A Model for the Stochastic Fracture Behaviour of Glass

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Abstract

The failure of glass is caused by initial flaws that are induced during the manufacturing process. These micro-cracks are randomly distributed on the surface of glass, which is why failure is a random process and stress at failure is a non-deterministic parameter. In the present work, a model for the stochastic fracture behaviour of glass is proposed and implemented as a user subroutine in LS-DYNA® for shell elements.
Due to the stochastic fracture behaviour of glass, a large scattering can be expected when determining the head injury criteria (HIC) in the case of a pedestrian head impact on an automotive windscreen. In this case a high experimental effort would be necessary to evaluate the stochastic scattering. This can be reduced by numerical simulation using a stochastic failure model.
The present model generates failure strengths out of a Weibull distribution obtained by coaxial ring-on-ring tests. The generated stresses are used to calculate initial crack lengths by recalculate the subcritical crack growth during experiments. These initial cracks slowly grow, i.e. subcritical, in dependence on the applied stress rate until the critical stress intensity is reached and failure occurs.
In order to validate the model, coaxial ring-on-ring tests with different test setups are simulated and compared to experimental values and analytical solutions using the Weibull surface shift.

Introduction

The strength of glasses has been studied and discussed in several publications and books, e.g. Wiederhorn [1], Haldimann et. al. [2], Wachtman et. al. [3] and Quinn [4]. The stress at failure of glass and other almost ideally brittle materials is determined by the number and length of microdefects. These microdefects occur during the production and handling of the glass. As these cracks are randomly distributed over the glass surface in their geometry and length, the failure stress is also subject to statistical scatter. At a loading below the failure stress $\sigma < \sigma_f$, these micro defects are subject to subcritical crack growth. With a varying stress rate, the duration of subcritical crack growth also varies. At a low stress rate, cracks grow slower and a lower failure stress is required to reach the critical stress intensity. Accordingly, at a higher stress rate, a higher stress is required to reach the critical stress intensity. Experience shows that the failure stress of glass under identical conditions can differ by a factor of up to 20 and more.
Several numerical models already exist which can represent the failure behaviour of glass, e.g. by stress-dependent failure (***MAT_280**) or by physically modelling of crack growth, see Alter et. al. [5]. Both models are capable to simulate experimental values such as the head injury in case of a pedestrian head impact on an automotive windscreen. However, due to the stochastic fracture behaviour of glass, a large scattering is to be expected when determining injury criteria (e.g. HIC).
The present model considers the stochastic fracture behaviour of glass based on initial crack lengths from coaxial ring-on-ring tests. These initial cracks are used as input for the computation. Hereby, subcritical crack growth is simulated in dependence on stress and stress rate until the critical stress intensity is reached and failure occurs.

Stochastic Failure Algorithm

The algorithm for calculating the stochastic failure is divided into two parts. Firstly, initial crack lengths are determined during the initialization phase of the finite element simulation for each integration point at each element surface. Subsequently, the subcritical growth of these initial flaws during the simulation is calculated until the combination of stress and flaw size triggers failure.

Determination of Initial Crack Lengths

During the initialization of each simulation, an initial crack length must be calculated for each integration point on the surface of the shell elements. These initial cracks serve as starting value and will grow during the simulation until failure occurs.

If the crack growth velocity is represented as a function of the stress intensity $\mathrm{K_I}$,

$$\mathrm{K_I} = Y\sigma\sqrt{\pi a} \tag{1}$$

with the geometric correction factor Y, the applied stress σ and the crack size, the subcritical crack velocity passes through four growth regions Wiederhorn [1]. The subcritical crack velocity v is often approached by the ordinary differential equation for the linear approximation of all growth regions by

$$\mathrm{v}(\mathrm{K_I}) = \frac{da}{dt} = \mathrm{v_0}\left(\frac{\mathrm{K_I}}{\mathrm{K_{Ic}}}\right)^{\mathrm{n}}, \tag{2}$$

with the crack growth parameter n and $\mathrm{v_0}$. This formulation is originally proposed by Evans and Johnson [6] with further improvement by Maugis [7]. It is a description of subcritical crack growth that has already been used in several models and studies, e.g. Haldimann [8], Overend and Zammit [9], Alter et. al. [5] and Kinsella and Persson [10]. Figure shows the original subcritical crack growth data from Wiederhorn [1] as a function of the stress intensity and the linear approximation through all four crack growth regions.
By using the stress intensity formulation from Eq. (1) and separating the variables, Eq. (2) can be expressed by

$$\mathrm{da} = \mathrm{v_0}\left(\frac{Y\sigma\sqrt{\pi a}}{\mathrm{K_{Ic}}}\right)^{\mathrm{n}} dt. \tag{3}$$

