Statistical Energy Analysis for High Frequency Acoustic Analysis with LS-DYNA®

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Abstract

The present work concerns about the new capability of LS-DYNA® in solving high frequency vibration and acoustic problem, using statistical energy analysis (SEA). As the frequency increases, the number of modes increases. As the result, the traditional numerical methods like finite element (FEM) and boundary element method (BEM) are difficult to use due to the large number elements requirement. It is more practical to consider average responses and their distribution over the structure, using a technique such as statistical energy analysis (SEA). In this paper, several numerical examples are investigated by SEA method with LS-DYNA®. The numerical results are in good agreement with other code.

Introduction

Statistical Energy Analysis (SEA) is a structural-acoustic method that is widely used for high frequency analysis. SEA arose during the 1960´s in the aerospace industry to predict the vibrational behavior when designing space craft. During this time computational methods were available but the size of the models that could be handled and the computational speed were such that only few of the lowest order modes could be predicted [1]. Furthermore, in traditional analysis of mechanical vibration the lowest modes are usually of most interest because these modes normally have the greatest displacement response. But when designing and constructing large and lightweight aerospace structures it is apparent that also high frequency broad-band loads is important in the process of predicting structural fatigue, equipment failure and noise production. The name SEA points out certain aspects of the method. *Statistical* accents that the system being considered are a member of a population of similar systems with known distributions of the subsystem parameters. *Energy* is the variable of interest. It describes the behavior of the system in terms of stored, dissipated and exchanged energies of vibration. Other often used variables for acoustic and structural vibration, such as displacement, pressure etc. can be derived from the energy of vibration. *Analysis* emphasizes that SEA is a framework rather than a specific technique. The development and use of SEA proved to be a good method to predict high frequency loads and the analysis technique has since then been applied, extended and developed for a growing number of applications. For example it has been used to model sound and vibration transmission in buildings, cars, aircrafts, ships and trains [2] and [3].

The General Procedures of SEA

One of the fundamental principles of SEA is that the average power flow between coupled groups of dynamical modes is proportional to the difference I the average modal energies. This makes it possible to analyze the dynamical response of a system consisting of many resonant modes in a certain frequency range by dividing the modes into groups (subsystems) and considering a power balance equation of the form $P_{in} = P_{out}$ for each mode group. The analysis is statistical in that the expected value and variance of the power flow are evaluated assuming a statistical distribution in the resonance frequencies of the subsystem modes [2].

The general procedures of SEA can be illustrated as shown in Figure 1 below. This system consists of two connected subsystems.

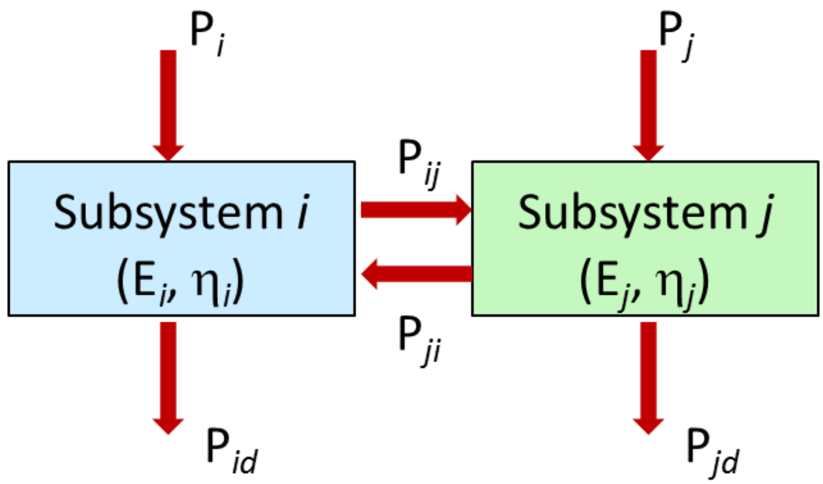


Figure1. A two subsystems SEA model.

