Sound Absorbing Porous Material in Statistical Energy Analysis

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Abstract

For high frequency analysis, Statistical energy analysis (SEA) has proved to be a promising approach to the calculation of sound transmission in complex structures. In automotive industry and also in civil engineering, most of noise transmission is due to high-frequency structural vibrations, where the characteristic wavelength is small compared to the dimensions of the structure. For these applications classical methods of structural analysis, such as the finite element method (FEM), and Boundary Elements Method (BEM), cannot be used due to the large number of degrees of freedom required to model structural deformation. Statistical Energy Analysis (SEA) considers the vibrations of the structure in terms of elastic waves which propagate through the structure and are partially reflected and partially transmitted at structural connections. For the last few years, there has been an increase in the application of SEA techniques to study noise transmission in motor vehicles.

In this paper we present new development on porous material for sound absorbing structure used with SEA formulation in LS-DYNA®. Numerical results in term of acoustic pressure inside the cavity are in good agreement compared to analysis using other software and published in the literature.

Introduction

Statistical Energy Analysis (SEA) is a structural-acoustic method that is widely used for high frequency analysis. SEA arose during the 1960´s in the aerospace industry to predict the vibrational behavior when designing space craft. During this time computational methods were available but the size of the models that could be handled and the computational speed were such that only few of the lowest order modes could be predicted [1]. Furthermore, in traditional analysis of mechanical vibration the lowest modes are usually of most interest because these modes normally have the greatest displacement response. But when designing and constructing large and lightweight aerospace structures it is apparent that also high frequency broad-band loads is important in the process of predicting structural fatigue, equipment failure and noise production. The name SEA points out certain aspects of the method. *Statistical* accents that the system being considered are a member of a population of similar systems with known distributions of the subsystem parameters. *Energy* is the variable of interest. It describes the behavior of the system in terms of stored, dissipated and exchanged energies of vibration. Other often used variables for acoustic and structural vibration, such as displacement, pressure etc. can be derived from the energy of vibration. *Analysis* emphasizes that SEA is a framework rather than a specific technique. The development and use of SEA proved to be a good method to predict high frequency loads and the analysis technique has since then been applied, extended and developed for a growing number of applications. For example it has been used to model sound and vibration transmission in buildings, cars, aircrafts, ships and trains [2] and [3].

The General Procedures of SEA

One of the fundamental principles of SEA is that the average power flow between coupled groups of dynamical modes is proportional to the difference I the average modal energies. This makes it possible to analyze the dynamical response of a system consisting of many resonant modes in a certain frequency range by dividing the modes into groups (subsystems) and considering a power balance equation of the form $P_{in} = P_{out}$ for each mode group. The analysis is statistical in that the expected value and variance of the power flow are evaluated assuming a statistical distribution in the resonance frequencies of the subsystem modes [2].

The general procedures of SEA can be illustrated as shown in Figure 1 below. This system consists of two connected subsystems.

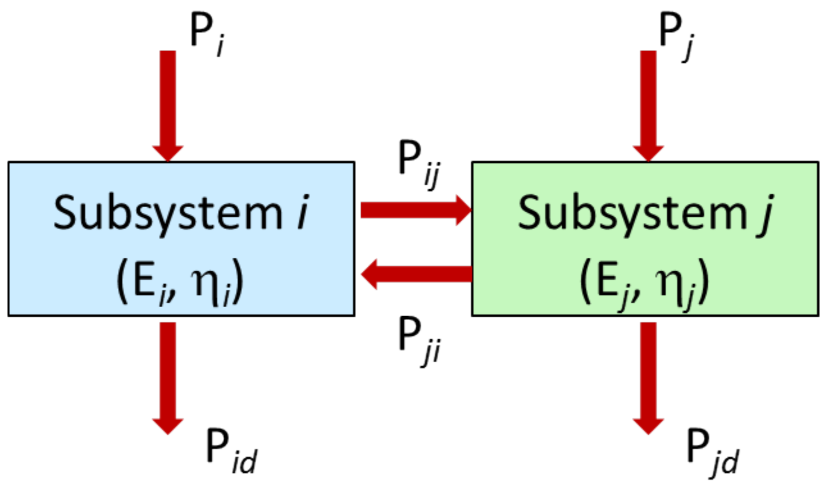

Figure 1. A two subsystems SEA model

Energy flows in and out of a subsystem. The energy that flows out consists of dissipation P_{id} *and* P_{jd}, radiation and transmission to other subsystems P_{ij} *and* P_{ji}. The energy that flows into a subsystem consists of external source excitations P_i *and* P_j and transmissions from other subsystems P_{ij} *and* P_{ji}.