Rearranging Eq (3) and integration from the initial flaw size a_{in}(t = 0) to the critical flaw size $a_f(t = t_f)$, we obtain

$$\int_{a_{in}}^{a_f} \mathrm{a}^{-\frac{n}{2}}\,\mathrm{da} = \mathrm{v_0} K_{Ic}^{-n} \pi^{\frac{n}{2}} Y^n \int_0^{t_f} \sigma(t)^n \; dt. \tag{4}$$

This only applies under the condition that the geometric factor Y remains constant during the entire experiment. The geometric factor Y approaches Y = 1.122 for cracks that are considerably smaller than the thickness of the considered object and are straight fronted, see e.g. Anderson [11], Haldimann [10]. For quarter-circle cracks on the glass surface Y = 0.722 by Porter [12] or for half penny shaped cracks on a flexure specimen Y = 0.713 by Halidmann [8] can be used. Due to this data situation and the still unknown geometry of the initial cracks, the geometric correction factor is set to Y = constant = 1 in the context of this investigation. This simplification should be considered when evaluating the results from this failure model. The development of the geometric factor for natural flaws is currently the subject of further research by the authors.

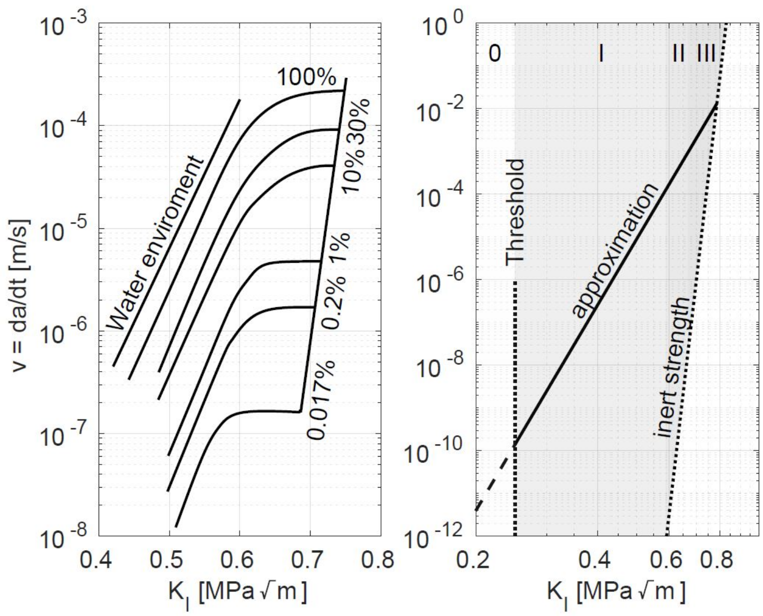

Figure 1: Sub-critical crack velocity in soda-lime-silica glass (SLS) according to Schula [14] on basis of Wiederhorn [1] for various humidity's (left) and the linear approximation of subcritical crack growth for all regions (right), see also [15].

The stress function $\sigma(t)$ in Eq. (4) can be expressed as product of time t multiplied with the function of the stress rate $\dot{\sigma}(t)$ to

$$\sigma(t) = \dot{\sigma}(t)t. \tag{5}$$

Hereby, the stress rate function is usually unknown. Therefore, a failure stress is needed, whose stress history is known. This is the case with coaxial ring-on-ring tests. These are carried out according to EN DIN 1288-5 [13] with a constant stress rate of $\dot{\sigma}$ = 2 MPa/s. Using this assumption, that $\dot{\sigma}(t)$ = constant, Eq. (5) can be integrated in Eq. (4) to

$$a_f^{1-\frac{n}{2}} - a_{in}^{1-\frac{n}{2}} = -\frac{n-2}{2}\mathrm{v}_0 K_{Ic}^{-n}\pi^{\frac{n}{2}}Y^n\frac{\dot{\sigma}^n t_f^{n+1}}{n+1}. \tag{6}$$