Energy flows in and out of a subsystem. The energy that flows out consists of dissipation P_{id} *and* P_{jd} , radiation and transmission to other subsystems P_{ij} *and* P_{ji} . The energy that flows into a subsystem consists of external source excitations P_i *and* P_j and transmissions from other subsystems P_{ij} *and* P_{ji} .

The dissipated power in a subsystem is given by equation 1 and 2 below:

$$P_{id} = \omega \eta_{id} E_i \tag{1}$$

$$P_{jd} = \omega \eta_{jd} E_j \tag{2}$$

Where η_{id} *and* η_{jd} are the damping loss factors and E_i *and* E_j are the total vibrational energy of the modes at frequency f .

The net transmitted power between subsystem *i* and *j* (P_{ij} *and* P_{ji}) are given by equation 3 and 4.

$$P_{ij} = \omega \eta_{ij} E_i \tag{3}$$

$$P_{ji} = \omega \eta_{ji} E_j \tag{4}$$

SEA calculation is based on energy flow equilibrium. The power balances for the two systems are given as in equations 5 and 6.

$$P_i + P_{ji} = P_{ij} + P_{id} \tag{5}$$

$$P_j + P_{ij} = P_{ji} + P_{jd} \tag{6}$$

When combining equations 1-6, the power balance equation for two subsystems can be expressed in matrix form shown in equation 7.

$$\left[\frac{P_i}{P_j}\right] = \omega \begin{bmatrix} (\eta_{id} + \eta_{ij}) & -\eta_{ji} \\ -\eta_{ij} & (\eta_{jd} + \eta_{ji}) \end{bmatrix} * \left[\frac{E_i}{E_j}\right] \tag{7}$$

For a more generalized case with & number of subsystems, the balance equations can be written in a general form as equation 8.

$$\begin{bmatrix} P_i \\ P_j \\ \vdots \\ P_n \end{bmatrix} = \omega \begin{bmatrix} \eta_i & -\eta_{ji} & \cdots & -\eta_{ni} \\ -\eta_{ij} & \eta_j & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ -\eta_{in} & \cdots & \cdots & \eta_n \end{bmatrix} * \begin{bmatrix} E_i \\ E_j \\ \vdots \\ E_n \end{bmatrix} \tag{8}$$

Where η_i equals the total loss factor and is the summation of the damping loss factor of the subsystem and the coupling loss factors representing energy transmission from the subsystem to others as equation 9.

$$\eta_i = \eta_{id} + \sum_{j=1,\, j\neq i}^{n} \eta_{ij} \tag{9}$$

Example 1: Three plates connection

The first example is three plates connected at a common line shown in Figure 2.

The plates are all made of steel. Each of the plates has a size of 1m x 1m. Plate 1 and Plate 3 are 2 mm thick, Plate 2 is 3 mm. Plate 1 is connected to plate 2 at an angle of 90°, plate 3 is connected at an angle of 210°. All Plates have constant damping of 0.01. Plate 1 is excited by an input power of 0.5 Watt (in bending wave).

We want to calculate the mean velocity amplitude of the bending wave at each plate. Figure 3 is the velocity of plate1 and plate 2. The results are compared with other code [4].

