A Unified SPH-DEM-FEM Approach for Modeling of Debris Flow Impacts on Protective Structures

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Abstract

Debris flows are rapid gravity-driven unsteady flows of highly concentrated mixtures of water and solid particle material, destroying numerous mountain building structures and traffic facilities. Investigation of debris flows is thus of significance to hazard prevention and mitigation. This paper aims to provide a numerical model capable of reproducing the debris flow impact estimation by accounting for complicated fluid-particle-structure interaction (FPSI) with a unified Smoothed Particles Hydrodynamics (SPH), Discrete Element Method (DEM) and Finite Element Method (FEM) approach. The fluid phase is represented by SPH. The solid phase consists of physical particle(s) is represented by DEM, and deformable solid structure is represented by FEM. The Wenjia gully debris flow is carried out to demonstrate the capability of the coupled model for simulating FPSI as the application of the debris flow impact simulations. Compared the actual situation, the propagation of the debris flow and destruction of structures were predicted. Then, the effectiveness of the treatment measures of the Wenjia gully debris flow was clarified from the impact evolution, impact force. The developed method will contribute to a better understanding of FPSI and is a promising tool for hazard analysis and mitigation.

Keywords: debris flow; impact estimation; coupled SPH-DEM-FEM approach; fluid-particle-structure interaction

Introduction

The mountainous areas in Southwest China have experienced numerous landslides, especially following the 2008 Wenchuan earthquake, which provide abundant material for the initiation of debris flows. When a heavy rainfall occurs, these landslides will collide with the mountains and slide down the slopes, leading to debris flows [1]. Debris flows are rapid gravity-driven unsteady flows of highly concentrated mixtures of water and solid particle material with large grain size distribution. These flows commonly occur without warning in mountainous regions around the world and usually cause massive losses of both human lives and properties. It has been reported that these post-earthquake debris flows destroyed numerous houses and traffic facilities in the earthquake-attacked areas [2].

![Fig. 1 Check dam in Qipan gully, Wenchuan county, Sichuan province, China](image-url)
Structures have to be designed to resist the dynamic impact force generated by the debris flow. To conduct the dynamic characteristics analysis, there are three key points to be addressed: (a) the debris fluid-debris particle interaction (FPI) process, (b) debris fluid-structure interaction (FSI) process, and (c) the debris particle-structure interaction (PSI) process. To date, due to various practical limitations and challenges, the three key points are rarely studied in detail simultaneously, that is, the debris fluid-debris particle-barrier structure interaction has rarely been studied in detail. In this study, a coupled model using a unified SPH-DEM-FEM approach for FSPI was implemented into the code of LS-DYNA® with the aim to take advantages of these three methods.

### Coupling schemes for the hybrid approach

#### Fluid–particle interaction

SPH and DEM are coupled in the following way: DEM model particles are inserted into SPH model for interpolation to obtain hydrodynamic interaction force, and then hydrodynamic interaction force from fluid is obtained according to "Newton's third law". A penalty method-based node to node contact model is introduced on the interfaces of the SPH part and DEM part. As shown in Fig. 2, all the SPH interpolations (density, pressure and so on) are carried out inside the local domains of each SPH part.

![Fig. 2 SPH and DEM interaction through node to node contact](image)

In this system, the contact force acting on particle due to contact, is directly proportional to the displacement or overlap between SPH particle and DEM particle $\delta_{sd}$:

$$F_{sd} = K_{sd} \delta_{sd}$$

where $\delta_{sd} = d - h_i - r_i$ and $K_{sd}$, is the linear-spring stiffness. In reality, some kinetic energy is dissipated in plastic deformation, and/or converted to heat or sound energy. To account for those energy losses, a contact damping force based on a dashpot model is also included:

$$F_{\eta} = \eta_{sd} v_{sd}$$

The contact damping force is proportional to the relative velocity $v_{sd}$ of the SPH particle and DEM particle, where the constant of proportionality $\eta_{sd}$ is known as the damping coefficient.

#### Fluid–structure interaction

To make the most of advantages of both SPH and FEM, the coupling between SPH and FEM is needed. In general, the coupling algorithm is based on a master-slave algorithm to account for the contact interaction of FE and SPH. SPH is defined as slave node and FEM is defined as master segment. The contact algorithm will detect when a SPH particle penetrates the segments of the surface of the FE. The contact thickness indicates the distance away from a contact face where physical contact is established (Fig. 3). Whenever a node enters this
detection region, it is repelled by a force that is a proportional function of the predicted penetration or contact spring stiffness.