Furthermore, the relationship between the failure stress σ_f as product of time to failure t_f and the constant stress rate $\dot{\sigma}$ can be used to express Eq.(6) as

$$a_i = \left[\frac{n-2}{2(n+1)}v_0 K_{Ic}^{-n}\pi^{\frac{n}{2}}Y^n\dot{\sigma}^{-1}\sigma_f^{n+1} + a_f^{1-\frac{n}{2}}\right]^{-\frac{2}{n-2}} \tag{7}$$

The critical crack length a_f can be replaced by the critical stress intensity $\mathrm{K_{Ic}}$ and the relation from Eq. (1), so that the initial crack length can be calculated by

$$a_i = \left[\frac{n-2}{2(n+1)} v_0 K_{Ic}^{-n} \pi^{\frac{n}{2}} Y^n \dot{\sigma}^{-1} \sigma_f^{n+1} + \left(\frac{K_{Ic}^2}{Y^2 \sigma_f^2 \pi} \right)^{1-\frac{n}{2}} \right]^{-\frac{2}{n-2}} \quad (8)$$

In order to consider possible residual stresses of the glass, the failure stress from the coaxial ring-on-ring tests must be corrected by the residual stress σ_0 induced by e.g. the cooling process or chemical treatment, so that the initial crack length a_i of each integration point can be expressed by

$$a_i = \left[\frac{n-2}{2(n+1)} v_0 K_{Ic}^{-n} \pi^{\frac{n}{2}} Y^n \frac{(\sigma_f - \sigma_0)^{n+1}}{\dot{\sigma}} + \left(\frac{K_{Ic}^2}{Y^2 (\sigma_f - \sigma_0)^2 \pi} \right)^{1-\frac{n}{2}} \right]^{-\frac{2}{n-2}} \quad (9)$$

The advantage of this formulation is that, except for the stochastic scattering of the failure stress, all remaining values are constant parameters. Furthermore, there are already many values for different glasses that can be found in literature, so that a larger experimental effort is not necessary.
In order to consider the orientation of the cracks, an angle $\phi \in [0, \pi]$ is assigned to each initial crack by means of a uniform distribution.
To determine the initial crack lengths, a statistical distribution of the failure stress is required for taking the statistical scatter of the initial crack lengths into account. This is determined by coaxial ring-on-ring tests on glass [13]. It is assumed that the failure stresses are subject to a two-parameter Weibull distribution with the scale parameter η and shape parameter β. Using the inverse function of the Weibull distribution $P(\sigma_f)^{-1}$, a general failure stress σ_f value can be calculated for each surface integration point during initialization with

$$\sigma_f = P(\sigma_f)^{-1} = -\eta \sqrt[\beta]{\ln(1-P)}, \quad (10)$$

where $P \in [0,1)$, is a uniform distributed random number. The distribution parameters are valid only for the test surface $A_{P(\sigma_f)}$ during the coaxial ring-on-ring tests. By limiting P, special cases such as the 5% quantile ($P \in [0, 0.05]$) can be considered for the design of structures or other applications.
In order to take other surface dimensions into account, the failure stress of the integration point $\sigma_{f,IP}$ is calculated for the element surface via the so-called Weibull surface shift

$$\sigma_{f,IP} = \sigma_f \left(\frac{A_{Shell}\, nip}{A_{P(\sigma_f)}} \right)^{\beta}. \quad (11)$$

The area of the integration point is defined as the shell surface A_{Shell} multiplied by the number of integration points nip on the surface. With Eq. (9) and the failure stress belonging to the integration point $\sigma_{f,IP}$, the initial flaw size for each integration point can be calculated.

Failure Calculation

After the initialization phase, the initial calculated subcritical cracks grow as a function of the applied stress state at the crack. Since the crack velocity depends on the stress rate, rate-dependent failure effects are considered.