The dissipated power in a subsystem is given by equation 1 and 2 below:

$$P_{id} = \omega \eta_{id} E_i \tag{1}$$

$$P_{jd} = \omega \eta_{jd} E_j \tag{2}$$

Where η_{id} *and* η_{jd} are the damping loss factors and E_i *and* E_j are the total vibrational energy of the modes at frequency f.

The net transmitted power between subsystem *i* and *j* (P_{ij} *and* P_{ji}) are given by equation 3 and 4.

$$P_{ij} = \omega \eta_{ij} E_i \tag{3}$$

$$P_{ji} = \omega \eta_{ji} E_j \tag{4}$$

SEA calculation is based on energy flow equilibrium. The power balances for the two systems are given as in equations 5 and 6.

$$P_i + P_{ji} = P_{ij} + P_{id} \tag{5}$$

$$P_j + P_{ij} = P_{ji} + P_{jd} \tag{6}$$

When combining equations 1-6, the power balance equation for two subsystems can be expressed in matrix form shown in equation 7.

$$\left[\frac{P_i}{P_j}\right] = \omega \begin{bmatrix} (\eta_{id} + \eta_{ij}) & -\eta_{ji} \\ -\eta_{ij} & (\eta_{jd} + \eta_{ji}) \end{bmatrix} * \left[\frac{E_i}{E_j}\right] \tag{7}$$

For a more generalized case with & number of subsystems, the balance equations can be written in a general form as equation 8.

$$\begin{bmatrix} P_i \\ P_j \\ \vdots \\ P_n \end{bmatrix} = \omega \begin{bmatrix} \eta_i & -\eta_{ji} & \cdots & -\eta_{ni} \\ -\eta_{ij} & \eta_j & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ -\eta_{in} & \cdots & \cdots & \eta_n \end{bmatrix} * \begin{bmatrix} E_i \\ E_j \\ \vdots \\ E_n \end{bmatrix} \tag{8}$$

Where η_i equals the total loss factor and is the summation of the damping loss factor of the subsystem and the coupling loss factors representing energy transmission from the subsystem to others as equation 9.

$$\eta_i = \eta_{id} + \sum_{j=1,\, j \neq i}^{n} \eta_{ij} \tag{9}$$

Example 1: Three plates connection

This example consists of three plates which are connected at a common line as shown in figure 2. The 3 plates are all made linear elastic material with the following properties: density = 7800 kg/m^3, Young Modulus = 210x10^9 Pa, Poisson's ratio = 0.3. Each of the plates has a size of 1x1 m. The plate 1 and plate 3 are 2 mm thick and the plate 2 is 3 mm. The plate 1 is connected to plate 2 at an angle of 90° and the plate 3 is connected at an angle of 210°. All Plates have constant damping of 0.01. Plate 1 is excited by an input power of 1 Watt (in bending wave). The goal is to calculate the mean velocity amplitude of the bending wave at all three plates. Figure 3 and 4 are the velocity of plate1 and plate 2 of the results comparison between LS-DYNA and other code [4]. One can see that the results by the two codes match very well.

