An Engineering Approach to Estimating Partially Saturated Soil Constitutive Properties Using LS-DYNA®

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Abstract

Soil is perhaps the most common civil engineering material, and ironically one of the most difficult to model due to its variability. At any given site, soil samples taken at different depths and distances may show considerable variability. Even if such samples show relative uniformity, there is often the question of what happens if it rains and the soil saturation changes?

In part because of soil’s variability, experimental research involving soil often uses dry sand that is widely available and has been characterized sufficiently for use in constitutive models. Any soil constitutive model has two main components:

1. Compaction response, i.e. pressure versus volume strain response
2. Shear strength as a function of confining pressure, i.e. frictional material behavior.

This manuscript provides an engineering model for both of these constitutive components, such that given a representative constitutive response of dry sand, an estimate of the constitutive properties of a partially or fully saturated version of that dry sand can be made.

Depending on the loading rate and pressure, one or the other of these two constitutive components will be dominant. For high rate loading, e.g. buried charges, pore water does not have time to migrate and remains trapped in partially, or fully, water filled voids in the soil. For this loading rate the compression behavior is dominant and may be defined by a single Equation of State.

Shear strength of sandy soils varies inversely with saturation due to the lubricating effect of water on the interlocking sand particles. At low loading pressures where shear strength dominates, e.g. far-field ground “shock” (stress) propagation and soil-structure interaction, a simple bi-linear pressure-dependent yield surface may adequately capture the sandy soil behavior.

Compaction Model

Figure 1 shows an illustration of two springs in parallel used to represent the compaction behavior of partially saturated soil. This two spring model was motivated by a similar three spring model proposed by Barsotti et al. (2016).
The lower spring with stiffness represents the compaction of the dry soil and the upper spring with stiffness represents the compaction of water. The water spring represents the pores (voids in the soil) that are filled with water. The empty pores must be fully compressed before the water filled pores will be compressed.

In this one dimensional representation of compaction, the applied force is a proxy for the pressure in a hydrostatic compression test and the change in length of the soil spring represents the volume strain, i.e.

$$\varepsilon_{kk} = \ln \left( 1 - \frac{\Delta L}{L_0} \right)$$  \hspace{1cm} (1)
Figure 2 is an illustration of the two extremes for soil compaction of saturated and dry soil with an intermediate case of partially saturated soil. For the saturated case there are no empty pores so both the water and soil springs are engaged simultaneously. For the dry soil case all the pores are empty so there is no water spring and only the soil spring provides resistance to compaction. For the partially saturated case, the empty pores are compacted without resistance, so the dry soil compaction response is followed. Once the water filled pores are engaged both the water and soil springs provide resistance to additional compaction.

The compaction curve for water is obtained using the LS-DYNA *EOS_GRUNEISEN with parameters: C=1480 m/s, $S_1 = 1.92$ and $\gamma_0 = 0.1$ combined with *MAT_NULL that provides the density of water.

The compaction curve for dry soil must be obtained from laboratory hydrostatic compression testing. Unfortunately, dry soil is seldom tested as some moisture is needed to help mold and fill the test specimens. The addition of water to such “dry” test samples is prescribed by the Procter Compaction Test as defined by the American Society for Testing Materials (ASTM D698 & D1557).

“The Proctor compaction test is a laboratory method of experimentally determining the optimal moisture content at which a given soil type will become most dense and achieve its maximum dry density.”

Wikipedia

Also see the derivation of the dry density $\rho_d$ in a subsequent section.

**Partially Saturated Compaction Algorithm**

The basic idea of the algorithm is to combine the two spring stiffnesses accounting for the empty and water filled voids. This is accomplished by shifting the start of the water spring to when the empty void cells have been compacted.

The saturation ratio, $S_w$, provides the percentage of voids that are filled with water. It follows that the percentage of voids that are empty is $1 - S_w$. The total amount of voids, or porosity, $\phi$, is given by

$$\phi = \frac{V_v}{V} = 1 - \frac{\rho_d}{\rho_s}$$  \hspace{1cm} (2)

where $V_v$ is the volume of voids and $V$ is the total volume. Also, $\rho_d$ is the dry density and $\rho_s$ is the density of the soil grains, often taken as 2640 kg/m$^3$.