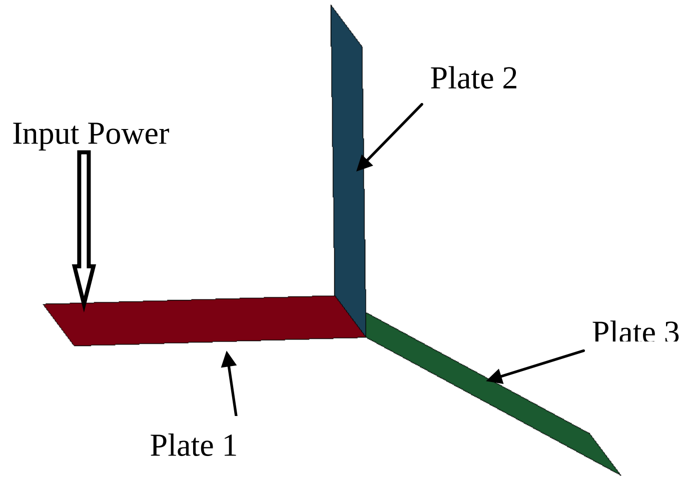
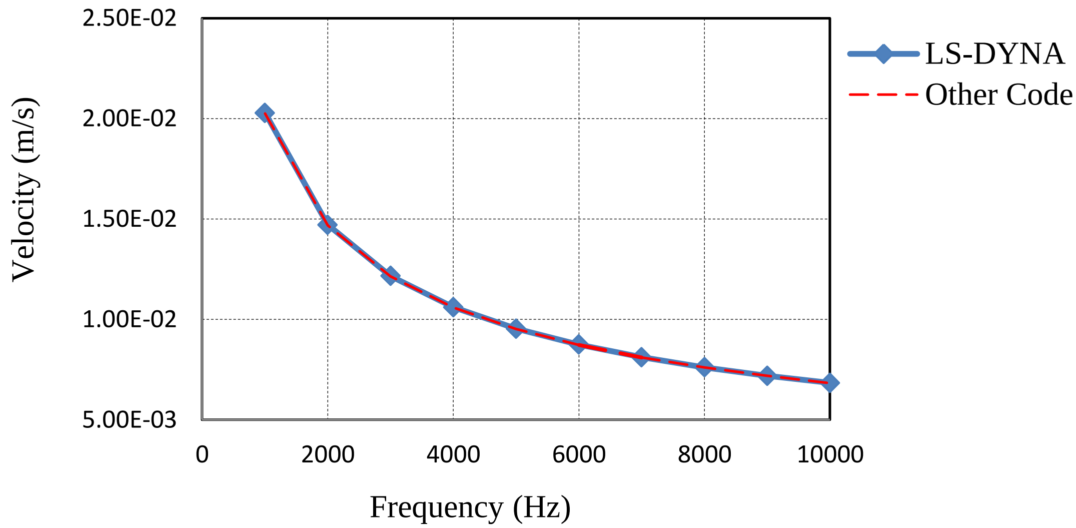


Figure2. Three plates connection model



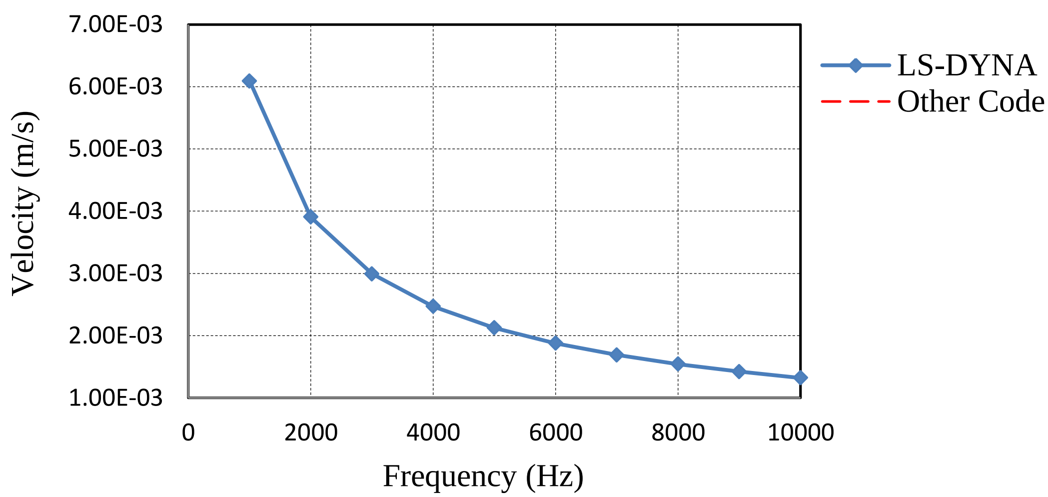


Figure3. Velocity on Plate 1 and plate 2

Example 2: Sound transmission inside a car

This simple car example shown in the figure 4 consists only of plates welded together and is not a very realistic construction. A real car would have some sort of a frame, pillars and stiffeners.

The SEA model has 26 subsystems, 18 steel plates, 6 glass plates and 2 acoustic cavities. The steel plates and glass plates are modelled as linear elastic materials. Input power are applied on floor, fender right front, fender left front and windshield.

