Adaptive Smoothed Particle Hydrodynamics and Higher Order Kernel Function in LS-DYNA®

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Abstract

This paper presents the implementation of an adaptive smoothed particle hydrodynamics (ASPH) method for high strain Lagrangian hydrodynamics with material strength in LS-DYNA. In standard SPH, the smoothing length for each particle represents the spatial resolution scale in the vicinity of that particle and is typically allowed to vary in space and time so as to reflect the local value of the mean interparticle spacing. However, in the presence of strongly anisotropic volume changes which occur naturally in most of the applications the local mean interparticle spacing varies not only in time and space, but in direction as well. In ASPH, the isotropic kernel in the standard SPH is replaced with an anisotropic kernel whose axes evolve automatically to follow the mean particle spacing as it varies in time, space, and direction around each particle. By deforming and rotating these ellipsoidal kernels so as to follow the anisotropy of volume changes local to each particle, ASPH can capture dimension-dependent features such as anisotropic deformations with a more generalized elliptical or ellipsoidal influence domain. Some numerical examples are investigated using both SPH and ASPH, also higher order kernel function is studied for both SPH and ASPH formulation. The comparative studies show that ASPH has better accuracy than the standard SPH when being used for high strain hydrodynamic problems with inherent anisotropic deformations, also higher order kernel function has better accuracy than the standard cubic kernel function.

Introduction

SPH is a Lagrangian method for solving partial differential equations. Essentially, the domain is discretized by approximating it by a series of roughly equal spaced particles. They move and change their properties (such as temperature) in accordance with a set of ordinary differential equations derived from the original governing PDEs. SPH was first applied by Lucy (1977) to astrophysical problems, and then was extended by Gingold (1982). Cloutman (1991) used SPH to model hypervelocity impacts. Libersky and Petschk have shown that SPH can be used to model materials with strength. In recent years it has been developed as a method for incompressible isothermal enclosed flows by Monaghan (1994).

The standard SPH method uses an isotropic smoothing kernel, which is characterized by a scalar smoothing length. One of the problems associated with the standard SPH is that the isotropic kernel of SPH can be seriously mismatched to the anisotropic volume changes that generally occur in many problems. To closely match the anisotropic volume changes, an anisotropic smoothing kernel that can be characterized by a matrix (2D) or a tensor (3D) smoothing length can be efficacious. This leads to the development of the adaptive smoothed particle hydrodynamics in which the smoothing length can be adapted with the volume changes or other dimension-dependent features. The idea of using anisotropic kernel with SPH dates back to Bicknell and Gingold. Shapiro et al. first began investigating a generalized approach using an ellipsoidal kernel in SPH. Fulbright et al. also presented a three-dimensional SPH designed to model systems dominated by deformation along a preferential axis using spheroidal kernels. Later Shapiro et al. systematically introduced anisotropic kernels, tensor smoothing and shock tracking to SPH to create ASPH. Owen et al. presented an alternative formulation of the ASPH algorithm for evolving anisotropic smoothing kernels. Except for problems with anisotropic deformations, the concept of elliptical kernel has also been applied to channel flows with very large length width ratio for saving computational efforts. The numerical results presented in the references further demonstrated that ASPH has better performance than the standard SPH in terms of resolving ability for a wide range of problems.

The cubic spline function has been, so far, the most widely used smoothing function in the emerged SPH literatures since it resembles a Gaussian function while having a narrower compact support. However, the second derivative of the cubic spline is piecewise linear functions, and accordingly, the stability properties can be inferior to those of smoother kernels. In addition, the smoothing function is in pieces, which is slightly more difficulty to use compared to one piece smoothing functions. Morris introduced higher order (quartic and quantic) splines that are more closely approximating the Gaussian and more stable.

This paper presents the implementation of an adaptive smoothed particle hydrodynamics (ASPH) method for high strain Lagrangian hydrodynamics with material strength in LS-DYNA. Some numerical examples are investigated using both SPH and ASPH, also higher order kernel function is studied for both SPH and ASPH formulation. The comparative studies show that ASPH has better accuracy than the standard SPH when being used for high strain hydrodynamic problems with inherent anisotropic deformations, also higher order kernel function has better accuracy than the standard cubic kernel function.

Standard SPH Formulation

Fundamentals of the SPH method.