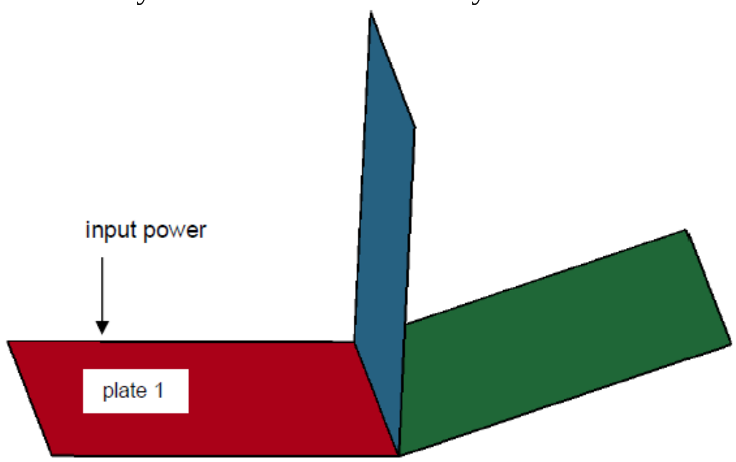

Figure 2. Description of the 3 plates model

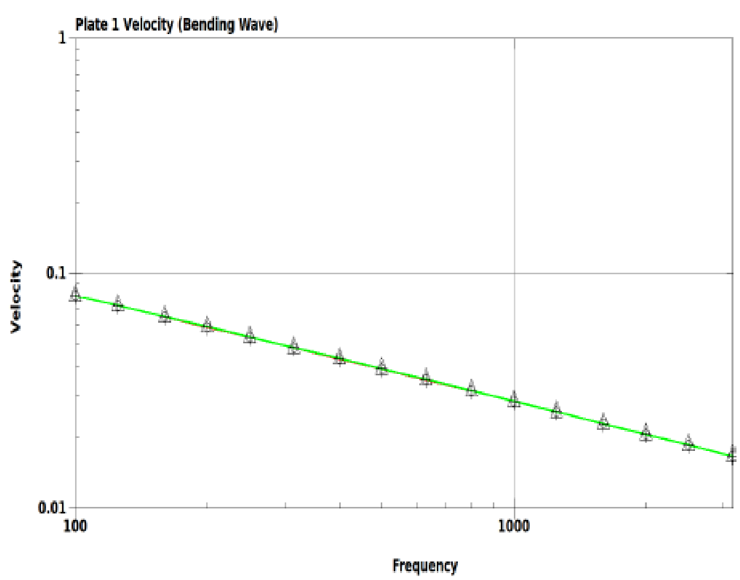

Figure 3. Comparison for Mean Velocity (Log-Log) on plate 1

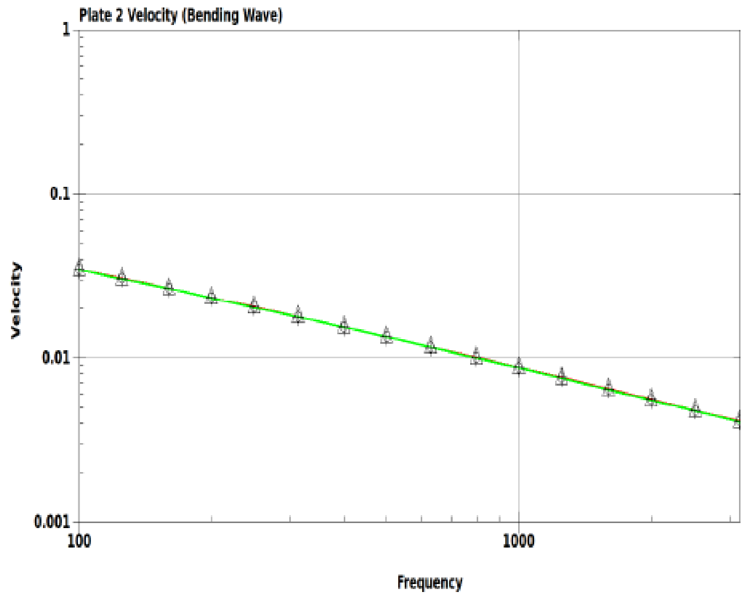

Figure 4. Comparison for Mean Velocity (Log-Log) on plate 2

Example 2: Sound Transmission between 2 rooms

This example consists of 35 subsystems, 32 plates (walls) and 2 acoustic rooms as shown in figure 5. The 32 plates are all made concrete material modelled as linear elastic with thickness 0.19 m, area 13.95 m, and the following material properties: density = 2300 kg/m^3 , Young Modulus = 2.8x10^{10} Pa, Poisson's ratio = 0.3. The 2 rooms are modelled with ambient air material. An acoustic source of Power input 1 Watt into room 1 is assumed. The goal is to calculate the acoustic sound pressure in both rooms. The results comparison of LS-DYNA and other code [4] are shown in figure 6. One can see that the results by the two codes match very well.