The results are compared with other code [4] shown in figure 5 and figure 6.

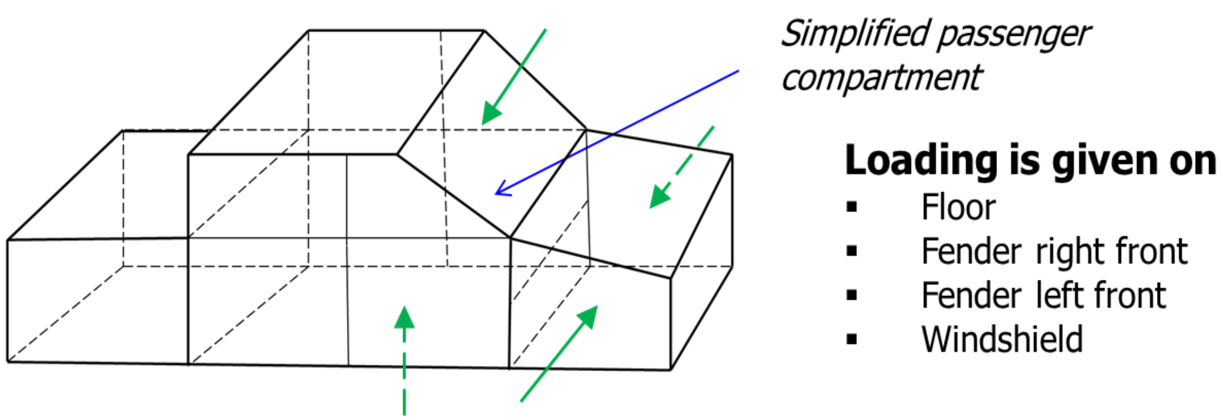


Figure4. Simple car SEA model

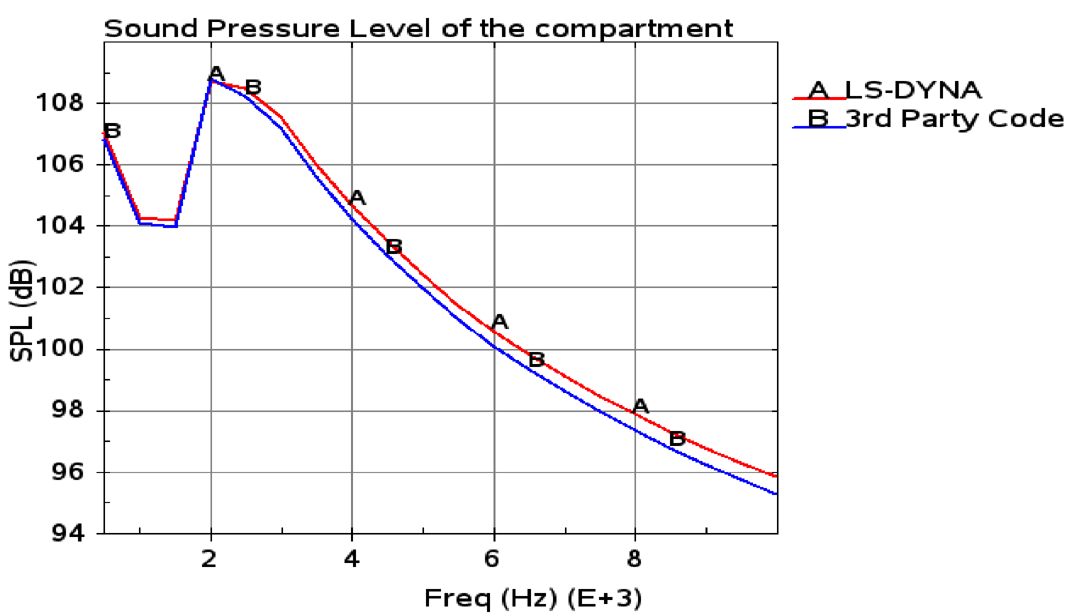


Figure5. Sound pressure inside compartment

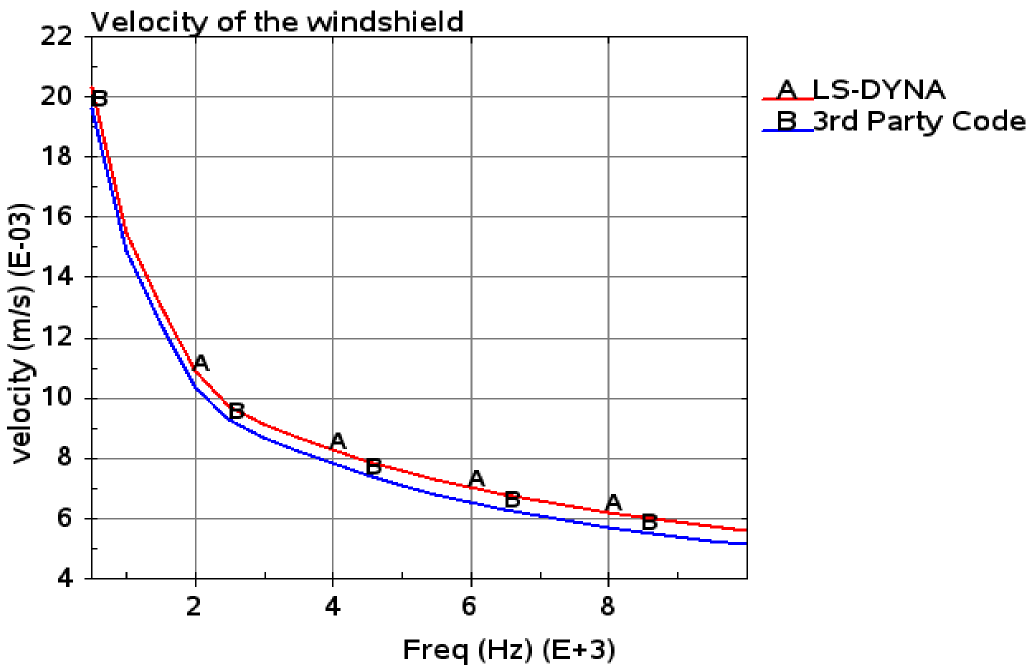


Figure6. Velocity of the windshield

Example 3: Acoustic analysis of a simple ship model

This simple ship model is courtesy of Numerical Engineering Solutions, Australia. The SEA model is shown in the figure 7.