Particles methods are based on quadrature formulas on moving particles $(x_i(t), w_i(t)) i \in P$, P is the set of the particles. $x_i(t)$ is the location of particle i and $w_i(t)$ is the weight of the particle i. The quadrature formulation for a function can be written as:

$$\int_{\Omega} f(x)dx = \sum_{j \in P} w_j(t) f(x_j(t)) \tag{1}$$

The quadrature formulation (1) together with the definition of smoothing kernel leads to the definition of the particle approximation of a function. The interpolated value of a function: $u(X)$ at position X using the SPH method is:

$$\prod^h (u(x_i)) = \sum_{j \in \Omega} w_j(t) u(x_j) W(x_i - x_j, h) \tag{2}$$

Where the sum is over all particles inside Ω and within a radius $2h$, W is a spline based interpolation kernel of radius $2h$. It mimics the shape of a delta function but without the infinite tails. It is a C^2 function. The kernel function is defined as following:

$$W(x_i - x_j, h) = \frac{1}{h} \theta \left\{ \frac{x_i - x_j}{h(x, y)} \right\} \tag{3}$$

$W(x_i - x_j, h) \to \delta$ when $h \to 0$, δ is Dirac function, h is a function of x_i and x_j and is the so-called smoothing length of the kernel.

Isotropic kernel function options.

The cubic B-spline function has been so far the most widely used smoothing function in the emerged SPH literatures since it resembles a Gaussian function while having a narrower compact support. The cubic B-spline function is defined:

$$\theta(d) = C \times \begin{cases} 1 - \frac{3}{2}d^2 + \frac{3}{4}d^3 & \text{when } 0 \le d \le 1 \\ \frac{1}{4}(2-d)^3 & \text{when } 1 \le d \le 2 \\ 0 & \text{elsewhere} \end{cases} \tag{4}$$

The second derivative of the cubic spline is piecewise linear functions, and accordingly, the stability properties can be inferior to those of smoother kernels. In addition, the smoothing function is in pieces, which is slightly more difficulty to use compared to one piece smoothing functions. Morris (1994) introduced higher order (quintic) splines that are more closely approximating the Gaussian and more stable and have bigger support size too. The quintic spline is:

$$\theta(d) = C \times \begin{cases} (3-d)^5 - 6(2-d)^5 + 15(1-d)^5 & when\ \ 0 \le d < 1 \\ (3-d)^5 - 6(2-d)^5 & when\ \ 1 \le d < 2 \\ (3-d)^5 & when\ 2 \le d \le 3 \\ 0 & d > 3 \end{cases} \tag{5}$$

Where C is $\frac{120}{h}, \frac{7}{478\pi h^2}\ and\ \frac{3}{359\pi h^3}$ in one, two- and three-dimensional space, respectively. This kernel can help to reduce the tensile instability due to Eulerian kernel.

The gradient of the function $u(X)$ is given by applying the operator of derivation on the smoothing length:

$$\nabla \prod^h (u(x_i) = \sum_j w_j u(x_j) \nabla W(x_i - x_j, h) \tag{6}$$

Evaluating an interpolated product of two functions is given by the product of their interpolated values.

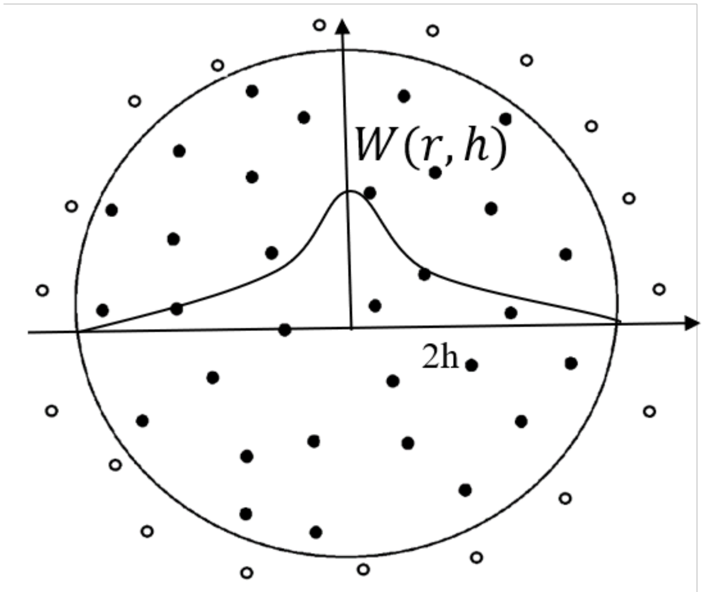

Fig 1. Two-dimensional isotropic kernel function.

