

# Advances in Fatigue Analysis with LS-DYNA<sup>®</sup>

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## Abstract

*Fatigue analysis is critical to the design and optimization of metal structures and components. This paper reviews the recent development in fatigue analysis with LS-DYNA. Both frequency domain and time domain fatigue solvers have been implemented to LS-DYNA. They can be used towards different simulation situations. Some examples are provided in this paper, to illustrate how to use these fatigue analysis methods. The plan on future development of the fatigue solvers in LS-DYNA, is also discussed.*

## Introduction

Fatigue analysis is important in many industry areas, including automotive industry, aircraft industry and pressure vessel design. This is because for many structures and components, they are subjected to continuous and cyclic loading during their service life and fatigue failure is the dominant failure mode in these cases.

Since version 971 R6, a set of fatigue solvers was provided in LS-DYNA in frequency domain, including random vibration fatigue (\*FREQUENCY\_DOMAIN\_RANDOM\_VIBRATION\_FATIGUE) and SSD (Steady State Dynamics) fatigue (\*FREQUENCY\_DOMAIN\_SSD\_FATIGUE).

Sometimes users need to run fatigue analysis in time domain. For example, they may need to study the fatigue of gun pipes due to cyclic thermal loading. Another example is the fatigue of the aircraft turbine blades, which are made of alloy and subject to combined loads of high temperature and high air pressure. In these cases, the loading level is high and the plastic deformation is observed in the materials. So it is more appropriate to use low cycle fatigue criterion and solve the problems in time domain. To help these uses, we developed time domain fatigue solvers in LS-DYNA. A new keyword \*FATIGUE is introduced to run the time domain fatigue. This keyword provides user with the opportunity to perform fatigue and durability analysis following the transient analysis in LS-DYNA. The transient analysis, which provides stress or strain cycles for fatigue analysis, can be linear or nonlinear, can use a wide variety of material models (elastic, plastic, ...), and can work with a variety of loading conditions (thermal, mechanical, ...).

The results of fatigue analysis, in both frequency domain and time domain, are provided in the binary plot database D3FTG, which can be easily accessed by LS-PrePost<sup>®</sup>.

Some examples are provided in the paper to demonstrate the fatigue analysis capabilities of LS-DYNA, and show how to use these frequency domain or time domain fatigue solvers towards different cases. Some of the examples mentioned in this paper were taken from previous publications or presentations [1].

## Database and post-processing for fatigue analysis

The fatigue analysis results are saved in a binary plot database D3FTG. A keyword \*DATABASE\_FREQUENCY\_BINARY\_D3FTG is used to activate the output of D3FTG database. Currently this keyword is also used even if it is a time domain fatigue analysis. D3FTG can be easily accessed by LS-PrePost. A typical GUI for D3FTG is shown in Figure 1.

For random vibration fatigue analysis, the following results are provided in D3FTG:

1. Cumulative damage ratio
2. Expected fatigue life
3. Zero-crossing frequency with positive slope
4. Peak-crossing frequency
5. Irregularity factor
6. Expected fatigue cycles