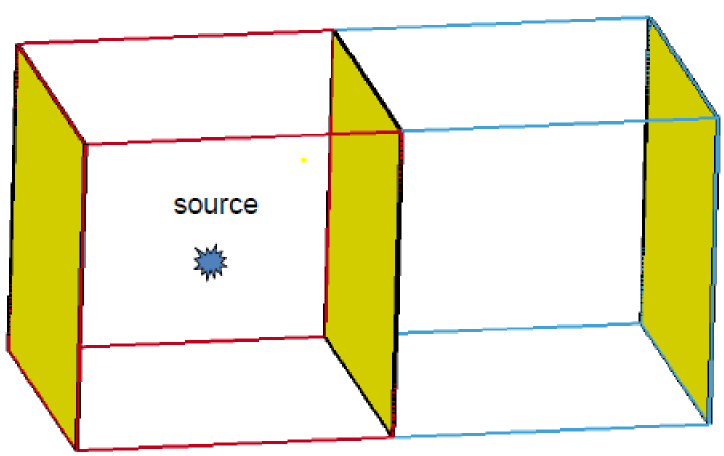

Figure 5. Problem Description for room 1 and 2

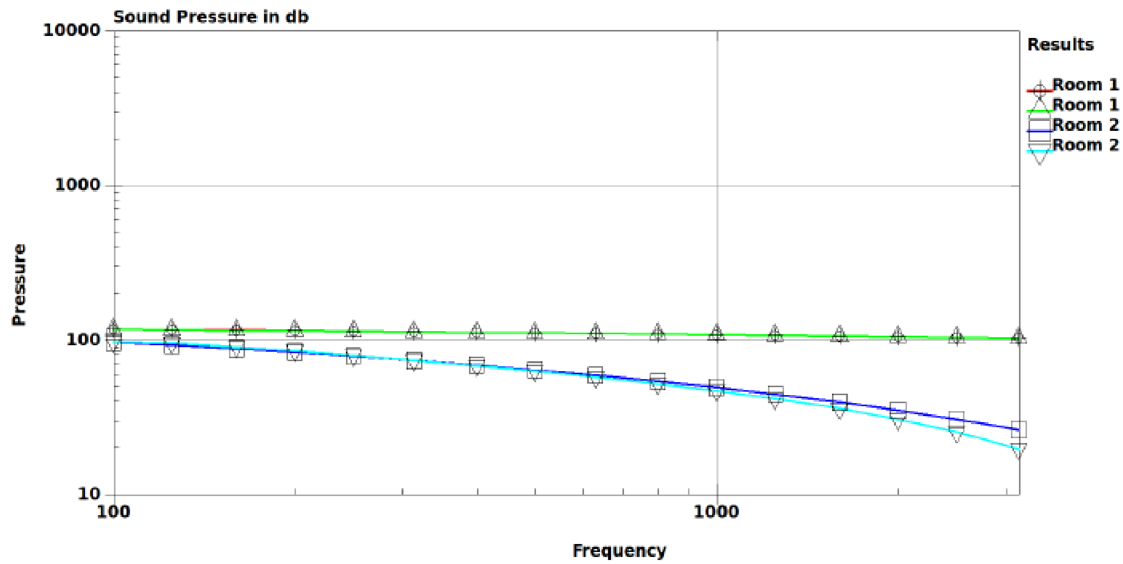

Figure 6. Comparison for acoustic pressure (Log-Log) in room 1 and room 2

Example 3: Sound Transmission inside a car

The car example consists of 26 subsystems, 18 Steel plates, 6 glass plates and 2 acoustic rooms as shown in figure 7. The steel plates and glasses are modelled as linear elastic, and the following material properties: density 7800 kg/m^3 , Young Modulus = 2.1x10^{11}Pa, Poisson's ratio = 0.3 and density = 2500 kg/m^3 , Young Modulus = 6.0x10^{10}Pa, Poisson's ratio = 0.2. A real car would have some sort of a frame, pillars and stiffeners. However with only the information in build a simple SEA model. Input power is applied in bending on the fender left and right front parts of the car as described in figure 7. Sound Pressure in dB units in the passenger compartment of comparison between LS-DYNA and other code [4] is shown in figure 8. One can see that the results by the two codes match very well.