The ASPH with anisotropic kernel.

In general, the local mean interparticle spacing varies in time, space, as well as direction. The influence domain of the smoothing function should represent the variation of the interparticle spacing. The standard SPH method with a variable scalar smoothing length can only reflect the interparticle spacing variation in time and space but not the direction. It can lose neighbor information in some directions and is not suitable for simulating problems with anisotropic deformations.

The ASPH models use an anisotropic algorithm that employs an ellipsoidal smoothing function characterized by a different smoothing length along each axis of the ellipsoidal. The smoothing length long each axis is evolved so as to follow the variation of the local interparticle separation surrounding each particle. By deforming and rotating the ellipsoidal smoothing function so as to follow the anisotropic volume changes associated with each particle, ASPH adapts its spatial resolution scale in time, space, and direction. Hence, ASPH was shown to significantly improve the spatial resolving capability over that of the standard SPH method for the same number of particles used.

The main idea of the ASPH is that in three-dimensional space, the smoothing function is of ellipsoidal shape, which can be arbitrarily oriented. A smoothing tensor H can be used to characterize the influence domain of the smoothing function

$$H = \begin{pmatrix} h_{xx} & h_{yx} & h_{zx} \\ h_{xy} & h_{yy} & h_{zy} \\ h_{xz} & h_{yz} & h_{zz} \end{pmatrix} \quad (7)$$

$H$is a second order, real and symmetric tensor, $h_{yx} = h_{xy},\ h_{zx} = h_{xz},\ h_{yz} = h_{zy}$. The eigenvectors of H are the directions along the three axes of the ellipsoid and the corresponding eigenvalues are the dimensions of the ellipsoid along each axis. SPH can be regarded as a special case of ASPH, with each diagonal element of H

equal to h while other elements equal to zero. Therefore, one has more freedom with ellipsoidal smoothing functions that one has with spherical smoothing functions.

The smoothing function in ASPH can be written as a function of the tensor smoothing length H and the normalized position vector:

$$\bar{\kappa} = \frac{1}{H} \cdot \bar{r} = G \cdot \bar{r}$$

The H tensor can be evolved both spatially and temporally, nine components of tensor define three vectors that adapt their special resolution scale in time, space and direction, the anisotropic volume changes represented by a smoothing ellipsoid can be transformed through a local, linear transformation of coordinates into those in which the underling anisotropic volume changes appear to be isotropic. This tensor for each particle is dynamically evolved by using the components of the deformation tensor $\frac{\partial v_i}{\partial x_j}$ to follow the local deformation and vorticity of the flow. All the SPH equations can now be rewritten in terms of the H tensors and these expressions for W and ∇W. Three principal axes based on the principal direction of the deformation tensor can be defined for an ellipse support domain, three principal vector values (h_x, h_y, h_z) are updated based on the principal values of deformation tensor.