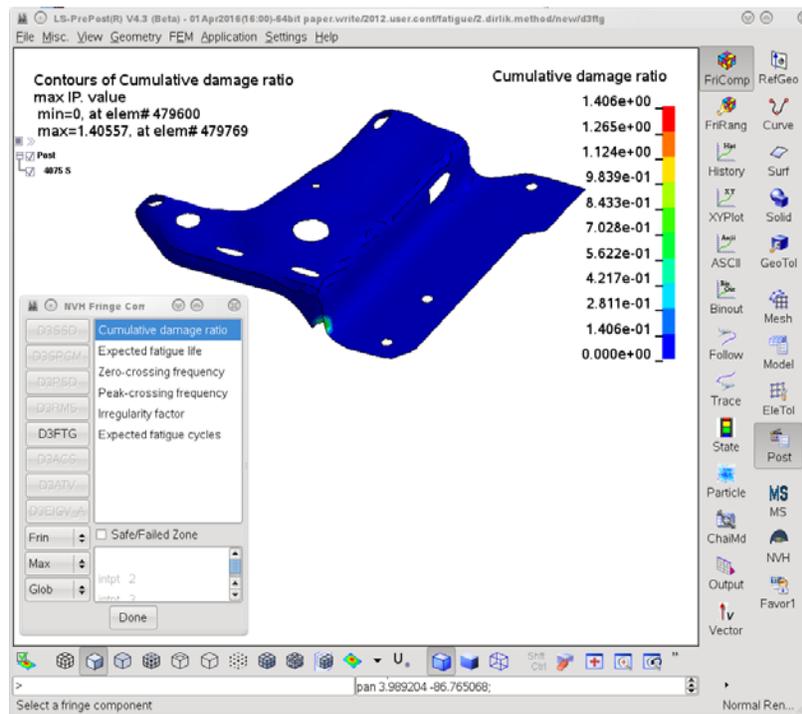


Figure 1. GUI in LS-PrePost for fatigue analysis results

For fatigue analysis based on \*FREQUENCY\_DOMAIN\_SSD, and fatigue analysis in time domain, only the cumulative damage ratio is provided in D3FTG. The other fringe components are zero.

More details about the post-processing of the fatigue analysis results can be found in [1].

## Frequency domain fatigue analysis with LS-DYNA

A series of frequency domain analysis features have been implemented to LS-DYNA, since version 971 R5. A comprehensive review of the frequency domain features, and their applications in auto NVH and acoustic problems can be found in [2, 3].

Solutions for both deterministic and non-deterministic vibration are provided in frequency domain. For non-deterministic vibration, one can define the loads in terms of PSD (Power Spectral Density) and run random vibration analysis (\*FREQUENCY\_DOMAIN\_RANDOM\_VIBRATION); for deterministic vibration, one can run steady state dynamics (\*FREQUENCY\_DOMAIN\_SSD) and get the response for each excitation frequency.

With the knowledge of stress cycles from the vibration analysis, one can proceed to fatigue analysis, upon receiving material's fatigue properties (S-N fatigue curve). A Miner's rule of cumulative damage ratio, is used:

$$E(D) = \sum_i \frac{n_i}{N_i} \quad (1)$$

Where,  $E(D)$  is the expected cumulative damage ratio,  $n_i$  is the number of cycles at stress level  $S_i$ , and  $N_i$  is the number of cycles for failure at stress level  $S_i$ , given by a material's S-N curve.

### *Fatigue analysis in random vibration environment*

In this section, we describe the fatigue analysis method in random vibration environment.

With random vibration analysis, PSD of stress components are provided (and saved in D3PSD). Various methods (Three-band method, Dirlik method, etc.) have been developed to calculate the PDF (Probability Density Function) of stresses. The number of stress cycles can be calculated for a given time of exposure [4].

The material's S-N curve can be defined by \*MAT\_ADD\_FATIGUE. More details about defining the materials' S-N curve can be found in [4].

The example in this section considers a metal bracket shown in Figure 2. It is constrained to a shaker table through the two small holes (marked as red in Figure 2) for a random vibration test.



Figure 2. A bracket model constrained to shaker table

The S-N curve is represented by the equation

$$N \cdot S^{2.5} = 1.56 \times 10^8 \tag{2}$$

where,  $N$  is the number of cycles for failure (or endurance) under the stress level  $S$ .

The model is subject to base acceleration. The acceleration PSD curve is shown in Figure 3.

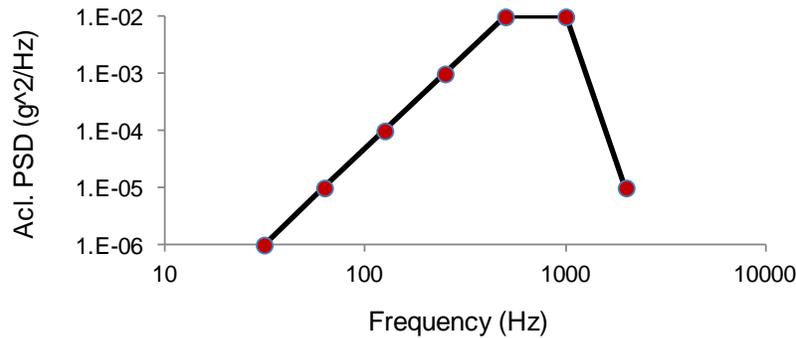


Figure 3. The base acceleration PSD curve

The duration of excitation is 1 hour. For the first loading case, we consider the acceleration in z-direction only.