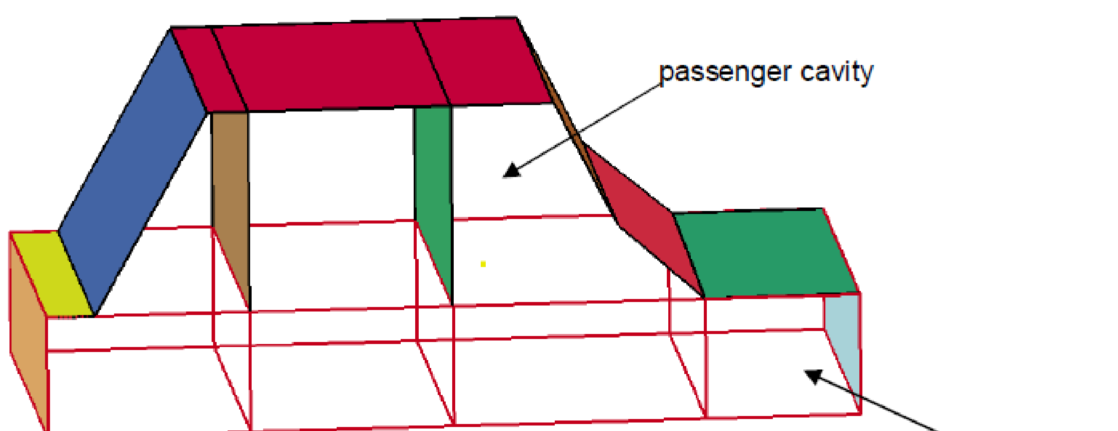

Figure 7. Problem Description of the subsystems

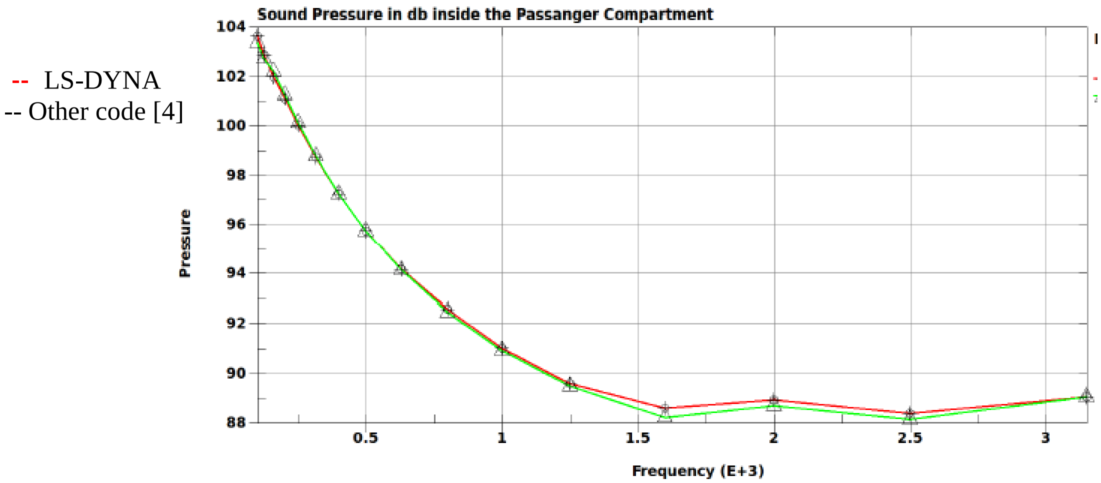

Figure 8. Comparison of acoustic pressure in passenger cavity

The absorbing materials are the very commonly used in industry to lower noise by disseminating acoustic energy. One of the important parameters governing the absorption of a porous material is its flow resistance. It is defined by the ratio of the pressure differential across a sample of the material to the normal flow velocity through the material. The flow resistivity σ is the specific (unit area) flow resistance per unit thickness.

The complex wave number k and the characteristic impedance Zc have been measured by Delany and Bazley [5] for a large range of frequencies in many fibrous materials with porosity close to 1. According to these measurements, the quantities k and Zc depend mainly on the angular frequency ω and on the flow resistivity σ of the material. A good fit of the measured values of k and Zc has been obtained with the following expressions:

$$Zc = \rho_o c_o[1 + 0.057X^{-0.754} - j0.087X^{-0.732}] \tag{10}$$

$$k = \omega/c_o[1 + 0.0978X^{-0.700} - j0.189X^{-0.595}] \tag{11}$$

Where ρ_o and c_o are the density of air and the speed of sound in, and X is a dimensionless parameter equal to

$$X = \rho_0 f / \sigma \tag{12}$$

The f is the frequency related to ω by $\omega = 2\pi f$.

The layer of a fibrous material is fixed to a rigid, impervious wall, on its rear face, and is in contact with air on its front face, as shown in figure 9. In numerical example, the absorbing material is added to the roof with thickness equal to 5 cm (figure 10). The figure 11 is the result Comparison of acoustic pressure with and without porous material. One can see that the sound pressure decrease with adding the absorbing material.