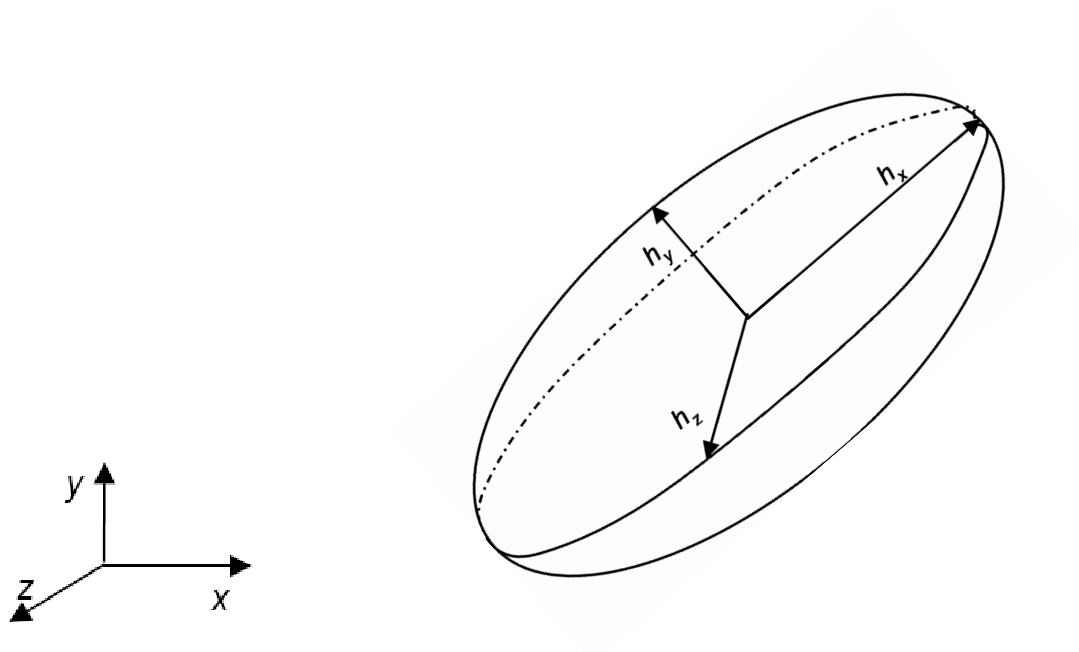

Figure 2. Three-dimensional anisotropic kernel support function

Coordinate transformation between local and global systems:

To search for the influencing nodes and calculate the shape function of the evaluation point, the coordinate of the evaluation point should be transformed to the principle orientation of the deformation tensor (local coordinate).

$$\mathbf{x}' = \mathbf{\Phi} \cdot \mathbf{x} \tag{8}$$

Where $\mathbf{x}'$ is the position vector under the local coordinate, $\mathbf{x}$ is the position vector under the global coordinate, $\mathbf{\Phi}$ is the transformation matrix between two coordinate systems.

The weight function should be calculated in the local coordinate system along the principle orientation. And the space derivation of the weight function should be transformed back to global coordinate system.

$$\frac{\partial w}{\partial \mathbf{x}} = \frac{\partial w}{\partial \mathbf{x}'} \cdot \frac{\partial \mathbf{x}'}{\partial \mathbf{x}} = \frac{\partial w}{\partial \mathbf{x}'} \cdot \mathbf{\Phi} \tag{9}$$

$$\frac{\partial^2 w}{\partial \mathbf{x}^2} = \mathbf{\Phi}^{\boldsymbol{T}} \cdot \frac{\partial^2 w}{\partial \mathbf{x}'^2} \cdot \mathbf{\Phi} \tag{10}$$

Where w is weight function.

Continuity equation and Momentum equation.

The particle approximation of continuity equation is defined as:

$$\frac{d\rho_i}{dt} = \rho_i \sum_j \frac{m_j}{\rho_\mathrm{j}} \left(v^\beta{}_i - v^\beta{}_j \right) W_{ij'\beta} \tag{11}$$

It is Galilean invariant due to that the positions and velocities appear only as differences and has good numerical conservation properties. $v^\beta{}_i$ is the velocity component at particle i.

The discretized form of the SPH momentum equation is developed as:

$$\frac{dv_i^\alpha}{dt} = -\sum_j \frac{m_j}{\rho_i \rho_j} (\sigma_i^{\alpha\beta} \pm \sigma_j^{\alpha\beta}) W_{ij,\beta} \tag{12}$$

The above formulation ensures that stress is automatically continuous across material interfaces. Different types of SPH momentum equations can be achieved through applying the identity equations into the normal SPH momentum equation. Symmetric formulation of SPH momentum equation can reduce the errors arising from particle inconsistency problem.

From equation (6), the following particle body forces were derived:

$$F_i^{pressure} = -\sum_j m_j \frac{p_i + p_j}{2\rho_j} \nabla W(r_{ij}, h)$$
$$F_i^{vis\cos ity} = \mu \sum_j m_j \frac{v_i - v_j}{2\rho_j} \nabla^2 W(r_{ij}, h) \tag{13}$$

Where $r_{ij} = x_i - x_j$, μ is the viscosity coefficient of the fluid. The pressure p_i are computed via the constitutive equation:

$$p_i = k(\rho_i - \rho_0) \tag{14}$$

where k is the stiffness of the fluid and ρ_0 is its initial density.