Figure 4. Cumulative damage ratio for the case of z-acceleration loading



Figure 5. Safe / Failed zone for the case of z-acceleration loading

Dirlik method is employed to perform fatigue analysis in this example. Von-mises stress is used as the stress index.

Figure 4 shows the cumulative damage ratio plot under the z-acceleration excitation. The peak value of the cumulative damage ratio is around 2.48. This indicates that the model will fail due to fatigue in the test. The peak value appears at the edge of one constrained hole. This suggests that the initial crack will take place at that location.

Using the keyword `*INITIAL_FATIGUE_DAMAGE_RATIO`, one can accumulate the damage on a structure from multiple loading cases. For example, the same structure shown in Figure 2 can be subject to base acceleration in x-, y- and z-direction sequentially. When we run fatigue analysis for the last loading case (e.g. acceleration in z-direction), we can define the keyword `*INITIAL_FATIGUE_DAMAGE_RATIO` and set the path and name for the binary databases for the previous fatigue results (due to acceleration in x- and y-directions). The combined cumulative damage ratio plot, due to the three loading cases, can be found in Figure 6.



Figure 6. Cumulative damage ratio for x-, y- and z-acceleration loading cases

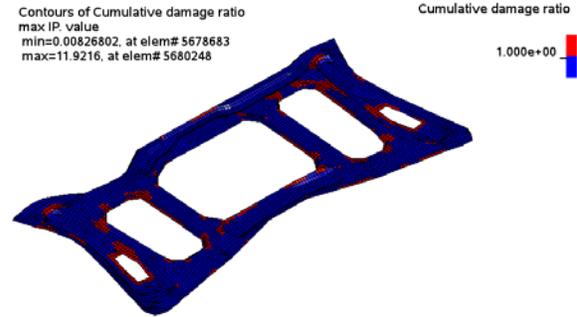


Figure 7. Safe / Failed zone for x-, y- and z-acceleration loading cases

Figure 7 shows the safe / failed zone as the final result of the three loading cases. As we can see, the hole edge area has a higher chance for failure than other areas in this test.

Sometimes it is important to account for the effect of pre-stress or nonzero mean stress on fatigue analysis. For random vibration analysis, one can use a predefined initial stress as the mean stress. The initial stress is introduced by keywords \*INITIAL\_STRESS\_SHELL, \*INITIAL\_STRESS\_SOLID, etc. To define the mean stress correction method, use the keyword \*FATIGUE\_MEAN\_STRESS\_CORRECTION. The following mean stress correction methods are available in LS-DYNA.

Goodman method 
$$S = \frac{\sigma_a}{1 - \sigma_m / \sigma_u} \tag{3}$$

Soderberg method 
$$S = \frac{\sigma_a}{1 - \sigma_m / \sigma_y} \tag{4}$$

Gerber method 
$$S = \frac{\sigma_a}{1 - (\sigma_m / \sigma_u)^2} \tag{5}$$

Goodman tension-only method, and Gerber tension-only method are also available. For the two tension-only methods, LS-DYNA performs mean stress correction for tensile mean stress only, using the equations (3) and (5).

In the equations (3)-(5),  $S$  is the corrected stress magnitude or alternating stress;  $\sigma_a$  is the original stress magnitude or alternating stress;  $\sigma_m$  is the mean stress;  $\sigma_u$  is the ultimate tensile strength and  $\sigma_y$  is the yield strength of the material.