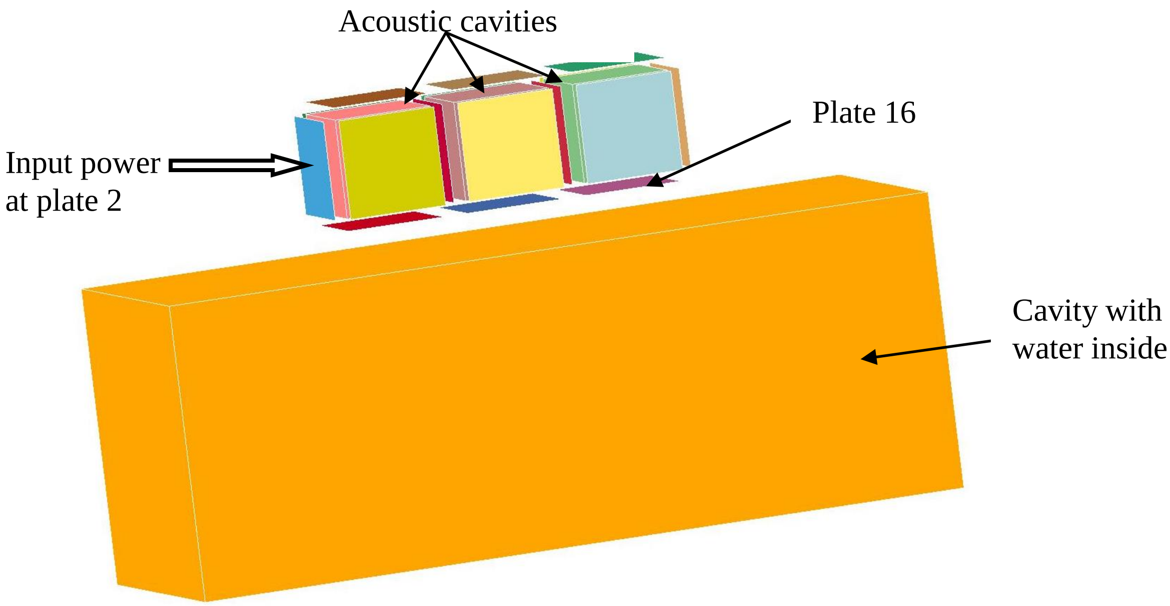


Figure7. SEA ship on water

The SEA model has 20 subsystems, 16 steel plates, 3 acoustic cavities with air inside and one acoustic cavity filled with water. The steel plates are modelled as linear elastic materials. Input power is applied on plate 2.

The results are compared with other code [4] shown in figure 8, 9 and 10. .