Thus the water-spring should be engaged when the empty voids are collapsed

$$\frac{\Delta V}{V_0} = \phi (1 - S_w)$$  \hspace{1cm} (3)

or in terms of the natural volume strain

$$\ln \left[1 - \frac{\Delta V}{V_0}\right] = \ln \left[1 - \phi (1 - S_w)\right]$$  \hspace{1cm} (4)
In the two spring model LS-DYNA implementation, the water-spring volume strains are offset by the amount of volume strain given by Equation (4), i.e. the *DEFINE_CURVE parameter \( \text{OFFA} = \ln \left[ 1 - \phi (1 - S_w) \right] \).

### Compaction Model Illustration

Laboratory provided hydrostatic compression data for a sandy soil at three water ratios: 10, 25 and 45%, with the first called “dry” and the last called “saturated.” The model input parameters are:

- Grain (particle) density \( 2.641 \times 10^{-3} \text{ g/mm}^3 \)
- Water density \( 1.000 \times 10^{-3} \text{ g/mm}^3 \)
- Water pressure versus natural volume strain (from EOS)
- “Dry” soil pressure versus natural volume strain \(^1\)
- Water ratio
- Soil dry (no water) density
- Soil density as tested (“wet”)
- Soil spring length \( L_0 = \phi \) (see footnote \(^2\))

<table>
<thead>
<tr>
<th>Description</th>
<th>Water Content (%)</th>
<th>Saturation ( S_w )</th>
<th>Dry Density (kg/m(^3))</th>
<th>Wet Density (kg/m(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry</td>
<td>10</td>
<td>0.23</td>
<td>1219</td>
<td>1342</td>
</tr>
<tr>
<td>Partially Saturated</td>
<td>25</td>
<td>0.57</td>
<td>1219</td>
<td>1525</td>
</tr>
<tr>
<td>Saturated</td>
<td>45</td>
<td>1.02</td>
<td>1219</td>
<td>1770</td>
</tr>
</tbody>
</table>

Table 1 provides the reported water percentage and densities of the tested soil. Figure 3 shows the corresponding data with the two spring model comparison for the three water ratios.

---

\(^1\) If “dry” soil pressure versus volume strain is not available from a laboratory test, the data provided by Laine and Sandvik (2001) may be used.

\(^2\) A spring length equal to the porosity is an “over kill” for the saturated case where the offset OFFA is near zero. A more general spring length of OFFA + 0.2 is recommended.
The model reproduces the saturated data as the computed water curve offset, from Equation (4), is only 0.01 because all the pores are water filled. For the partially saturated case with 25% water content the model over predicts the water curve offset as 0.265 rather than the measured 0.2, or an error of about 33%. Finally, for the dry 10% water case the model under predicts the water curve offset as 0.54 compared to the measured 0.6 or an error of -10%.

As this is the only laboratory data used to compare with the model, it is inclusive if the model would perform better or worse for a different soil.

**Saturation Modification of Shear Failure Surface**

Figure 4 shows the laboratory shear failure data for the saturation levels corresponding to the above compaction data. As indicated by Anderson et al. (2010), the addition of water to dry sand dramatically reduces the shear strength. In the present case, the presence of the water essentially eliminates the frictional response of the sand.

![Figure 4 Shear failure data for three degrees of saturation.](image)

**References**


**Appendix Saturation Ratio**

The saturation ratio is the fraction of pores that are filled with water, often expressed as
where $V_w$ is the volume of water and $V_v$ is the volume of voids. Typically the volume of water is determined by weighing the sample and then oven drying the sample to determine $M_w$ the mass of water. The volume of water is calculated via

$$V_w = \frac{M_w}{\rho_w} \quad (6)$$

where $\rho_w = 1000 \text{ kg/m}^3$. The void volume is obtained by subtracting the soil particle volume from the total volume

$$V_v = V - \frac{M_s}{\rho_s} = V - \frac{M_s}{G\rho_w} \quad (7)$$

Where $M_s$ is the mass of the dry soil and $\rho_s$ is the grain density often take to be $2640 \text{ kg/m}^3$. The second form uses the specific gravity of the soil particles $G = \rho_s / \rho_w$.