For the model shown in Figure 2, an initial stress (mean stress) state is assumed (see Figure 8)

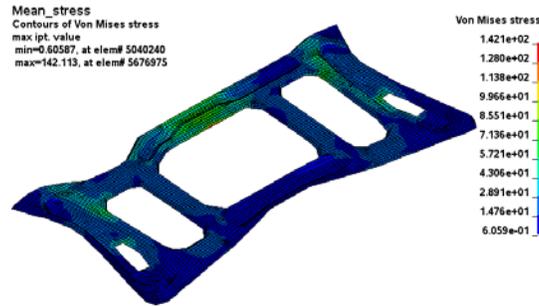


Figure 8. Initial Von mises stress (mean stress)

for the loading case with z-acceleration, the updated cumulative damage ratio results, with mean stress correction by Goodman’s method (equation 3), and by Soderberg’s method (equation 4), are shown in Figures 9 and 10.



Figure 9. Cumulative damage ratio for z-acceleration PSD loading only (use Goodman’s equation for mean stress correction)

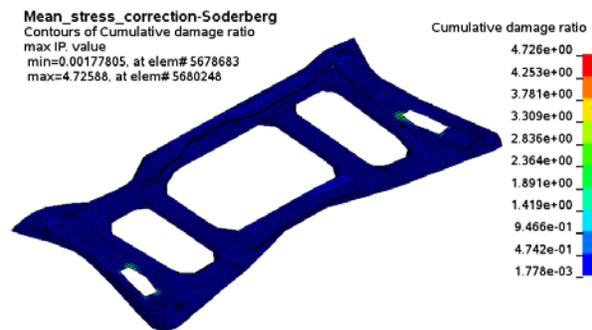


Figure 10. Cumulative damage ratio for z-acceleration PSD loading only (use Soderberg’s equation for mean stress correction)

Table 1 shows the maximum cumulative damage ratios with and without mean stress correction.

Mean stress correction method	Maximum damage ratio
No mean stress correction	2.480
Goodman’s equation	2.767
Soderberg’s equation	4.726
Gerber’s equation	2.510

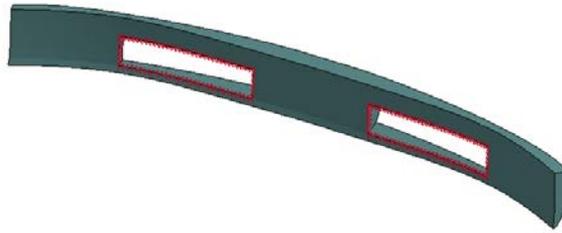
Table 1: Maximum damage ratio with and without mean stress correction

One can see, with the mean stress correction, the damage ratio of the structure is increased, which means that the fatigue life of the structure becomes shorter due to the existence of the mean stress.

### *Fatigue analysis in steady state vibration environment*

Sometimes we have steady state vibration environment, for example, the vibration in a sine sweep test. With the keyword `*FREQUENCY_DOMAIN_SSD_FATIGUE` one can run fatigue analysis based on steady state vibration. This approach is also based on Miner's rule of cumulative damage and we sum up the fatigue damage from different stress level.

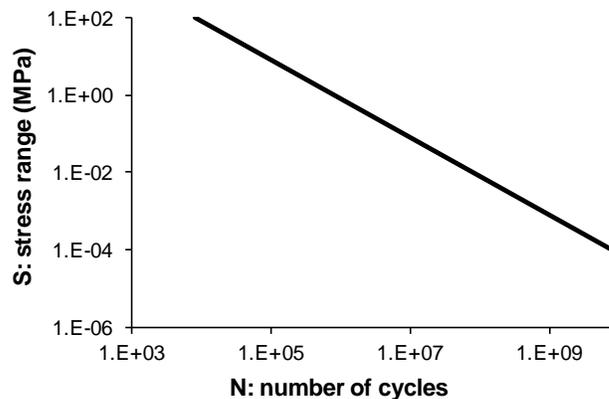
The example shown here is about the fatigue analysis of a solid structure (a simplified auto front bumper model), in a steady state vibration test.



*Figure 11.* A front bumper model in steady state vibration

The bumper is subjected to continuous vibration from ground excitation during driving. It is assumed that the bumper is constrained to auto frame by the edge of the two holes (see the red nodes shown in Figure 11).

The material's S-N curve is defined `*MAT_ADD_FATIGUE` using a curve shown in Figure 12.