![Contact model diagram](image)

**Fig. 3** SPH particles contact with finite elements

The overlap \( \delta_{sf} \) is calculated as follows:

\[
\delta_{sf} = \begin{cases} 
  h_1 - d_{min}; & \text{for solid} \\
  h_1 + h_2 / 2 - d_{min}; & \text{for shell}
\end{cases}
\]

where \( d_{min} \) is the distance between the center of the SPH particle and the surface of the contact segment.

The normal contact force \( f_{sf,n} \) between interacting particles is calculated as follows:

\[
f_{sf,n} = (-k_{sf,n}\delta_{sf} + c_{sf,n}\delta_{sf})n
\]

where \( k_{sf,n} \), \( c_{sf,n} \), \( \delta_{sf} \), and \( n \) are the normal spring stiffness, normal damping coefficient, the relative normal velocity, and the unit normal displacement vector respectively.

The tangential contact forces \( f_{sf,t} \) between interacting particles is calculated as follows:

\[
f_{sf,t} = \begin{cases} 
  (-k_{sf,t}\delta_{sfa} + c_{sf,t}\delta_{sfa})n; & \text{if } |f_{sf,n}| \mu > -k_{sf,t}\delta_{sfa} + c_{sf,t}\delta_{sfa} \\
  (-k_{sf,t}\delta_{sfa} + c_{sf,t}\delta_{sfa})n; & \text{if } -k_{sf,t}\delta_{sfa} + c_{sf,t}\delta_{sfa} > |f_{sf,n}| \mu \\
  -k_{sf,t}\delta_{sfa} + c_{sf,t}\delta_{sfa}; & \text{otherwise}
\end{cases}
\]

where \( k_{sf,t} \), \( c_{sf,t} \), \( \delta_{sfa} \), and \( \mu \) are the tangential spring stiffness, tangential damping coefficient, the incremental tangential displacement, and the friction coefficient respectively. \( k_1 \) is taken as \( k_n \).

\( k_n \) is calculated as follows (Feng et al. 2019):

\[
k_n = \begin{cases} 
  k_1 \frac{Ks}{\text{max}(\text{shell diagonal})}; & \text{for shell} \\
  k_1 \frac{Ks^2}{V}; & \text{for solid}
\end{cases}
\]

where \( k_1 \) is a penalty scale factor; \( K \) is the material bulk modulus; \( s \) is the segment area; \( V \) is the element volume.
Particle–structure interaction

The interaction between contact surfaces is handled following the penalty method (Fig. 4). In this approach, the finite-element program searches for intersections between the particles and the solid or shell. In general, the coupling algorithm is based on a master-slave algorithm to account for the contact interaction of FE and DEM. DEM is defined as slave node and FEM is defined as master segment. The contact algorithm will detect when a DEM particle penetrates the segments of the surface of the FE. The contact thickness indicates the distance away from a contact face where physical contact is established. Whenever a node enters this detection region, it is repelled by a force that is a proportional function of the predicted penetration or contact spring stiffness [3]. The DEM-FEM contact force calculation is same to the SPH-FEM contact force calculation. These specific formulas are not expanded.

Fig. 4 DEM particles contact with finite elements

As before-mentioned, the combined finite-discrete element method proposed in this paper is focused on dynamic simulation, and the Central Difference Method (CDM) is employed to solve the equations. Since CDM is conditional convergence, the step must satisfy the numerical stability conditions. SPH, DEM and FEM adopt the conditional stable central difference method, and their coupling requires that their integrals must be synchronized, which requires both to adopt the same time step for under the same calculation framework. The time step $\Delta t_{\text{SPH-DEM-FEM}}$ takes the smaller value of both.

$$\Delta t_{\text{SPH-DEM-FEM}} = \min(\Delta t_{\text{SPH}}, \Delta t_{\text{DEM}}, \Delta t_{\text{FEM}})$$

where $\Delta t_{\text{SPH}} = \beta_s h / c$, $\Delta t_{\text{DEM}} = \beta_d 0.2\pi \sqrt{m_0 / K_{\text{spring}}}$ and $\Delta t_{\text{FEM}} \leq \beta_f L_{\text{min}} / c$. $c$ is the material sound speed. $\beta_s$, $\beta_d$ and $\beta_f$ are the scaling coefficient of time step length. $m_0$ is the DEM particle mass. $K_{\text{spring}}$ is the contact spring stiffness of DEM particle, and $L_{\text{min}}$ is the minimum finite element size.