Now the saturation ratio can be expressed as

$$S_w = \frac{V_w}{V} = \frac{\frac{M_w}{\rho_w}}{V - \frac{M_s}{\rho_s}} \quad (8)$$

Noting the volume can be expressed in terms of the sample density and mass

$$V = \frac{M}{\rho} = \frac{M_w + M_s}{\rho} \quad (9)$$

The saturation ratio can be rewritten as

$$S_w = \frac{V_w}{V_v} = \frac{\frac{M_w}{\rho_w}}{(M_w + M_s)/\rho - M_s/\rho_s} = \frac{\frac{\rho_s M_w}{\rho w}}{\rho_s (M_w + M_s) - M_s} \quad (10)$$

$$= \frac{\rho_s M_w}{\rho w} \frac{1}{\rho_s (M_s + 1)} - 1 \quad = \frac{G\omega}{\rho w (1+\omega)} - 1$$

where the notation $\omega = M_w / M_s$ i.e. gravimetric water content, has been introduced.
Appendix Dry Density

Starting from the total mass

\[ M = M_w + M_s \quad (11) \]

rearranging

\[ M = M_s (1 + \omega) \quad (12) \]

where \( \omega = M_w / M_s \) is the gravimetric water content, and finally

\[ M_s = \frac{M}{(1 + \omega)} \quad (13) \]

The definition of the dry density is

\[ \rho_d = \frac{M_s}{V} = \frac{M}{V(1 + \omega)} = \frac{\rho}{(1 + \omega)} \quad (14) \]

Appendix – Two Spring LS-DYNA Input Template

*Keyword

$    C:\All-Mine\LS-DYNA\Version971\ls-dyna_smp_d_DEV_115126_winx64.exe
$    i=Two-Springs.k
$

******************************************************************************
$    Len Schwer
$    Schwer Engineering & Consulting Services
$    Len@Schwer.net
$    Revised 4 July 2017
******************************************************************************

$    UNITS:
$    length -> millimeters
$    time -> milliseconds
$    mass -> grams
$    force -> Newtons
$    stress -> F/A = Newtons/mm^2 = MPa
$

$ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
$    Input:
$    GrainD -- grain density typically 2640kg/m^3
$    Density -- density of tested soil, i.e. wet density
$

$    Output:
$    NODOUT -- Node 1 X-Displacement

---

\textsuperscript{3}\textsuperscript{ Derivation provided by Dr. Lance Besaw.}
SPCFOR -- Node 2 X-Force

*TITLE
Spring Model for Partially Saturated Soil

*Parameter
Tstart 0.0  r Tend  5.00  i States 20  r Tmax  1.00e-4
r TSSFAC 0.6  i LCTM  500  i Binary 1

PORO -- Porosity = 1 - dry density / grain density
GrainD -- grain density typically 2640kg/m^3
H2ODen -- water density typically 1000 kg/m^3
Density -- density of tested soil, i.e. wet density
H2OPer -- water percentage -- w = mass of water / mass of soils

GrainD 2.641E-3  r H2ODen 1.0E-3
DenDry 1.219E-3
Density 1.342E-3  r H2OPer 0.10
Density 1.525E-3  r H2OPer 0.25
Density 1.770E-3  r H2OPer 0.45
Density 1.750E-3  r H2OPer 0.45

*Parameter Expression

TDplot (Tend-Tstart)/(States-2)
TASCII TDplot/100.0
TLBE 4.0*TDplot
TASCITR TDplot/100.0

Tend2 2.0*Tend

PORO 1.0 - DenDry/GrainD
SpecG GrainD/H2ODen

RhoRat GrainD/Density
SatRat SpecG*H2OPer/(RhoRat*(1 + H2OPer) - 1.0)
VoidR PORO*(1.0 - SatRat)

VStrain log(1.0 - VoidR)
Compres (-1.0*VStrain) + 0.2
SprLen (-1.0*VStrain) + 0.2