In each time step m and for each integration point at the shell surface, the crack stress $\sigma_{flaw,m}$ is calculated with the crack angle ϕ by

$$\sigma_{flaw,m} = \frac{\sigma_{x,m} + \sigma_{y,m}}{2} + \frac{\sigma_{x,m} - \sigma_{y,m}}{2}\cos(2\phi) + \tau_{xy,m}\sin(2\phi) - \sigma_0. \quad (12)$$

Residual stresses resulting from the cooling process or a chemical strengthening can be considered by the residual stress σ_0. Subsequently, the corresponding stress intensity is calculated by

$$K_{I,m} = \sigma_{flaw,m}\sqrt{\pi a_{m-1}} \quad (13)$$

As during the determination of the initial crack lengths, the linear approximation from Figure 1 is again used to calculate the subcritical crack growth. The crack velocity multiplied by the size of the time step dt gives the increment between the new crack length a_m and the old crack length a_{m-1} according to

$$a_m = a_{m-1} + v_0\left(\frac{K_{I,m}}{K_{Ic}}\right)^n dt \quad (14)$$

Finally, failure occurs if the current stress intensity $K_{I,m}$ reaches the critical stress intensity K_{Ic}.

Due to its dependence on the time step, this model can only be used for explicit or implicit simulations with a small timestep at present. Furthermore, the growth of cracks after the initial failure is not considered so far. The Gauss-Lobatto integration is recommended since the outer integration points are then located at the surface of the element and no stress shift towards the surface is necessary.

Validation of the Stochastic Failure Model

The validation of the stochastic failure model is divided into two parts. Firstly, the mesh dependency is investigated by a constant surface test, see Fig. 2. The purpose of this test is, to check if the element edge length has an influence on the results.

Subsequently, coaxial ring-on-ring tests are performed to provide an experimental basis which serve as comparison for the numerical results.

Constant Surface Test

In order to proof the mesh dependency of the failure model, constant surface tests are simulated. A constant surface may not influence the stochastic failure results. Also, the failure of glass depends on the size of the glass surface, the input parameters do not need to be converted between different simulations with different element sizes if they have the same surface in sum. Accordingly, a constant surface test is performed instead of a single element test. For all simulations, a surface of 10x10 mm was discretised. A FE-mesh for 1x1 and 2x2 elements is shown in Fig. 2. The simulations were carried out with a discretisation of 1x1, 2x2, 4x4 and 10x10 elements for the quadratic surface of 100 mm^2. Element edge lengths of 1, 2.5, 5 and 10 mm were achieved accordingly. With the constant simulation surface, the failure distribution achieved after several runs should be able to reproduce the input distribution.

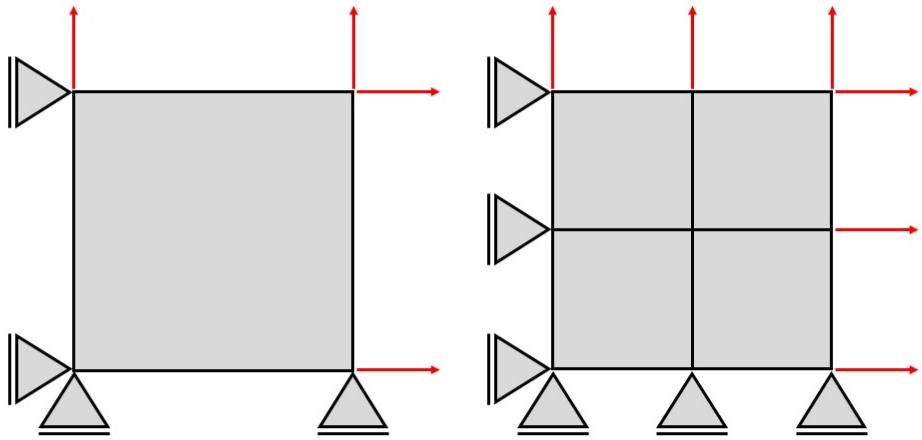

Figure 2: Constant surface test discretised with one (1x1) and four (2x2) elements.
Both tests have the same surface.

A biaxial ($\sigma_x = \sigma_y, \tau_{xy} = 0$) stress field with a constant stress rate was applied. As input parameters, the two parameter Weibull distribution was used with $\eta = 283.03$ MPa and $\beta = 4.01$ and the corresponding surface of $A_1 = 113.097$ mm^2. These values were determined by means of coaxial ring-on-ring tests and are explained in the next section in detail.
The scale parameter of the input values can be shifted for the expected simulation results by the Weibull surface shift by

$$\sigma_2 = \sigma_1 \left(\frac{A_1}{A_2}\right)^{\beta}. \qquad (15)$$

With the Weibull shift and the above-mentioned input parameters, a value of $\eta = 294.04$ MPa is to be expected as result from the simulations. The results from the different simulations are shown in Tab. 1. The deviation of the scale parameter is at maximum 4.56 %. Also, no systematic deviation is observable. Respectively, an influence of the element edge length can be neglected.

