

# Acoustic Radiated Power and Radiation Efficiency Calculation with LS-DYNA<sup>®</sup>

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## Abstract

*The keyword \*FREQUENCY\_DOMAIN\_SSD in LS-DYNA not only provides convenient solution for steady state vibration analysis for structures, but also raises the possibility for acoustic simulation. For example, it can be combined with acoustic boundary element method (keyword \*FREQUENCY\_DOMAIN\_ACOUSTIC\_BEM) or acoustic finite element method (keyword \*FREQUENCY\_DOMAIN\_ACOUSTIC\_FEM), to compute the acoustic pressure and sound pressure level for vibro-acoustic problems. In addition, with the option ERP for this keyword, one can perform ERP (Equivalent Radiated Power) analysis to get a quick solution for radiated noise, based on the plane wave assumption for the acoustic waves.*

*A new parameter RADEFF has been added to the keyword \*FREQUENCY\_DOMAIN\_SSD\_ERP to run acoustic radiated power computation for baffled plates, and also computes the radiation efficiency.*

*With some examples, this paper explains the difference between the ERP (equivalent radiated power) and ARP (acoustic radiated power) and shows how to use this new parameter to compute the acoustic radiated power and radiation efficiency for vibrating structures.*

## Introduction

Starting from R7 of 971 version, a keyword \*FREQUENCY\_DOMAIN\_SSD has been introduced in LS-DYNA to run steady state dynamic analysis. It provides vibration response due to harmonic loading in frequency domain. This provides also possibility for some other simulations, for example, acoustic computation. Usually engineers and researchers use finite element method or boundary element method to run acoustic or vibro-acoustic simulation. These methods require explicit finite element or boundary element mesh for the acoustic domain. A complex variable equation system is established and solved for each excitation frequency. As a comparison, the SSD based acoustic approach is much simpler as it does not require any acoustic elements. Based on the vibration results from SSD, one can compute the acoustic intensity on the surface of structures, with some assumptions. Then with an integral on the elements on the surface, one can get estimated radiated acoustic power from the structure. This paper introduces three methods to compute the radiated acoustic power based on SSD: 1) ERP; 2) ERP with corrected radiation efficiency at lower frequencies; and 3) Using Rayleigh Integral.

## Review of the theory for computation of acoustic power

The acoustic intensity is defined as

$$I(r_p) = \text{Re} \left[ p(r_p) \cdot v_n(r_p)^* \right] / 2 \quad (1)$$

where  $p(r_p)$  is acoustic pressure,  $v_n(r_p)$  is the normal velocity at the surface.

For ERP, the plane wave assumption is used. As the result,

$$Z = \frac{P}{v_n} = \rho c \quad (2)$$

where  $\rho$  is the density of the fluid (e.g. air),  $c$  is the sound speed in acoustic fluid. For air,  $\rho=1.21 \text{ kg/m}^3$  and  $c = 340 \text{ m/s}$ .  $Z$  is acoustic characteristic impedance.

With some manipulation of the equation (2), one can get

$$p(r_p) = \rho c v_n \quad (3)$$

The ERP absolute value can be computed as integral of acoustic intensity over the surface  $S$ :

$$W_{ERP} = \int_S I(r_s) dS = \rho c \int_S v_n(r_s) v_n^*(r_s) dS \quad (4)$$

As pointed in reference [1], corrected radiation efficiency for low frequencies is given as

$$\sigma = 1 - \frac{J_1(2kR)}{kR} \quad \text{I} \quad (5)$$

Where,  $k = \omega/c$  is the wave number and  $R$  is the radius of a rigid circular piston. In equation (5), the Bessel Function  $J_1$  is dependent on the Helmholtz number  $kR$ . With the introduction of the corrected radiation efficiency, the equation (4) can be rewritten as

$$W_{ERP\_c} = \left(1 - \frac{J_1(2kR)}{kR}\right) \rho c \int_S v_n(r_s) v_n^*(r_s) dS \quad (6)$$

For pressure given by Rayleigh Integral, it is given as

$$p(r_p) = \frac{i\omega\rho}{2\pi} \int_S v_n(r_p) \frac{e^{-ikR}}{R} dS \quad (7)$$

And the radiated power by Rayleigh Integral is given as below

$$W_{RI} = \int_S I(r_s) dS = \frac{\omega\rho}{4\pi} \int_{S'} \int_S v_n(r_s) \frac{\sin kR}{R} v_n^*(r_s) dS dS' \quad (8)$$