*Figure 12.* S-N curve for the bumper model

The base acceleration spectrum is given in Table 2 (see below).

Frequency (Hz)	Amplitude (g)	Duration (sec.)
200	1.0	600
300	1.0	600
400	1.0	600
500	1.0	600
600	1.0	600
700	1.0	600
800	1.0	600
900	1.0	600
1000	1.0	600
1100	1.0	600
1200	1.0	600
1300	1.0	600
1400	1.0	600
1500	1.0	600

Table 2. Base acceleration spectrum

The cumulative damage ratio fringe plot can be found in Figure 13.

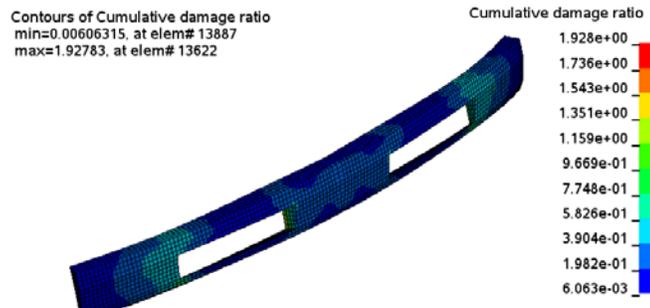


Figure 13. Cumulative damage ratio plot for the bumper

To locate the failed zone quickly, one can use the “Safe / Failed zone” check box and get the failed zone indicated by red elements as below (Figure 14).

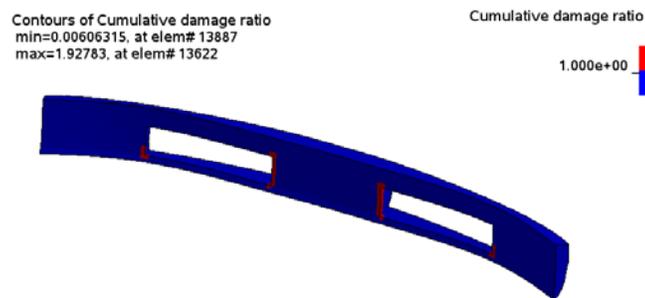


Figure 14. Safe / Failed zone under the steady state vibration

One can see that the failed elements are almost the same elements which are constrained to shaker table directly. For those elements we expect higher stress concentration due to the constraints and that is the reason for the higher chance for fatigue failure at the same location.

## Time domain fatigue analysis with LS-DYNA

For many applications, the loading conditions are defined in time domain and the response of the structures or parts are also computed in time domain. This is true for door slam, and for thermal loading on structures, and for many other cases where the nonlinear effect is significant and the problem cannot be linearized. To provide fatigue solution for those cases, time domain fatigue solvers are being implemented in LS-DYNA. The solvers can be put into 2 categories: one is based on stress index and the other is based on strain index.

A new keyword for general time domain fatigue analysis has been implemented

<b>*FATIGUE</b>			
\$#	pid	ptype	
	1	20	
\$#	dt		
	0.01		
\$#	strsn	index	restrt
	0		

To do mean stress correction, the keyword `*FATIGUE_MEAN_STRESS_CORRECTION` is needed.

A simple way to run fatigue analysis in time domain is to use the stress/strain data saved in ELOUT (see `*DATABASE_ELOUT` and `*DATABASE_HISTORY_SHELL`, `*DATABASE_HISTORY_SOLID`, etc.).

To use ELOUT, one just need to add option `_ELOUT` to `*FATIGUE` and skips the first two cards in this keyword. To save strain results to ELOUT, one can set `STRFLG = 1` in `*DATABASE_EXTENT_BINARY`.

The Material fatigue properties can be defined by keyword `*MAT_ADD_FATIGUE` (for SN curve) or `*MAT_ADD_FATIGUE_EN` (for EN curve). More details of these keywords can be found in LS-DYNA Users' Keyword manual.

### *Stress-based fatigue analysis*

For stress-based time domain fatigue solver, a rain flow counting method is adopted to get the number of stress cycles for each stress level. One can also make mean stress correction based on the Goodman method, the Soderberg method, the Gerber method and tension-only version of the Goodman method, and the Gerber method (see keyword `*FATIGUE_MEAN_STRESS_CORRECTION`).