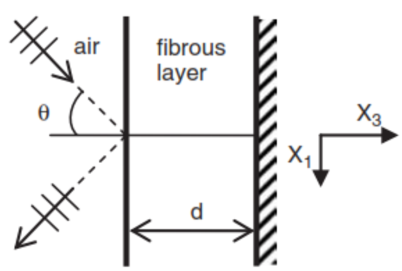

Figure 9. A layer of fibrous material backed by an impervious rigid wall

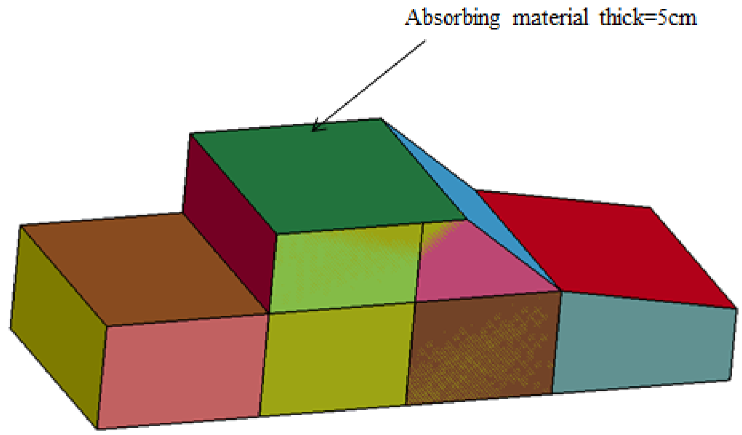

Figure 10. The absorbing material is added to the roof

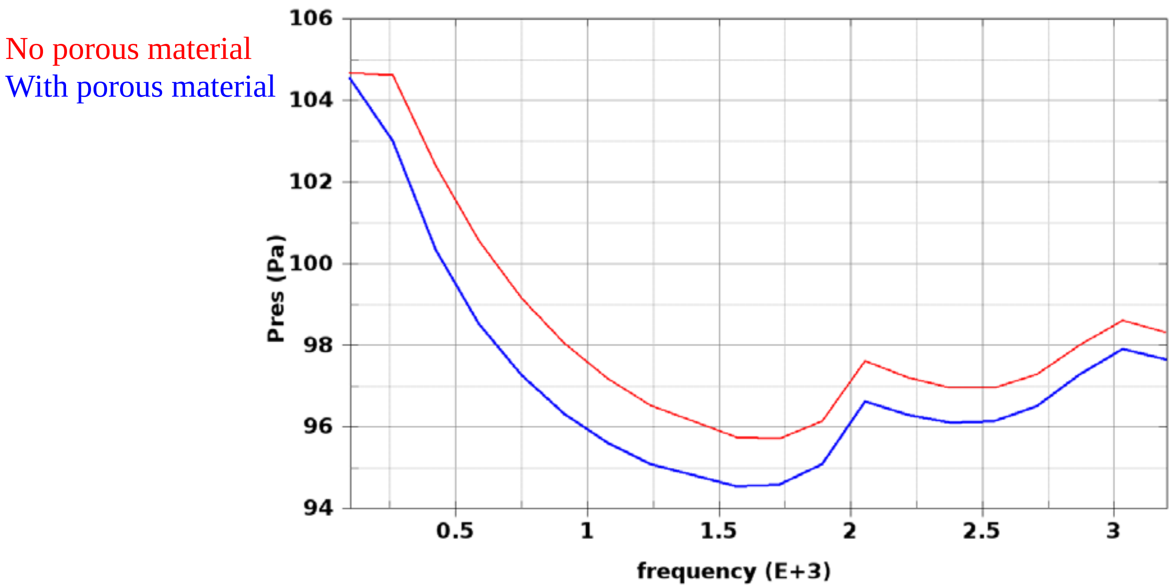

Figure 11. Comparison of acoustic pressure with and without porous material

Summary

A new SEA solver has been implemented in LS-DYNA including porous material for sound absorbing structure used with SEA formulation. It offers effective and efficient tools for high frequency vibration and acoustic analysis in complex structures. Three simple examples are included in the paper to show the procedure of running SEA analysis with LS-DYNA. The numerical results are in good agreement with the analysis using other code [4]. For the future development, the LS-PrePost® will be updated for user to speed up the generation of SEA model and to make it more convenient to review the results.

References

[1] Fahy, Frank J. Statistical energy analysis: a critical overview. *Philosophical Transaction: Physical Sciences and Engineering, Vol. 346*, No. 431-447, 1994.

[2] Richard H. Lyon and Richard G. DeJong. Theory and Application of Statistical Energy Analysis. Cambridge, MA. 1998.

[3] R.S. Langley and K.H Heron. A wave intensity technique for analysis of high frequency vibrations, Journal of Sound Vibration 159(3), 483-502, 1992.

[4] Ennes Sarradj. Statistical Energy Analysis Freeware.

[5] J.F. Allard and N. Atalla, Propagation of Sound in Porous Media, Wiley. 2009