Numerical Examples

ASPH option in the LS-DYNA: FORM=9,10 for ASPH formulation and ASPH with renormalization formulation respectively in *CONTROL_SPH keyword. Those two formulations must be used with the *SECTION_SPH_ELLIPSE keyword for the ellipsoidal support of domain, HXCSLH, HYCSLH, HZCSLH have to be defined to set the scale factor in each direction. The default smoothing kernel function is cubic B_spline kernel function with SPHKERN = 0 in *SECTION_SPH keyword. For higher order kernel function, set SPHKERN = 1 for quintic spline kernel function with a larger support size which is available for both SPH and ASPH formulation (recommend using with HMAX = 3.0 or larger in *SECTION_SPH keyword).

1. Three point bending test with SPH.

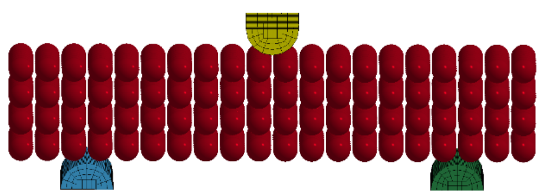

Figure 3. Three point bending test set up

A 3D isotropic plate was modeled by SPH particles under bending test. The plate has the dimension of 100x40x20, and is loaded as shown in figure 3 with two bottom rigid solid fixed in the space, top rigid solid moved with a prescribed motion along z direction. The plate was modeled with 640 SPH particles, and the deformation, stresses distribution are plotted and compared for SPH and ASPH method, higher order kernel was tested here and compared with standard cubic spline kernel function.

In the model, automatic_ node_ to surface contact was used for the interaction between SPH particles and rigid solid elements. *MAT_57 was used for SPH particles with density equal 1.7e-11, E=2.5. For ASPH formulation, *SECTION_SPH_ELLIPSE with hxcslh = hycslh = hzcslh = 1.2 was used.

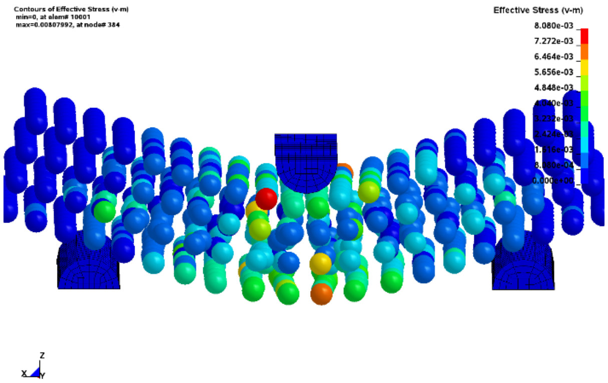

Figure 4. Particle distribution and stress contour for SPH with cubic spine kernel at t=1 form=1

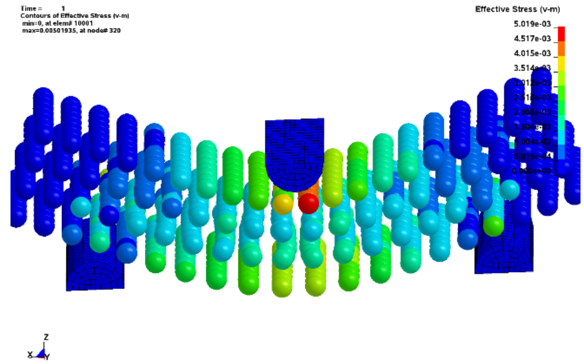

Figure 5. Particle distribution and stress contour for SPH with quintic spline kernel at t=1 Form=1

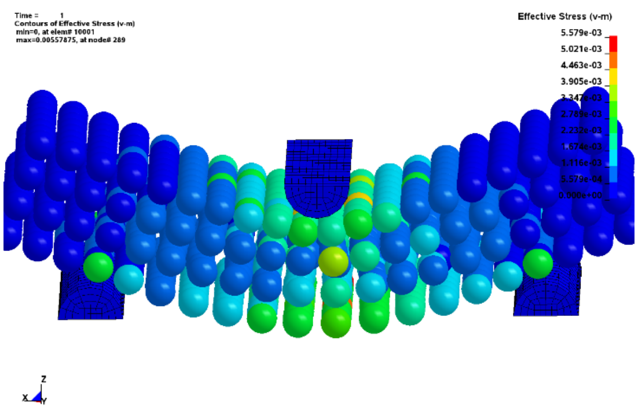

Figure 6. Particle distribution and stress contour for ASPH with cubic spine kernel at t=1 form=10