The example concerning the stress based time domain fatigue analysis is a metal pipe model (Figure 15), subject to temperature oscillation (see Figure 16). For real life application, this pipe can be cannon or gun pipe and the temperature oscillation can be due to continuous gun fires. The thermal stress cycle due to the temperature oscillation induces fatigue damage to the metal. After sufficient number of gun fires, the cumulative damage ratio can reach 1 and indicates the end of the service life of the gun pipe.

The pipe is modelled by `*MAT_ELASTIC_PLASTIC_THERMAL`. And the thermal loading is defined by `*LOAD_THERMAL_LOAD_CURVE`.

pipe simulation

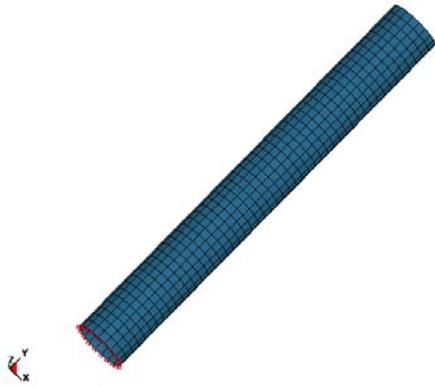


Figure 15. The metal pipe model

pipe simulation

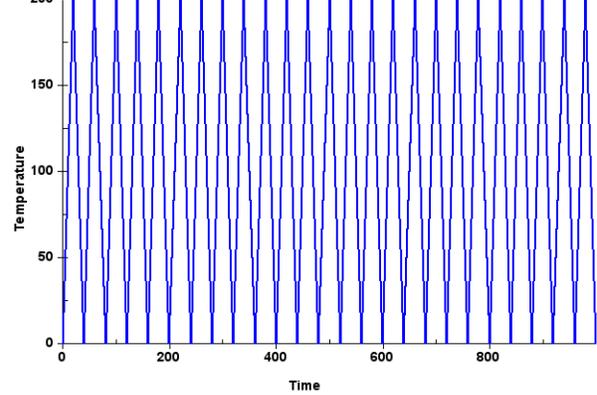


Figure 16. The temperature load curve

The material's fatigue property is defined by \*MAT\_ADD\_FATIGUE. The SN curve looks like

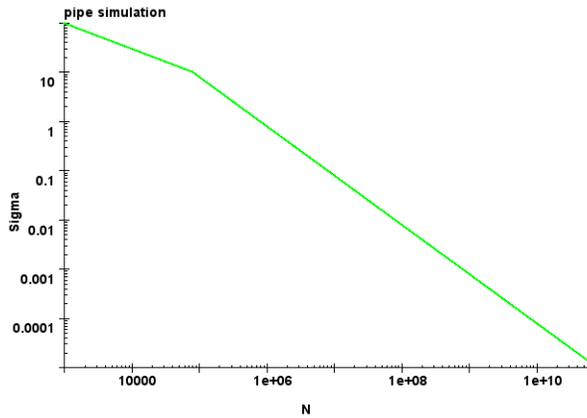


Figure 17. SN fatigue curve of the material

The cumulative damage ratio plot is given in Figure 18. One can see that the peak value of the cumulative damage ratio appears in the middle part of the pipe, which agrees with the location for peak value of the von mises stress (Figure 19).

pipe simulation  
Contours of Cumulative damage ratio  
max IP: vol%  
min=0.000231687, at elem# 1452  
max=0.0028434, at elem# 1076

