# Eigensolution Technology in LS-DYNA®

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**Abstract**

*LSTC has been adding additional eigensolution technology to LS-DYNA.*

*For several years LS-DYNA has a Block Shift and Invert Lanczos Eigensolver in both SMP and MPP implementations. But this capability did not cover the full spectrum of applications. We have supplemented the Lanczos solver with a Power Method solver for Implicit mechanic problems using the Inertia Relief Feature.*

*As we have been adding unsymmetric modeling features through materials, elements, and contact, we have added an eigensolver based on ARPACK for such problems. Important applications for the unsymmetric eigensolver are rotational dynamics and brake squeal analysis.*

*We are also developing an implementation of AMLS (Automated Multilevel Substructuring Method) for applications such as NVH that want hundred, even thousands, of eigenmodes quickly which are willing to have a less accurate solution compared to the Lanczos eigensolver.*

## Introduction

After the finite element discretization, LS-DYNA solves the following system of ODE's.

$$M\ddot{a} + Wv + Ku = f$$

Using the characteristic equation approach for the solution of ODE's we get a quadratic eigenvalue problem (QEP)

$$-\omega^2 M\Phi + i\omega W\Phi + K\Phi = 0$$

When W = 0 (no damping) this becomes the first order eigenproblem

$$K\Phi = M\Phi\Lambda$$

When K and M are symmetric this is the standard vibration analysis problem.

The rest of this paper will highlight the variations of eigenvalue problems that can now be posed and solved in LS-DYNA.

## Various Eigenvalue Problems

For many years LS-DYNA has used the Block Shift and Invert Lanczos software, both in SMP and MPP, to solve the standard vibration analysis problem. Lanczos assumes that K and M is symmetric positive semi-definite. And that some linear combination of K and M is nonsingular. The last requirement is equivalent to requiring K and M not to have a common null space. In the engineering world this means no massless mechanisms.

LS-DYNA also uses the Block Shift and Invert Lanczos software for the buckling analysis problem where K is symmetric positive definite and Kδ (the geometric stiffness matrix) is symmetric but indefinite. Because $K_\delta$ is indefinite K must be positive definite. This requirement causes a problem with the buckling analysis of structures in flight. To perform the time simulation Implicit uses the Inertia Relief feature that uses a LaGrange formulation to constrain out the rigid body motion. Leaving out these constraints makes K indefinite due to the rigid body motion. Removing the constraints by our usual approach turns the factorization dense. Leaving the constraints in K using the LaGrange formulation turns K symmetric indefinite and Lanczos no longer is applicable. To solve such problems we added a Power Method eigensolver. This method is chosen automatically for such problems (Buckling with Inertia Relief).

LS-DYNA usually ignores the Implicit Dynamic terms for the eigenvalue analysis. But EIGMTH=6 (see *CONTROL_IMPLICIT_EIGENVALUE) includes the dynamic terms to the stiffness matrix. This is particularly useful for model validation and debugging. EIGMTH=6 includes the mass matrix in the eigensolution. EIGMTH=5 analyzes only the stiffness matrix.

LSTC extended this model validation and debugging to our mechanical thermal applications using *CONTROL_THERMAL_EIGENVALUE. This performs a eigenvalue computation using just the thermal conductance matrix.

Recently LSTC has been adding the ability to add certain unsymmetric terms to the stiffness matrix (see *CONTROL_IMPLICIT_SOLVER). These terms can come from certain materials and certain elements and from certain contact types. These terms are required in some applications to properly model the physics. The unsymmetric stiffness matrix means that Lanczos is no longer applicable. For these problems we have added ARPACK, a public domain software from Rice University based on the Arnoldi method. A particular important application for such an unsymmetric eigenvalue analysis is brake squeal. For brake squeal capturing the unsymmetric terms from contact due to the warped rotor is critical to the modeling the physics. At the time of the writing of this paper the use of ARPACK in LS-DYNA is only available in the SMP versions. Our long term plans do include using the Parallel version of ARPACK, PARPACK, for the MPP version.

LS-DYNA can also add damping terms to the unsymmetric analysis. This requires going back to the original quadratic eigenvalue problem. Since the eigensolution software requires a first order eigenvalue problem we make a variable substitution and double the order of the eigenvalue problem. For such problems we again are using ARPACK.

Finally the addition of Rotational Dynamics modifies the stiffness matrix with spin softening terms and adds first order terms of gyroscopic effects. Again for such problems we are using ARPACK.

## Automatic Multi Level Substructing

Automatic Multi Level Substructing, known by its acronym AMLS, is used in the Noise, Vibration, and Harshness community to compute hundreds or thousands of eigenmodes for automotive models. These eigenmodes are used for frequency domain computations. AMLS reduces the full scale model using a recursive application of the Craig-Bampton substructing approach to a smaller model. The eigensolution is then computed on the smaller model. The eigenvalues and eigenvectors computed by AMLS are approximations to the eigenvalues and eigenvectors for the full scale model. But AMLS requires far less computing resources than Lanczos on the full scale model.

LSTC is actively implementing AMLS. At the time of the writing of this article we have a functional implementation for small and moderate size problems. We hope to provide some full scale results with the LSTC implementation of AMLS at the time of the conference.