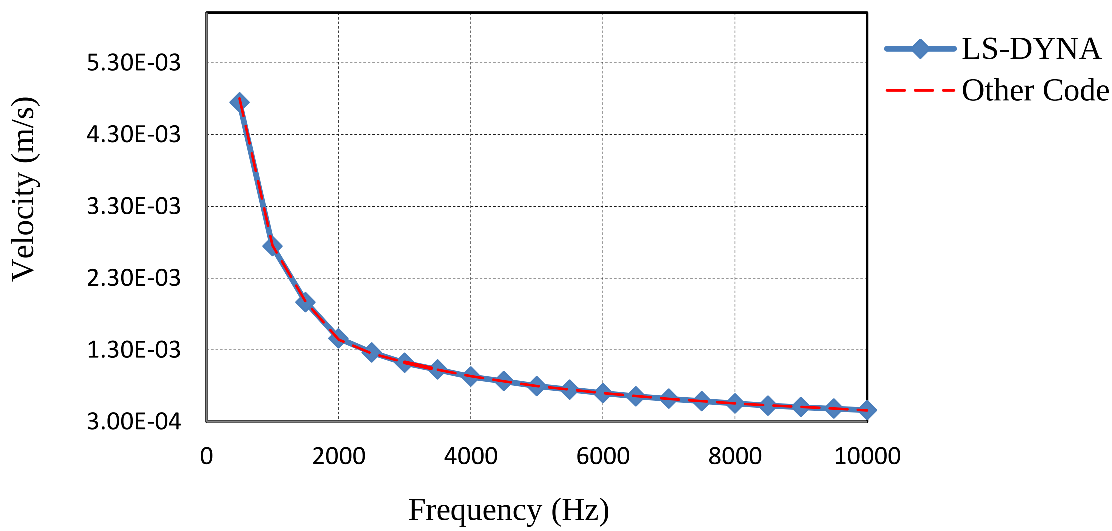


Figure8. Velocity of plate 16

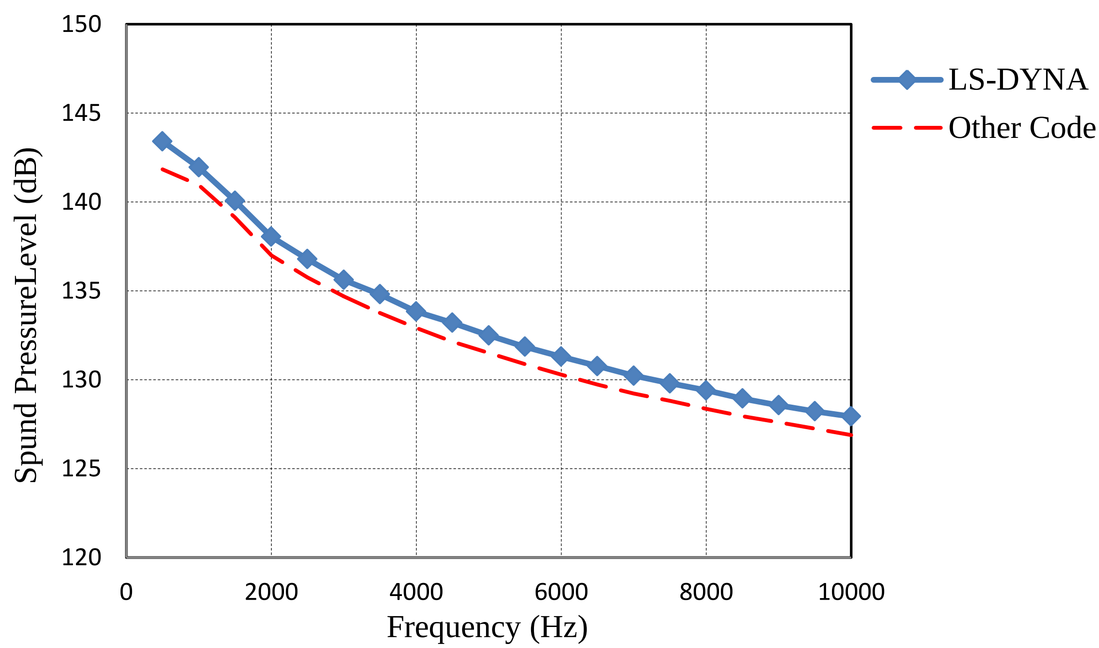


Figure9. Sound pressure level of cavity with water

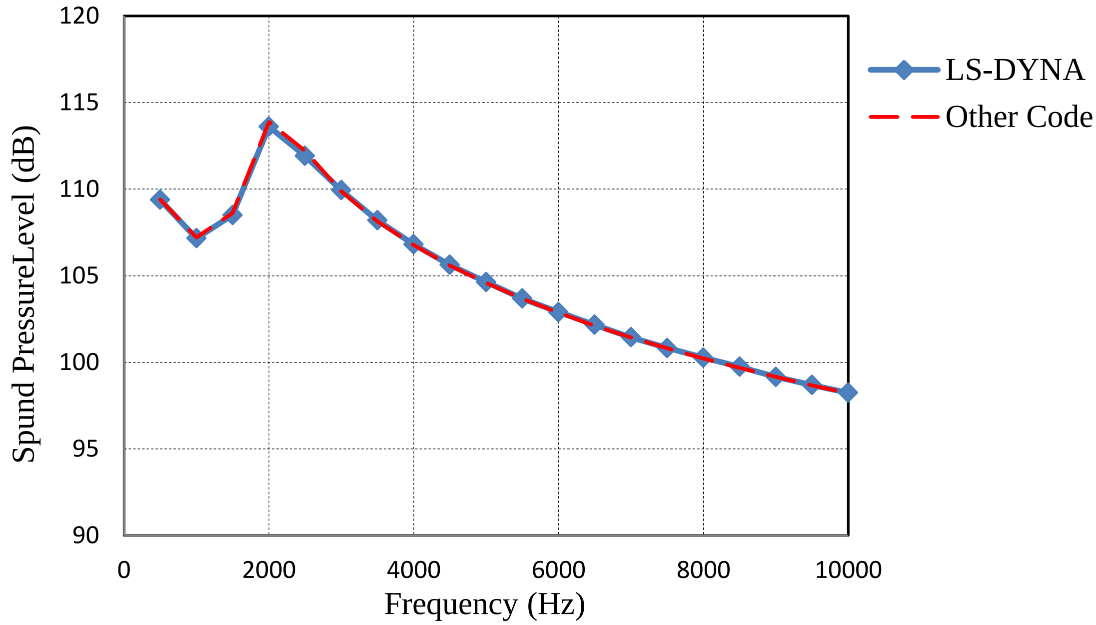


Figure10. Sound pressure level of cavity with air (next to the plate 2)

Conclusion

A new SEA solver has been implemented in LS-DYNA. It offers effective and efficient tools for high frequency vibration and acoustic analysis in complex structures. Three simple examples are included in the paper to show the procedure of running SEA analysis with LS-DYNA. The numerical results are in good agreement with the analysis using other code. For the future development, the LS-PrePost will be updated for user to speed up the generation of SEA model and to make it more convenient to review the results.

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References

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