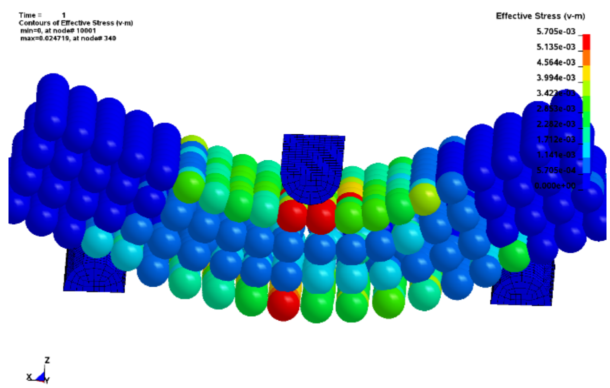

Figure 7. Particle distribution and stress contour for ASPH with quintic spline kernel at t=1 Form=10

As we can see that in fig 4, the standard SPH formulation have particle clustering and fracture problem around the center of the plate, the stresses distribution is non-smooth and irregular. The higher order kernel with standard SPH formulation help to reduce the particle clustering and fracture problem greatly, also have much smoother stresses distribution than normal cubic b-spline kernel function.

ASPH (form=10) formulation with standard cubic spline kernel function can capture dimension-dependent features such as anisotropic deformations with a more generalized elliptical or ellipsoidal influence domain. ASPH is shown here to significantly improve the spatial resolving capability over that of the standard SPH method for the same number of particles used, has much smoother particle distribution and stress contour results than the standard SPH method. Also, ASPH with cubic b-spline kernel help to reduce the particle clustering and fracture problem greatly.

2. Swegle rubber ring impact with SPH.

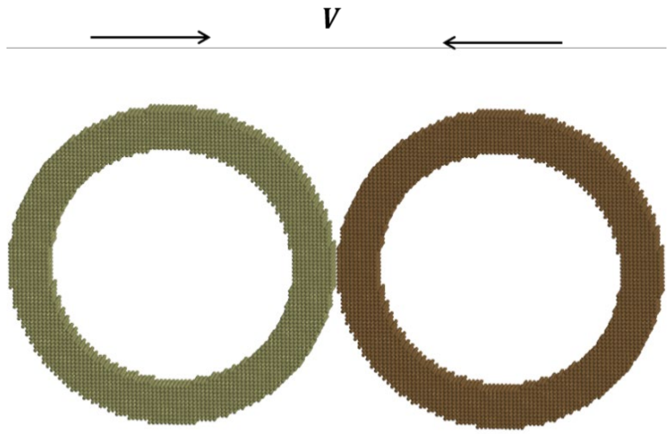

Figure 8. Swegle rubber ring impact problem setup.

Two 3D isotropic rubber rings modeled with SPH particles impact into each other with initial speed V=50m/s. Each ring has outer radius R=40mm, inner radius r=30.5mm and thickness H = 10mm, density of rubber ring rho=3.0e-3, E=1401. The whole setup modeled with 43920 SPH particles and the deformation, stresses distribution are plotted and compared for SPH and ASPH method, higher order kernel was tested here and compared with standard cubic spline kernel function. The standard SPH integral method (Eulerian kernel) was used as interaction method between two rubber rings.

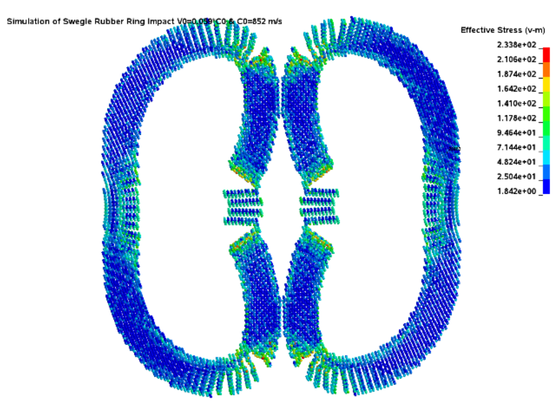

Figure 9. Particle distribution and stress contour for SPH with cubic spine kernel at t=0.54 form=1

