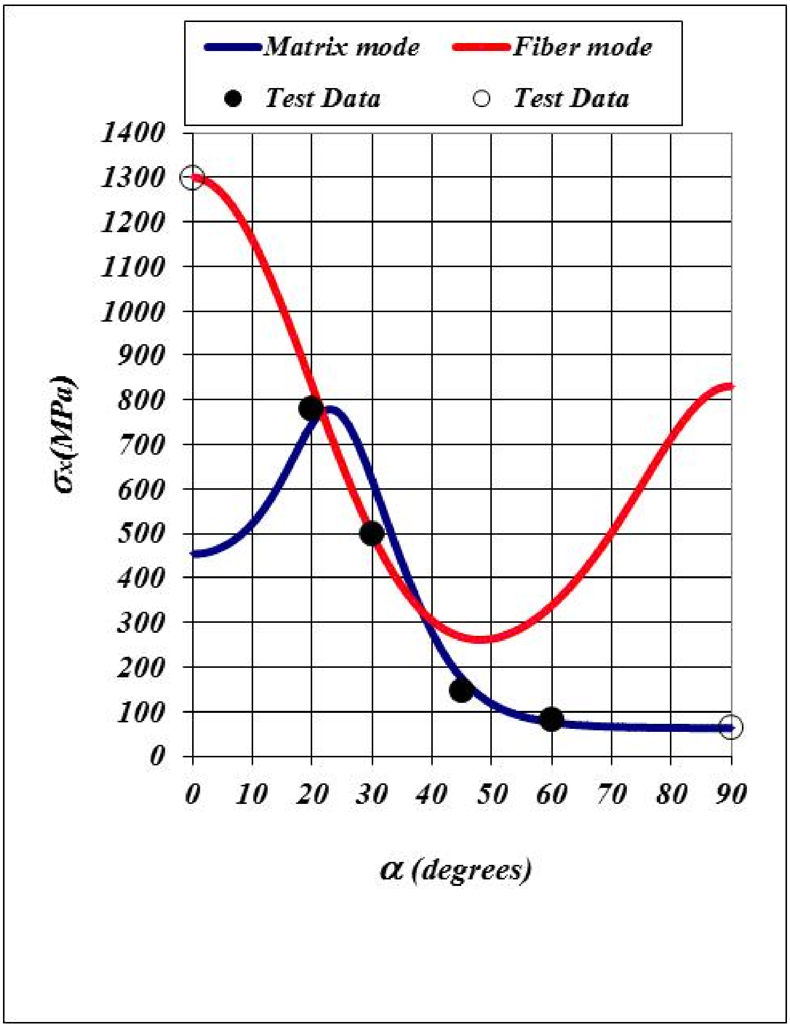


Figure 3. The strengths predicted by the Feng failure criterion and the experimental data for a boron/epoxy, symmetrical balanced, angle-ply laminate.

Conclusions

The predictions of the Feng failure criterion agree with the experimental data for a boron/epoxy, symmetrically balanced, angle-ply composite laminate. The results also indicate that the failure criterion can be used to predict the initial failure of a composite, whether it be fiber failure mode or matrix failure mode.

There are few failure criteria for composite materials subject to large deformation. The Feng criterion is derived from the finite deformation theory and is applicable to both finite deformation theory and to infinitesimal strain theory.

Based on these reasons, the Feng criterion has been implemented into LS-DYNA for composite materials, and is available to users.

Discussion

The Tsai-Wu failure criterion for transversely isotropic materials in strain space is

$$G_{xx}\varepsilon_x^2 + G_{yy}\left(\varepsilon_y^2 + \varepsilon_z^2\right) + G_{ss}\left(\varepsilon_s^2 + \varepsilon_\alpha^2\right) + G_{tt}\varepsilon_t^2 + 2G_{xy}\left(\varepsilon_y + \varepsilon_z\right)\varepsilon_x + 2G_{yz}\varepsilon_y\varepsilon_z + G_x\varepsilon_x + 2G_y\left(\varepsilon_y + \varepsilon_z\right) - 1 = 0 . \qquad (13)$$

The Tsai-Wu criterion was written in terms of a polynomial of infinitesimal strains. The Feng finite-strain-invariant failure criterion was derived from mathematical formulations. There were seven failure material constants in the Tsai-Wu criterion and these constants cannot be determined accurately. The Feng criterion decouples the fiber- and matrix-failure modes; it has five failure constants, and they can be determined simply and accurately. In the Tsai-Wu criterion the matrix and fiber failure modes are coupled. In the Feng failure criterion the matrix and fiber failure modes are decoupled.

References

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