## Keywords

A typical keyword for running ERP and acoustic radiated power by Rayleigh integral is given below. The option `_ERP` after `*FREQUENCY_DOMAIN_SSD` indicates that the calculation for ERP (Equivalent Radiated Power) is requested. With this option, cards 4, 5 for the keyword `*FREQUENCY_DOMAIN_SSD` are required [2]

```

*FREQUENCY_DOMAIN_SSD_ERP
$#  mdmin  mdmax  fnmin  fnmax  restmd  restdp
$#    1      20    0.000  0.000
$#  dampf  lcdam  lctyp  dmpmas  dmpstf
$#  0.010000  0      0      0.000  0.000
$#                                nerp  strtyp  nout  notyp  nova
$#                                1
$#    ro      c  erprlf  erpref  radeff
$#  1.21    340.      1.      1
$#    pid  ptyp
$#    1      2
$#  nid  ntyp  dof  vad  lc1  lc2  lcflag  vid
$# horizontal base acceleration
$#                                1      3      200
*DATABASE_FREQUENCY_BINARY_D3SSD
$# binary
$#    1
$#  fmin  fmax  nfreq  fspace  lcfreq
$#  10.  1000.  100
    
```

To use the corrected radiation efficiency for low frequencies for ERP results, one can set “ERPREF” = -1.

### Example: a simplified engine model

This example considers a simplified auto engine model. It has 16041 nodes and 13484 Hexahedron elements. Elastic material is used for this model. The engine model is constrained to a shaker table from the base (see Figure 2). The harmonic excitation is given in x-direction through the shaker table. The range of excitation frequency is 10-1000 Hz, with frequency step 10 Hz. The excitation is given in the form of constant acceleration amplitude 0.02 g, for the whole range of frequency.

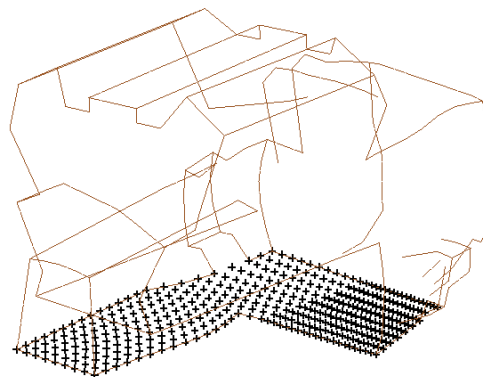
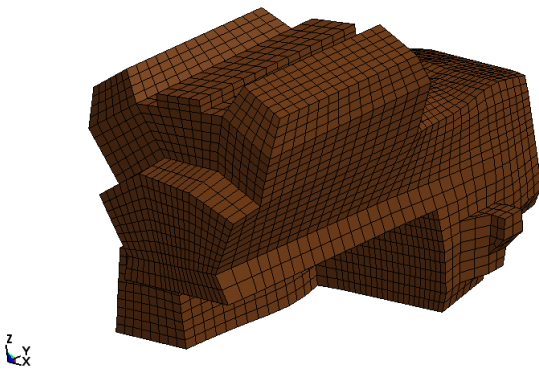


Fig 1. FEM mesh of a simplified auto engine model

Fig 2. Constraints of the engine model

The whole outer surface of the engine model is taken as the acoustic radiation surface. With ERP option and air density, sound speed in air, as well as the radiation panel defined in card 4 and 5, one can get the ERP results in the form of

- 1) Binary plot database D3ERP, which shows the ERP density on the surface, and
- 2) ERP\_ABS and ERP\_DB for the ERP absolute value, and dB values (if a reference ERP value is provided in the position of ERPREF in card 4).