Control CARDS

*CONTROL_CONTACT
0.900,0.000E+00,0,0,0,0,0,0
0,0,0,0,0.000E+00,0,0,0

*CONTROL_ENERGY
2,0,0,0
*CONTROL_OUTPUT

$ NPOUT  NEECHO  NREFUP  IACCOP  OPIFS  IPNINT  IKEDIT  IFLUSH
  1,    3,     0,       0,     0.0,    0,    99000,    0
$ IPRTF  IERODE  TET10S8  MSGMAX  IPCURV  GMDT  IP1DDBLT  EOCS
  ,      ,      ,      ,     0,    0,     0,      , 0

*CONTROL_TERMINATION

$ SOLN  NLQ  ISNAN  LCINT
  0,    0,    1

*CONTROL_TIMESTEP

$ DTINIT  TSSFAC  ISDO  TSLIMT  DT2MS  LCTM  ERQDE  MS1ST
  0.0,   0.90,    0, 0.0,   0.0,  &LCTM,   0,    0

$========1=========2=========3=========4=========5=========6=========7=======
$                               Database CARDS
$========1=========2=========3=========4=========5=========6=========7=======

*DATABASE_ELOUT

&TASCII, &Binary

*DATABASE_GLSTAT

&TASCII, &Binary

*DATABASE_JNTFORC

&TASCII, &Binary

*DATABASE_MATSUM

&TASCII, &Binary

*DATABASE_NODOUT

&TASCII, &Binary

*DATABASE_RBDOUT

&TASCII, &Binary

*DATABASE_RCFORC

&TASCII, &Binary

*DATABASE_SLEOUT

&TASCII, &Binary

$*DATABASE_DEFORC

&TASCII, &Binary

*DATABASE_SPCFORC

&TASCII, &Binary

$*DATABASE_BINARY_D3PLOT

&TDplot

$*DATABASE_EXTENT_BINARY

$ NEIPH  NEIPS  MAXINT  STRFLG  SIGFLG  EPSFLG  RLTFGL  ENGFLG
  8,    14,    0,      1,     1,     1,     0,     0
$ CMPFLG  IEVERP  BEAMIP  DCOMP  SHGE  STSSZ  N3THDT  IALEMAT
  0,     0,      0,     0,     0,     0,     0,     0
$ NINTSLD  PKP_SEN  SCLP  HYDRO  MSSCL  THERM  INTOUT  NODOUT
  1

$*DATABASE_HISTORY_NODE

$ ID1  ID2  ID3  ID4  ID5  ID6  ID7  ID8
  1,    2

$*Comment *DATABASE_HISTORY_BEAM

$ ID1  ID2  ID3  ID4  ID5  ID6  ID7  ID8
  1,    2,    3

$========1=========2=========3=========4=========5=========6=========7=======
$ NODE & ELEMENT CARDS
$========1=========2=========3=========4=========5=========6=========7=======
$
*Node
1, 0.0, 0.0, 0.0
2, &SprLen, 0.0, 0.0
$
*Element_Discrete
$ EID  PID  N1  N2  VID   S   PF   OFFSET
 1, 100, 1, 2, , , 0
 2, 200, 1, 2, , , 0
$
*Set_Node_List
$ SID
12
$ NID1  NID2  NID3
 1, 2
$
*ELEMENT_MASS_Node_Set
$ eid  nid            mass    pid
 321      12          1.0E-3
$
$========1=========2=========3=========4=========5=========6=========7=======
$                             SECTION CARDS
$========1=========2=========3=========4=========5=========6=========7=======
$
*Section_Discrete
$ SECID DRO KD V0 CL FD
 100, 0
$ CDL TDL
 0.0, 0.0
$
$========1=========2=========3=========4=========5=========6=========7=======
$                             PART CARDS
$========1=========2=========3=========4=========5=========6=========7=======
$
*Part
Soil Spring
$ PID  SECID  MID EOSID HGID GRAV ADPOPT TMID
 100, 100, 100
$
*Part
Water Spring
$ PID  SECID  MID EOSID HGID GRAV ADPOPT TMID
 200, 100, 200
$
$
$========1=========2=========3=========4=========5=========6=========7=======
$                             BOUNDARY SPC CARDS
$========1=========2=========3=========4=========5=========6=========7=======
$
*BOUNDARY_SPC_Node
$ NID/NSID CID DOFX DOFY DOFZ DOFRX DOFRY DOFRZ
 1, 0, 0, 1, 1, 1, 1
 2, 0, 1, 1, 1, 1, 1
$
$========1=========2=========3=========4=========5=========6=========7=======
$                             MATERIAL CARDS
$========1=========2=========3=========4=========5=========6=========7=======
$
*MAT_SPRING_NONLINEAR_ELASTIC
$ MID LCD LCR
  100, 100
$