Numerical examples

Wenjia gully debris flow

The Wenjia Gully is located in the north of Qingpingchang town in the mountainous area northwest of Mianzhu city, Sichuan province. The Wenjia gully is 3.6 km from the Yinxiu-Beichuan fault. The strata exposed in the Wenjia Gully drainage area belong to the Qingping Group (Cambrian period) and the Guanwushan Group (Devonian period). The 2008 Wenchuan earthquake triggered a large-scale coseismic landslide in Wenjia gully. The 2008 Wenchuan earthquake left approximately $20 \times 10^6$ m$^3$ of loose geomaterials at the Deposition Area I (an altitude between 1599 and 1890 m), and approximately $30 \times 10^6$ m$^3$ of loose geomaterials at the Deposition
Area II (1300 platform and downstream) (Fig. 12b). The Deposition Area II is the source area for the debris flow outbreaks after the earthquake. The deposits have a maximum thickness of 150 m. The grain composition and grain-size distribution of deposits on the 1300 m platform are complex and poorly sorted. Material with a grain size $<1$ cm is 30%. Material diameter between 1 and 20 cm is 60%, and the part of the diameter more than 20 cm is 10%. The maximum diameter is above 160 cm [4]. Under continuous and heavy rainfall, these loose geomaterials were relatively extensive in size and finally formed the material source for debris flows.

On 12 August 2010, a heavy rainstorm began in Wenjia gully and finally resulted in the “8.13” debris flow in the gully with a peak discharge of 1530 m$^3$/s and a total volume of $3.1\times10^6$ m$^3$. The material was composed of limestone rock fragments, clay, and silty clay with an average density of 1970-2190 kg/m$^3$. The yield stress of debris flow slurry is very high, reaching 10.029 kPa, and the viscosity coefficient is 136 Pa·s (Zhang et al. 2013; Dai et al. 2016). This mixture finally flowed into the Mianyuan River and reached 1500 m from downstream. During the propagation, the debris flow lasted approximately 3 hours, destroyed most of the downstream dam in the catchment, caused the deposition of up to 15 m in the downstream river, killed seven people, left five people missing and 39 people injured, and buried 479 houses.

Fig. 5 The catchment of the Wenjia Gully: 1-Guanwushan group (Devonian), 2-Qingping group (Cambrian), 3-main deposition areas, 4-Dingziya fault, 5-boundary of landslide source area, 6- Last check dam, 7-deposition area of the August 13 debris flow, 8-houses [4]
In this study, a pseudo-three-dimensional numerical model was used for analysis, that is, the profile of Branch 2 is selected to make a two-dimensional model, and then the profile is stretched along the width of 7 m. The last check dams are 8 m high and 2.0 m thick. Ground sills are arranged at an altitude of 1075-985 m at the bottom of the trench at an interval of 20 m, with a height of 2 m. 300 m away from the last check dam, there are three same houses. Based on actual experience, the height of each floor is set to 3.6 m, and the length is set to 6 m. The width of the house is set to 4 m (x direction), and the number of floors is set to 3. The spacing of houses is 20 m. The numerical model is shown in Fig. 6.

1. Constitutive models of the material

1) Channel. Rigid shell elements are adopted for the simulation, which can greatly reduce the analysis time. The density of the material is 3000 kg/m$^3$, the modulus of elasticity is 30 GPa, and Poisson's ratio is 0.24. The LS-DYNA material model MAT_RIGID is used to model the channel shell elements. The mesh size shell elements is 2.0 m.

2) Debris fluid. Debris fluid belongs to Non-Newtonian fluid, and the constitutive relation and dynamic characteristic is very complex. LS-DYNA material model MAT_ELASTIC_PLASTIC_HYDRO was used to model the slurry [1]. The definition of EOS (equation of state) is a linear polynomial equation of state with weak compressibility. The fluid was simulated by 40712 SPH particles, and the particles spacing is 2.0 m. The material parameters are shown in Table 3.