Particularly, the ERP density fringe plot at frequency 100 Hz and 500 Hz are shown in Figures 3 and 4

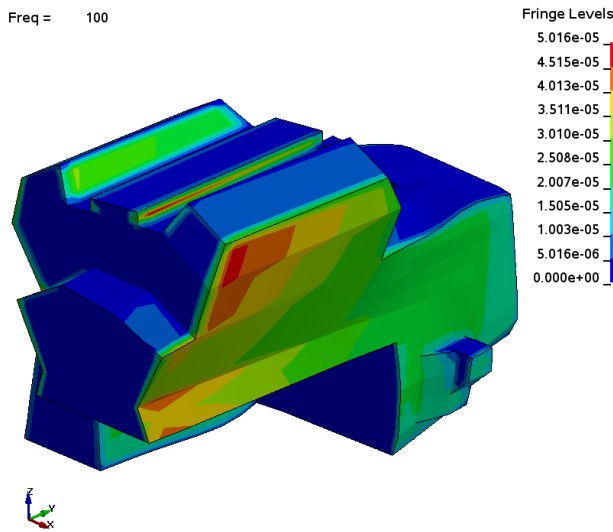


Fig 3. ERP density at frequency 100 Hz

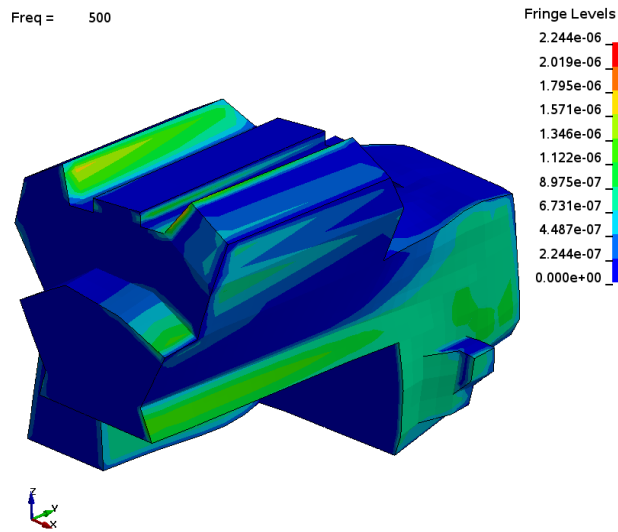


Fig 4. ERP density at frequency 500 Hz

The ERP absolute value for the whole range of frequency 10-1000 Hz can be found in Figure 5.

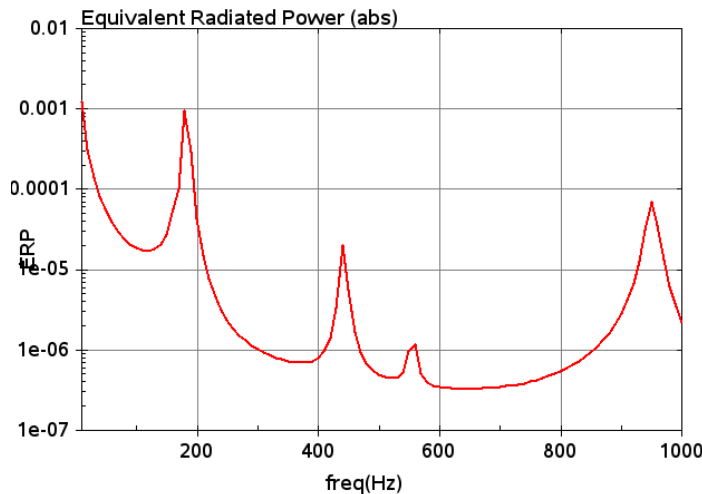


Fig 5. ERP absolute value (watt) vs. frequency

Since the “RADEFF” is set to be 1 in the input deck, LS-DYNA also runs radiated acoustic power calculation based on Rayleigh integral, and calculates the radiation efficiency, which is given as a ratio between the radiated acoustic power based on Rayleigh integral, and the acoustic power based on plane wave assumption (which is ERP absolute value computed before). By setting “ERPREF” = -1, one can also introduce the correction on the radiation efficiency for low frequencies for ERP and get a “corrected” ERP absolute value. Figure 6 below shows the radiated acoustic power, given by the three methods: 1) classic ERP; 2) ERP with corrected radiation efficiency for lower frequencies; and 3) Rayleigh integral.