*MAT_SPRING_NONLINEAR_ELASTIC
$ MID LCD LCR
  200, 200
$

$========1=========2=========3=========4=========5=========6=========7=======
$                             DEFINE CURVE CARDS
$========1=========2=========3=========4=========5=========6=========7=======
$
*DEFINE_CURVE
$ LCID SIDR   SFA   SFO OFFA OFFO DATTYP LCINT
  100,     , 1.0, 1.0
$ Displacement  Force  Soil 10% H2O
-0.6216,  -49.7094
-0.6211,  -49.6443
-0.6184,  -48.6499
-0.6156,  -47.6154
-0.6128,  -46.6062
-0.6099,  -45.5696
-0.6070,  -44.5981
-0.6040,  -43.5279
-0.6011,  -42.5334
-0.5983,  -41.5765
-0.5954,  -40.9492
-0.5918,  -39.6900
-0.5885,  -38.7228
-0.5850,  -37.7700
-0.5813,  -36.7753
-0.5777,  -35.7534
-0.5738,  -34.8091
-0.5699,  -33.9614
-0.5652,  -32.6370
-0.5605,  -31.5353
-0.5559,  -30.5384
-0.5508,  -29.4261
-0.5455,  -28.3453
-0.5400,  -27.2266
-0.5347,  -26.1814
-0.5292,  -25.1466
-0.5234,  -24.1223
-0.5173,  -23.1022
-0.5110,  -22.0925
-0.5043,  -21.0597
-0.4972,  -20.0269
-0.4897,  -19.0192
-0.4818,  -17.9842
-0.4733,  -16.9094
-0.4642,  -15.9394
-0.4545,  -14.8519
-0.4439,  -13.8460
-0.4325,  -12.8108
-0.4206,  -11.7735
-0.4092,  -10.9039
-0.3953,  -9.8665
-0.3826,  -9.0286
-0.3668,  -8.0628
-0.3489,  -7.1998
$*DEFINE_CURVE
$ LCID SIDR SFA SFO OFFA OFFO DATTYP LCINT
200 , 1.0, 1.0, \&VStrain
$ Displacement Force Water
-0.218, -1098.580
-0.202, -948.338
-0.186, -815.029
-0.170, -697.454
-0.154, -592.631
-0.138, -499.765
-0.123, -416.641
-0.107, -342.727
-0.091, -276.353
-0.076, -217.157
-0.061, -163.861
-0.045, -116.222
-0.030, -73.245
-0.015, -34.763
0.0, 0.0
-\&VStrain, 1.0E-6
1.0, 0.0
$
$
$*DEFINE_CURVE
$ LCID SIDR SFA SFO OFFA OFFO DATTYP LCINT
400
$ Time Displacement Loading
0.0, 0.0,
&\text{Tend}, &\text{Compres}
&\text{Tend2}, &\text{Compres}
$ &\text{Tend}, &\text{PORO}
$ &\text{Tend2}, &\text{PORO}
$
$
$*DEFINE_CURVE
$ LCID SIDR SFA SFO OFFA OFFO DATTYP LCINT
&\text{LCTM}
$ Time Time Step Time Step
0.0, 0.01
&Tend2, 0.01
$
$========1=========2=========3=========4=========5=========6=========7========$
$                             NODAL LOAD CARDS
$========1=========2=========3=========4=========5=========6=========7========$
$
*BOUNDARY_PRESCRIBED_MOTION_Node
$  typeID DOF VAD LCID SF VID DEATH BIRTH
   1,  1,  2,  400
$
$
*$End