Table 1: Results from the single surface test.

Discretization [-]	No. Of. Sim. [-]	Weibull Parameter		Scale Deviation [%]
		Scale – η [MPa]	Shape – β [-]	
1x1	125	297.91	4.00	-0.27
2x2	125	298.20	4.17	3.92
4x4	125	294.12	4.20	4.56
10x10	125	308.07	3.99	-0.4

Coaxial Ring-on-Ring Test - Experimental Part
Soda-lime silicate float glass is investigated, which was originally intended for use as automotive windshields. Glass panes with the dimension of 1480x1000x1.8 mm were cut into circular samples with a radius of 40 mm. All specimens were heated to 520°C prior to testing and cooled to room temperature at a maximum rate of 2 K/min to remove any residual stress.

In order to validate the determined crack growth parameters, 300 coaxial ring-on-ring tests were performed. The tests consisted of constant stress rates of 0.2, 2 and 20 $MPas^{-1}$, with 100 tests each. The load ring has a radius of 6 mm, the support ring a radius of 15 mm following EN 1288-5 [5]. These distributions are determined to validate the crack growth parameters on the failure probability of distributions with different stress rates. The results are shown in Table 22.
Out of 300 tests performed, 127 can be considered as valid. A coaxial ring-on-ring test is considered valid, if the initial failure occurs within the load ring. The value of the scale parameter η rises with an increasing stress rate. This was to be expected, since at higher stress rates the cracks in the glass have less time for subcritical growth and a higher stress is required for failure. The shape parameter β describes the scatter of the values and does not show a dependence on the loading rate.

Table 2: Results for coaxial ring-on-ring tests with different stress rates

Stress Rate [$MPas^{-1}$]	Valid Tests [-]	Weibull Parameter	
		Scale – η [MPa]	Shape – β [-]
0.2	30	233.39	5.14
2	55	285.15	4.01
20	42	340.15	4.45

Coaxial Ring-on-Ring Test – FE Model

The experimental coaxial ring-on-ring tests were finally simulated for validation of the model. The finite element model is shown in Figure . The sample was divided into the test area (green) and the rest of the sample (blue) so that failure occurs solely in the test area.
The entire sample was modelled linearly elastic (***MAT_001**) with a Young's modulus of E = 74 GPa and a Poisson's ratio of υ = 0.23. The test area has been extended by the failure model using the present user material subroutine in LS-DYNA R11. The load and support ring were made of hardened steel during the experiments and are assumed to be rigid (***MAT_020**). The shell elements for the glass plies are modelled as fully integrated, four node shell elements (**ELFORM=16**) with five integration points through the shell thickness using Gauss-Lobatto integration. The loading rate was chosen so that the surface integration points in the test area are subjected to constant stress rate.

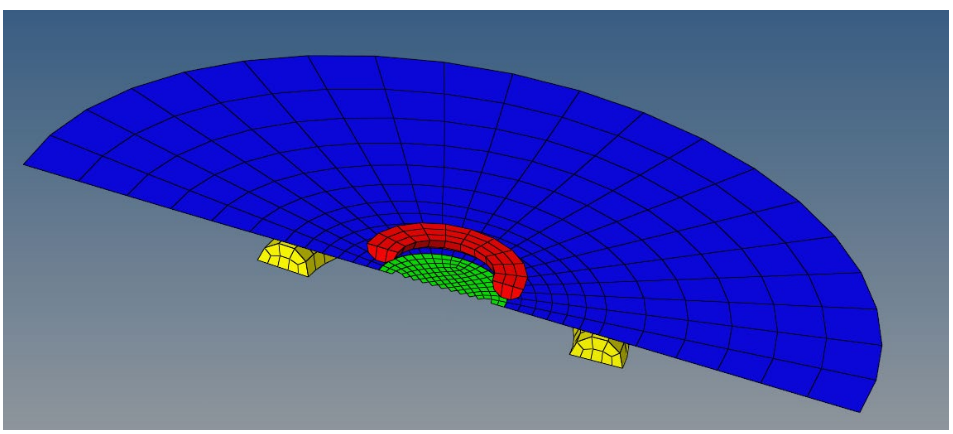

Figure 3: Cross-sectional view of the coaxial ring-on-ring model with load ring (red), support ring (yellow) and test surface. The specimen is divided into the non-failure (blue) and the failure area (green). A constant stress rate is realised by a displacement-controlled movement of the load ring.