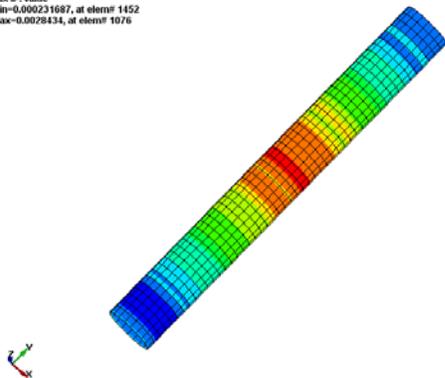


Figure 18. Cumulative damage ratio plot

Cumulative damage ratio  
2.843e-03  
2.582e-03  
2.321e-03  
2.060e-03  
1.799e-03  
1.538e-03  
1.276e-03  
1.015e-03  
7.540e-04  
4.929e-04  
2.317e-04

pipe simulation  
Contours of Effective Stress (v-m)  
max IP: vol%  
min=0.00242283, at elem# 1499  
max=0.0219156, at elem# 1176

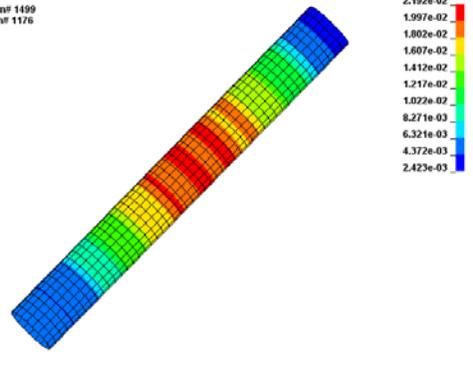


Figure 19. Distribution of von mises stress at termination time

Figure 20 shows the xy- shear stress cycles at an element in the middle of the pipe. One can see a lot of stress cycles have taken place in the 1000 seconds period.

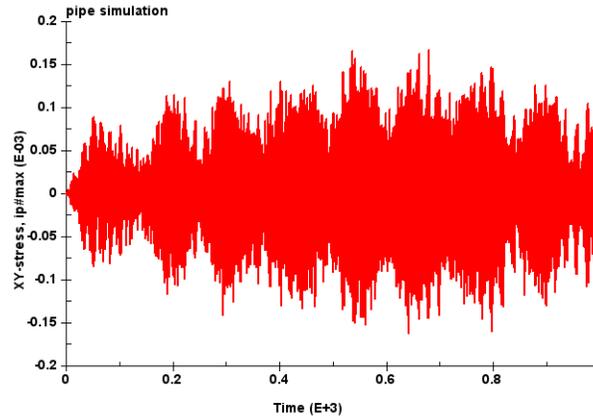


Figure 20. The xy-stress history at one element

### Strain-based fatigue analysis

For strain-based fatigue analysis, we use the local strain life equation

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c \quad (6)$$

With Smith-Watson-Topper mean stress correction, equation (6) is changed to

$$\frac{\Delta\varepsilon}{2} \sigma_{\max} = \frac{(\sigma'_f)^2}{E} (2N_f)^{2b} + \sigma'_f \varepsilon'_f (2N_f)^{b+c} \quad (7)$$

With Morrow's mean stress correction, equation (6) is changed to

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f - \sigma_m}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c \quad (8)$$

Here is the definition of the variables used in the equations

- $\Delta\varepsilon$  strain amplitude
- $N$  endurance, or number of cycles for failure
- $b$  the fatigue strength exponent (Basquin's exponent)
- $\sigma'_f$  the fatigue strength coefficient
- $\sigma_{\max}$  maximum stress
- $\sigma_m$  mean stress
- $c$  the fatigue ductility exponent (the Coffin-Mson exponent)
- $\varepsilon'_f$  the fatigue ductility coefficient, which is the plastic strain amplitude at  $2N_f = 1$
- $E$  the elastic modulus

If pure elastic materials are used in the stress-strain cycle computation (e.g. using an elastic finite element analysis), Neuber’s rule is used to convert the elastic stress/strain to local true stress/strain

$$\sigma \varepsilon = \sigma^e \varepsilon^e \tag{9}$$

Where,  $\sigma$  and  $\varepsilon$  are the local stress and strain,  $\sigma^e$  and  $\varepsilon^e$  are the elastic stress and strain.

For illustration purpose, we consider a bracket model shown in Figure 21. The bracket is modeled by \*MAT\_PIECEWISE\_LINEAR\_PLASTICITY.