3) Debris particle. In the debris flow, the content of particles with a diameter greater than 1 m is assumed to be 0.5%. The density of the material is 3000 kg/m$^3$. The elastic modulus is 30 GPa, and Poisson's ratio is 0.3. The LS-DYNA material model MAT_ELASTIC is used to model the boulders [3]. There are 213 spherical particles with diameter distribution between 1 m and 2 m. The values of the parameters between the particles are as follows: the normal damping ratio $\eta_n$ is set at 0.7, and the tangential damping ratio is set at 0.4. The sliding friction coefficient is set at 1.0, and the rolling friction coefficient is set at 0.1. The normal spring stiffness is set

![Fig. 6 Numerical model of Wenjia gully debris flow](image)

**Table 1** model parameters of slurry material

<table>
<thead>
<tr>
<th>Density (kg/m$^3$)</th>
<th>Shear modulus (MPa)</th>
<th>Yield stress (kPa)</th>
<th>$C_1$</th>
<th>$C_0$, $C_2$, $C_3$, $C_4$, $C_5$, $C_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>1.68</td>
<td>10</td>
<td>20 GPa</td>
<td>0</td>
</tr>
</tbody>
</table>


4) Check dam. The LS-DYNA material model MAT_PLASTIC_KINEMATIC is used to model the check dams. This model is adopted to simulate building elements. This model can describe strain hardening, strain rate hardening, thermal softening, and fracture/failure of materials, which is commonly used in impact simulations [6]. The dams are all simulated with solid elements. The material density is 2300 kg/m³. The elastic modulus is 8 GPa, and Poisson’s ratio is 0.23. The yield strength is set as 8 MPa [5], and the maximum principal strain of 0.0033 is selected for the dam.

5) Ground sills. The LS-DYNA material model MAT_PLASTIC_KINEMATIC is used to model the ground sills. The ground sills are all simulated with solid elements. The material density is 2300 kg/m³. The elastic modulus is 8 GPa, and Poisson’s ratio is 0.23. The yield strength is set as 8 MPa, and the maximum principal strain of 0.0033 is selected for the dam.

6) Houses. To simplify the houses model, the walls of the houses are made by brick with a thickness of 240 mm. The roof slab is made by concrete slab with a thickness of 100 mm. The houses are all simulated with shell elements. The LS-DYNA material model MAT_PLASTIC_KINEMATIC is used to model the houses. For concrete material, the material density is 2400 kg/m³. The elastic modulus is 28 GPa, and Poisson’s ratio is 0.2. The yield strength is 16.7 MPa [6], and the maximum principal strain of 0.05 is selected [1]. For brick material, the material density is 2300 kg/m³. The elastic modulus is 8 GPa, and Poisson’s ratio is 0.23. The yield strength is set as 8 MPa [5], and the maximum principal strain of 0.0033 is selected.

2. Boundary conditions

Six degrees of freedom (DOFs) of the channel are fixed. Six degrees of freedom (DOFs) of the houses bottom are fixed. Six degrees of freedom (DOFs) of the dam bottom and side are fixed. The friction coefficient between debris flow and building varies widely. In this paper, the friction coefficient between the debris flow and the channel is set as 0.3, and the friction coefficient between the debris flow and the dams is also set as 0.4. The friction coefficient between the debris flow and the houses is set as 0.4. The gravitational acceleration is 9.8 m/s². The calculation time is set as 120 s, which can simulate the complete impact process of the debris flow.

Comparative analysis with the actual situation

As shown in Fig. 7, the debris flow impingement process caused the destruction and burial of check dams and houses. The check dam is the first to be impacted. Due to the comprehensive action of debris fluid and debris particles and the low material strength of the dam, the whole dam is destroyed, which is similar to the destruction of the check dam in the actual process (Fig. 7a). Secondly, the debris flow spreads to the houses. The whole wall is destroyed, and then the houses are buried (Fig. 7b). The debris flow eventually deposited at the mouth of gully, and the deposition height reaches 17 m, which is only 13% different from the actual situation (deposition height 15 m) (Fig. 7c). The above results fully show that the coupled SPH-DEM-FEM model in this study can successfully predict the debris flow propagation and the impact destruction on the structures.
Fig. 7 Comparative analysis with the actual situation (a) damaged dam, (b) damaged houses, and (c) deposition height.
Treatment measure

Wenchuan earthquake caused a large landslide in Wenjiagou, which disintegrated into debris flow and accumulated in the gully, becoming a high frequency debris flow gully. According to the characteristics of debris flow in Wenjiagou, the treatment measures of "water and sediment separation, consolidation and barrier combination, monitoring and maintenance" were adopted in the late stage. The following will focus on introducing and discussing "consolidation and barrier combination" measure. The "consolidation and barrier combination" measure concludes two systems (Fig. 8). In the "artificial step-pool system", for the height difference of more than 50 m, 9 step-pools with 6 m high are set to dissipate energy. When it reaches the downstream, its potential energy and scale will be greatly reduced (an altitude between 1035 and 980 m). In the "slit dam system", the last line of defense is the three slit dams built in downstream. (an altitude between 980 and 950 m). The slit dams are 10 m high and 3 m thick.