Coaxial Ring-on-Ring Test – Failure Model Validation

To validate the stochastic failure model, the stress rate and the size of the simulated surface are varied. The crack velocity depends on the stress rate. At a low stress rate, cracks can grow subcritical for a longer time and a lower failure stress is required to reach the critical stress intensity. At a high stress rate, a higher stress is required to reach the critical stress intensity. As input parameters for the validation of the rate-dependent values from Table 22, 200 double-ring bending test simulations are performed for each stress rate. The Weibull parameters of the distribution with a stress rate of 2 $MPas^{-1}$ are used as input parameters for the failure distribution, these are the scale parameter $\eta = 287.45$ MPa and the shape parameter $\beta = 4.13$ with a corresponding test area of $A_{experiment}$ = 113.097 mm^2. As critical stress intensity $K_{Ic} = 0.75$ $MPam^{0.5}$ and as lower limit for crack growth $K_{th} = 0.25$ $MPam^{0.5}$ was used. The crack growth parameters were determined to $n = 15.098$ and $v_0 = 10.22$ mm/s in another study [15].

Table 3 shows the results of the stochastic simulations in dependence of the stress rate. The position parameter η of the experimental values could be simulated with an error of maximum 6.35 % deviation in comparison to the experimental values. The scale parameter β is nearly constant to the input value.

Table 3: Numerical Results for coaxial ring-on-ring tests with different stress rates and the deviation of the scale parameter to the experimental values in Table 22.

stress rate [$MPas^{-1}$]	Weibull parameters		scale deviation [%]
	scale – η [MPa]	shape – β [-]	
0.2	249.22	4.13	6.35
2	287.45	4.13	0.80
20	331.07	4.10	-2.74

In order to represent the stochastic failure of glass correctly, the size effect must also be considered as a kind of regularisation. The failure of glass is dominated by randomly distributed cracks on the surface. As the test area becomes larger, the probability of observing a larger and therefore more critical crack increases accordingly. Vice versa, a larger test area correlates with a decreasing failure stress. Analytically, the size effect can be determined via the Weibull shift by Eq. (15).

The stress σ_2 of a reference area A_2 can be determined via a stress σ_1 with associated area A_1 and Weibull shape parameter β. In the present work three different sizes of the test surface are simulated and the scale parameter of the Weibull distribution is compared between simulation and analytical values using the Weibull shift. The results are shown in Table 2. It can be seen that the presented failure model also takes size effects into account which is important to avoid mesh dependencies. The maximum deviation between numerical and analytical values is 3.47 %.

Table 4: Numerical Results for coaxial ring-on-ring test simulations with different simulation surfaces and the deviation to the analytical values according to Eq. (15).

Surface – A [mm2]	$Scale_{Analytical}$ – η [MPa]	$Scale_{Simulation}$ – η [MPa]	Deviation [%]
253.42	249.22	225.42	-3.47
505.69	287.45	196.53	0.08
913.41	331.07	171.77	1.34

Summary

A new model for the stochastic fracture behaviour of glass is presented. The model is based on the subcritical crack growth of glass and can reproduce the initial point of failure and its scattering. As input parameters the Weibull distribution and crack growth parameters are required, which are already available in the literature for several glass types.

The present model was capable to reproduce the rate-dependent failure of glass with an error of maximum 6.35 % in comparison to the experimental values from coaxial ring-on-ring tests. The influence of the size of the test area could be simulated with a maximum error of 3.47 % in comparison to analytical values. Furthermore, it could be proven, that the element size has no systematic influence on the results. The model can also consider residual stresses due to the cooling process or the chemical strengthening of glass.

Further development

In future investigation, the failure will be validated as a function of stress state in order to perform stochastic simulations of the head impact of a pedestrian on an automobile windshield. The final aim is to predict the scatter of the injury probability by numerical simulation.

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