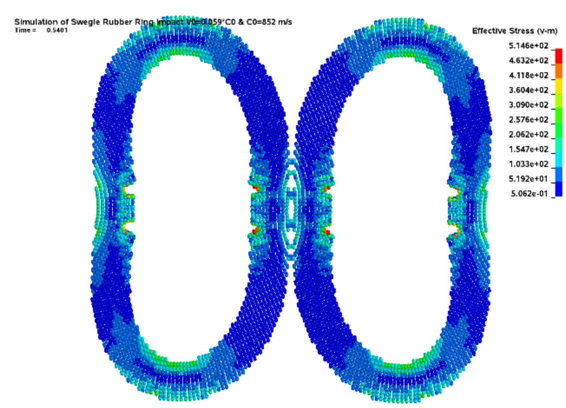

Figure 10. Particle distribution and stress contour for SPH with quintic spline kernel at t=0.54 Form=1

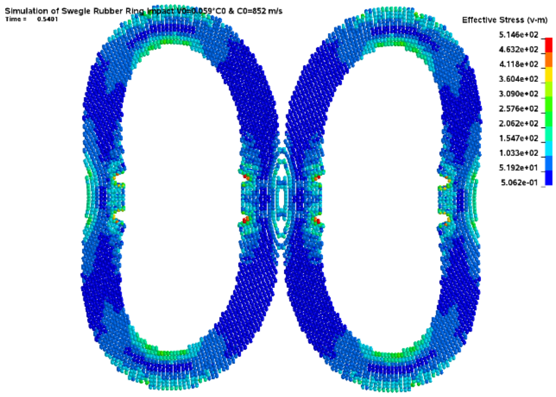

Figure 11. Particle distribution and stress contour for ASPH with cubic spine kernel at t=0.54 form=10

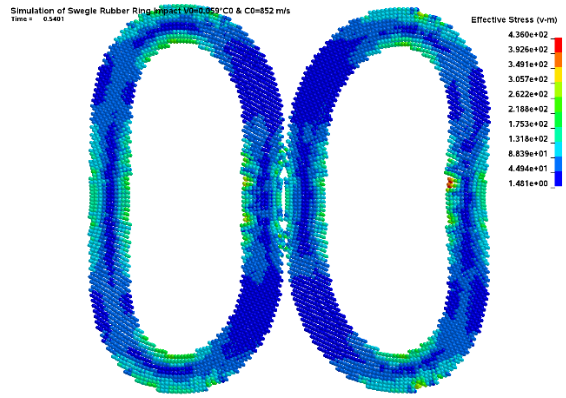

Figure 12. Particle distribution and stress contour for ASPH with quintic spline kernel at t=0.54 Form=10

As we can see that in fig 9, the standard SPH formulation have particle clustering and fracture problem around the corners that under tension pressure, the stresses distribution is non-smooth and irregular. Those phenomena are call the tensile instability since the fractures are not caused by the material failure but by the numerical errors. The higher order kernel with standard SPH formulation help to reduce the tensile instabilities around the corners greatly, also have much smoother stresses distribution than normal cubic b-spline kernel function.

ASPH (form=10) formulation with standard cubic spline kernel function can capture dimension-dependent features such as anisotropic deformations with a more generalized elliptical or ellipsoidal influence domain. ASPH is shown here to significantly improve the spatial resolving capability over that of the standard SPH method for the same number of particles used, has much smoother particle distribution and stress contour results than the standard SPH method (i.e. help to reduce the tensile instability around the corner too). ASPH formulation with quintic spline kernel help to reduce the tensile instabilities issue even more compared to the ASPH with cubic b-spline kernel option. The results are very close to the results with Lagrangian kernel option (form=8, which can totally avoid the tensile instability issue in this case).

Conclusion

This paper presents the implementation of an adaptive smoothed particle hydrodynamics (ASPH) method for high strain Lagrangian hydrodynamics with material strength in LS-DYNA. In ASPH, the isotropic kernel in the standard SPH is replaced with an anisotropic kernel whose axes evolve automatically to follow the mean particle spacing as it varies in time, space, and direction around each particle. By deforming and rotating these ellipsoidal kernels so as to follow the anisotropy of volume changes local to each particle, ASPH can capture dimension-dependent features such as anisotropic deformations with a more generalized elliptical or ellipsoidal influence domain. Some numerical examples are investigated using both SPH and ASPH, also higher order kernel function is studied for both SPH and ASPH formulation. The comparative studies show that ASPH has better accuracy than the standard SPH when being used for high strain hydrodynamic problems with inherent anisotropic deformations, also higher order kernel function has better accuracy than the standard cubic kernel function

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