Figure 21. A bracket model with sinusoidal nodal force excitation

The model is constrained at the two rings. The sinusoidal nodal force is defined as

$$F(t) = 5 \sin(1000t) \text{ N} \tag{10}$$

In the example, the material’s fatigue constants are defined as

$$E = 209,000 \text{ MPa}; \quad K' = 1230 \text{ MPa}; \quad n' = 0.161; \quad \sigma'_f = 1006 \text{ MPa}$$

$$b = -0.0809; \quad \varepsilon'_f = 1.104; \quad c = -0.6207$$

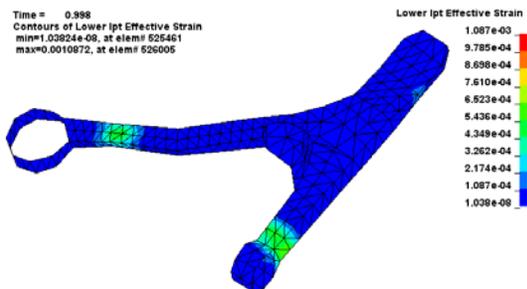


Figure 22. Effective strain at lower layer integration points

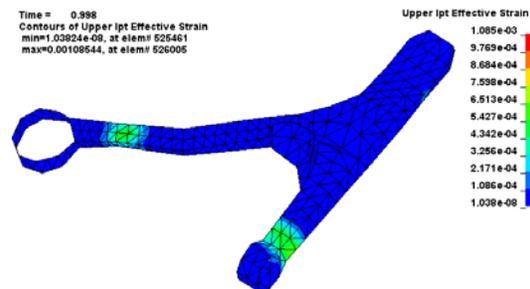


Figure 23. Effective strain at upper layer integration points

The cumulative damage ratio due to the sinusoidal nodal force for 1 second is computed and plotted in Figure 24. The location for the peak value of the cumulative damage ratio matches well with the location for highest strain values. Figure 24 shows the strain history at the element which has the highest cumulative damage ratio.

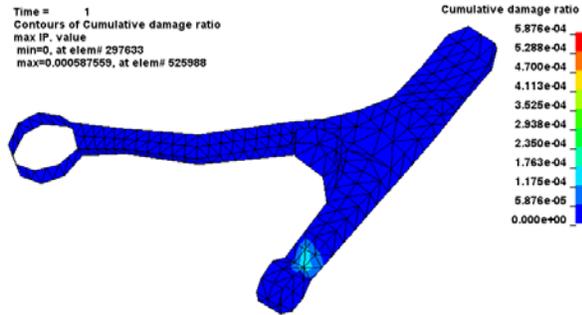


Figure 24. Cumulative damage ratio

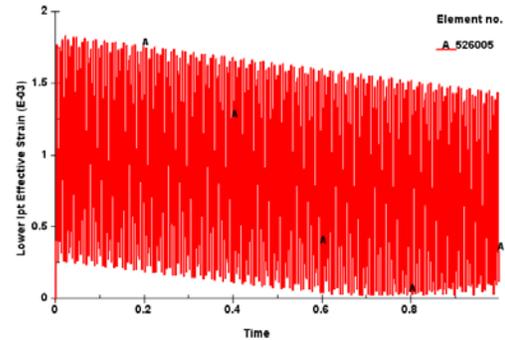


Figure 25. Effective strain at upper layer integration points

## Conclusion and future work

Some basic frequency domain and time domain fatigue analysis methods have been implemented to LS-DYNA. For frequency domain fatigue analysis, the index variable is stress; for time domain analysis, both stress and strain can be used as the index variable. These methods provide user the opportunity to run fatigue and durability analysis on the structures, following the vibration analysis in frequency domain or time domain. The results not only provide user an estimate of the fatigue life of the structures, but also suggest the location of “hot spot” for future revision or optimization.

Regarding future work, firstly, more testing and validations are needed, to check the accuracy, reliability of the current fatigue solvers. Secondly, an approach to consider multi-axial stress /strain state is needed, for more realistic simulation of the fatigue failure process. The current methods are based on the stress or strain invariants (e.g. the Von mises stress / strain, or the maximum shear stress / strain). We are interested in exploring the new methods based on critical plane approach.

## References

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