![Fig. 8 "Consolidation and barrier combination" treatment measure of Wenjia gully debris flow](image)

1 Impact process

Fig. 9 presents the simulated propagation of the Wenjia gully debris flow. When t = 26 s, the front velocity of the debris flow reaches 31 m/s. The front of the debris flow passes through the deep pool system, and the debris particles and debris fluid move together. Obviously, due to the friction effect of the channel bottom, on the same cross section of downstream, the debris flow near the bottom of the channel bottom moves slowly while the debris flow away from the bottom of the channel moves quickly (Fig. 9a). When t = 34 s, the front of the debris flow reaches slit dam 1, and the front velocity of the debris flow reaches 34 m/s (Fig. 9b). When t = 46 s, slit dam 1 has been filled with debris flow and overflow occurs. The remaining debris flow flows to slit dam 2 and the front velocity of the debris flow reaches about 30 m/s (Fig. 9c). When t = 160 s, the debris flow is basically in a stable and static state. At this time, slit dam 1 and 2 have been filled, and slit dam 3 has also consumed part of the storage capacity. The debris flow did not bury the houses, which shows the effectiveness of the treatment measures. The debris flow deposited in the slit dams formed static pressure load, and the debris particles were successfully intercepted in the interior of the slit dams to avoid harm to the downstream houses and traffic facilities. The simulated interception state is consistent with the actual debris flow interception state. When the debris flow occurs again, the slit dams have almost no interception ability due to the presence of these sediment deposit, so the slit dams must be cleaned in time. (Fig. 9 d).
Fig. 9 Simulated propagation of the Wenjia gully debris flow with artificial step-pool system and slit dams after treatment
2. Impact force

The evolution of the impact force is shown in Fig. 10. As shown in Fig. 17a, the debris fluid impact force of dam 1 is the largest ($65.7 \times 10^3$ kN) and that of dam 3 is the smallest ($12.5 \times 10^3$ kN). Because the velocity of debris fluid impacting dam 1 and dam 2 is relatively large, the impact force values have obvious peak value. When the debris fluid reaches dam 3, the impact velocity of the debris fluid is very small, so there is no obvious peak of its impact force. Comparing the three curves, due to the different deposition height of debris flow in the dams, the final steady impact force values are also slightly different. As shown in Fig. 17b, the impact of dam 1 is the largest ($15.9 \times 10^3$ kN), followed by that of dam 2 ($5.7 \times 10^3$ kN), and that of dam 3 is almost 0. Because the impact force of debris particles is close to the concentrated load, it has a great influence on the local damage of the dam. Compared with the impact force of the debris particles and the debris fluid, the impact effect of the debris fluid is the overall distributed load, while the impact effect of the debris particles is the local concentrated load, so the damage degree of the debris particles to the dam may be greater.

![Fig. 10 Impact force evolutions of the slit dams (a) debris fluid and (b) debris particles](image)

Conclusions

A rational prediction of debris flow propagation and impact behavior of debris flow is required in hazard analysis and mitigation design. In this work, a coupled fluid-particle-structure numerical model based on coupled SPH-DEM-FEM was established to predict the propagation of the debris flow and destruction of structures. The Wenjia gully debris flow was simulated as examples to apply the coupled SPH-DEM-FEM model. Compared the actual situation, the propagation of the debris flow and destruction of structures were predicted accurately. The effectiveness of the treatment measures of the Wenjia gully debris flow was clarified from the impact evolution, impact force and energy dissipation mechanism. After the treatment, the debris flow did not bury the houses during impact process, which shows the effectiveness of the treatment measures. Compared with the impact force of the debris particles and the debris fluid, the impact effect of the debris fluid could be regarded as the overall distributed load, while the impact effect of the debris particles could be regarded as the local concentrated load, so the damage degree of the debris particles to the dam may be greater. The numerical results could provide a basis for the design of appropriate protective measures.
References