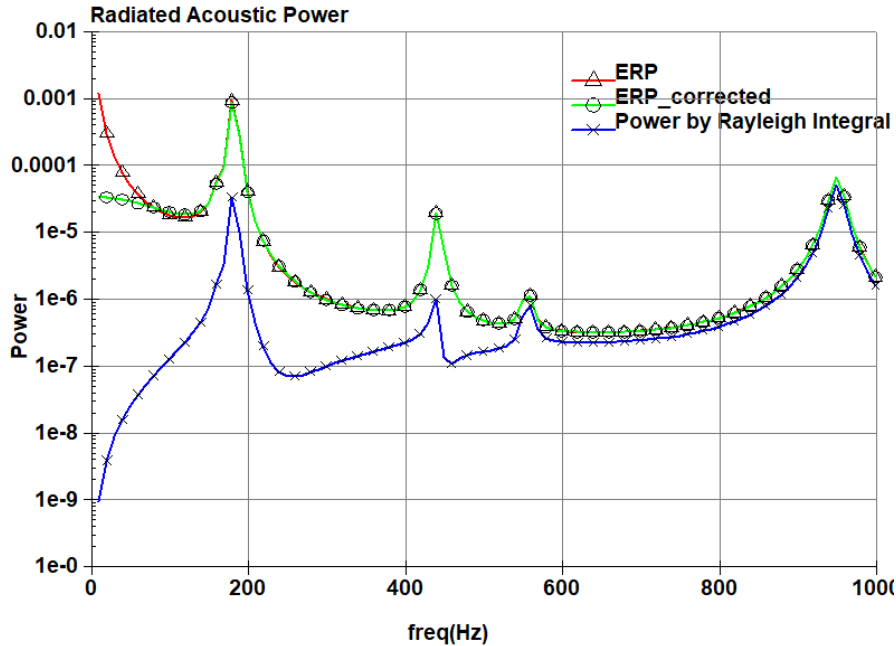


Fig 6. Radiated acoustic power (watt) by classic ERP, corrected ERP and Rayleigh integral

As can be seen in Figure 6, the radiated acoustic power by the classic ERP, the ERP with corrected radiation efficiency at low frequencies, and Rayleigh integral method have a good match at higher frequencies (e.g. frequency over 600 Hz). For the lower frequencies, the radiated acoustic power calculated by the Rayleigh Integral method is much lower than those given by ERP methods (classic one and the one with low frequency radiation efficiency correction). Besides, from Figure 6, one can see that the correction on the ERP absolute value by equation (6) has only influence on the results in low frequency range.

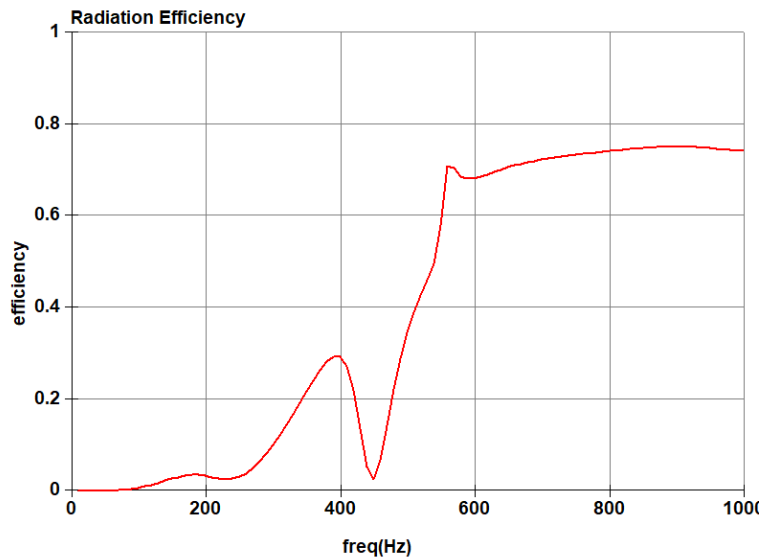


Fig 7. Radiation efficiency

Figure 7 shows radiation efficiency, which is given as a ratio between acoustic power by Rayleigh integral and ERP. One can see that the radiation efficiency is much higher and approaching to 1 for the higher frequency range.

## **Conclusion**

This paper introduces three acoustic radiated power computation methods for vibrating structures, based on the keyword \*FREQUENCY\_DOMAIN\_SSD. They are all much cheaper and faster than the standard finite element or boundary element acoustic methods. They help to characterize the structure borne noise quickly. They can be very useful in the early design phase of product development. The three methods are based on different theories and assumptions. It is important for user to understand the difference in the corresponding assumptions in order to use them appropriately.

## **References**

- [1] Münch, H. Equivalent Radiated Power - Sensibilisierung für Grenzen und Potenziale einer akustischen Berechnungsmethode, Master of Science Study Work, Friedrich-Alexander University, Erlangen, (2014).
- [2] Livermore Software Technology Corporation, LS-DYNA Keyword User's Manual, 2019.