

Worst-Case Topology Optimization

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1 Abstract

This paper presents a worst-case design approach for the multidisciplinary topology optimization of an automotive hood design. The study considers the impact of a pedestrian's head against the hood, static loads, and the minimum weight of the hood – all required to meet general design code requirements in automobile industry. Among the design code requirements of the hood design, the biggest challenge is to handle hundreds of head impact locations specified in the Euro NCAP pedestrian testing protocol, due to the high computational expense of hundreds and thousands of structural analyses demanded in the structural optimization. To overcome this challenge, we accordingly introduce a general framework for the worst-case topology optimization which investigates the worst impact locations on the hood by evaluating the maximum head injury criterion and the maximum deflection of the hood separately, reducing the burden to consider multiple disciplines simultaneously at hundreds of impact locations all at once. At the end, these worst impact locations are combined with a static load case and formulated into a single multidisciplinary design optimization problem that needs only tens of structural analyses per iteration for numerical gradients computation, enabling the proposed design framework suitable for large scale topology optimization problems.

Keywords: Topology Optimization; Multidisciplinary Design Optimization; Worst-Case Optimization; Automotive Hood Design; Head Injury Criterion;

2 Introduction

This research is an extension of our previous research [1], conducted to solve the topology optimization of an automotive hood combining static, impact, and eigen frequency load cases using LS-TaSC, while considering design requirements of multiple disciplines and multiple impact locations on the hood. For a detailed background review and discussion on this topic see paper [1]. This paper focuses on introducing the worst-case design approach to topology optimization for an automotive hood, which is required to meet the hard motor components protection of the hood and the pedestrian safety protection requirements at numerous impact locations specified in the European New Car Assessment Program (Euro NCAP) pedestrian testing protocol [2].

Design specifications for the inner panel of the automotive hood focus on the effect that places a limit on the severity of pedestrian head injury when struck by a motor vehicle. Other requirements are being made to use the hood as an active structure and push its surface several centimeters away from the hard motor components during a pedestrian crash. Thus, the design of the hood is typically expected to meet requirements such as HIC, deflection of the hood, and maximum energy absorption of the hood. These important requirements are from the crash groups at various automotive companies, and they can be achieved by Multi-disciplinary Design Optimization (MDO). MDO has been developed to address the interaction of multiple disciplines of engineering in designing complex structures by using various optimization methods. It has found increased applications in automotive industry, where the structures of automotives are required to consider crashworthiness and stiffness at the same time in the product development process for sizing or thicknesses of components [3-4] and topology optimization of parts or car bodies [5-7]. Our research has been focusing on multidisciplinary topology optimization of a crash box [8] and a hood [1] involving with a few load cases, and the current research has extended the capability to deal with such design problems involving with numerous of impact load cases.

The distribution of head impact locations on the automotive hood defined by the testing protocol are specified as the testing zones I and II described in Fig. 1, where each cross-dot point represents an impact location [2]. To liaise with this pedestrian testing protocol, the design for the hood is meant to conduct head injury severity assessments at all impact locations resulting from a low-speed crash (<30 km/h) of the vehicle striking a pedestrian. As one impact location denotes a load case which depicts the scenario of the pedestrian head striking onto the hood at the prescribed location, computation cost is very expensive to solve an optimization problem including hundreds of load cases drawn from all the impact locations. The larger the number of impact locations is, the more finite element (FE) analyses are needed, and the more expensive the computation cost is. Therefore, it's essential to have the number of load cases as low as possible by finding out a few worst impact locations out of hundreds of locations.

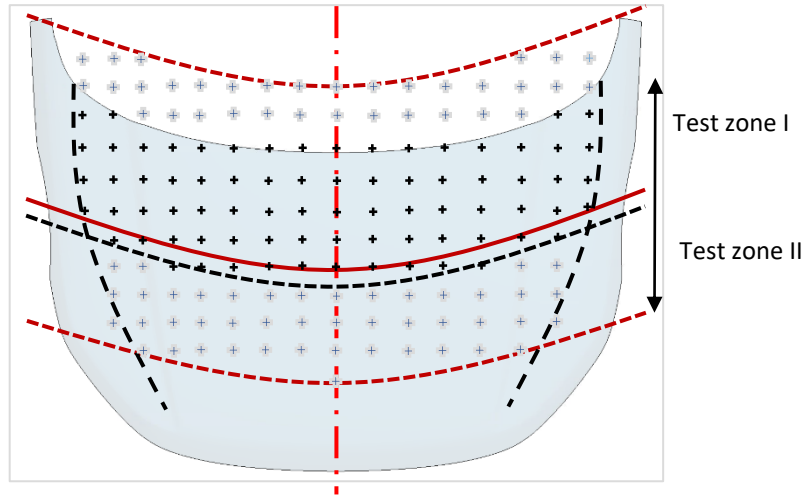


Fig. 1: Distribution of pedestrian head impact locations defined by Euro NCAP protocol

The worst-case design optimization is often used for robust optimization in literature [9-10]. It deals with the worst-case scenarios where structures are optimized by considering the most severe possible situations, such that the optimized structure can withstand the worst possible loads or manufacturing errors. This optimization strategy is desirable in situations where structural failure may result in potential disasters and accidents such as considerable economical damage or even loss of life. The worst-case topology optimization design can be a solution with minimum computation cost to improve head impact zone effectiveness in pedestrian impact protection. Because a worst-case design estimation of the hood can identify the impact locations leading to the weakest structure. The weakest hood structure is genuinely able to minimize the potential of violating the EURO NCAP pedestrian testing protocol under identified worst-case impact locations for locations within the test zones. This paper will describe the worst-case topology optimization for a hood considering both the HIC and deflection of the hood to meet the Euro NCAP testing protocol in detail.

In the following we first present solving the multidisciplinary topology optimization with constraints by using LS-TaSC and show the challenge in solving an optimization problem of the automotive hood. The principal idea of worst-case topology optimization is then specialized in Section 4, whereafter we proceed with the detailed description of the worst-case design strategy, and eventually end up with the conclusions.

3 MDO Design for An Automotive Hood

3.1 Multidisciplinary topology optimization with constraints using LS-TaSC

To solve for MDO problems with constraints, LS-TaSC developed the spacial kernel approach to address complex minimax problems, also known as the saddle point problems, which typically aims to maximizing energy absorption in automotive impact problems [11].

The MDO problem with constraints can be defined as

$$\begin{aligned} \min_x & f(x) \\ \text{s. t. } & g_{lj}(x) \leq 0 \quad (j = 1, 2, \dots) \end{aligned} \quad (1)$$

where x is the vector of topology variables, l and j indicate the load case index and constraint index, respectively. $g_{lj}(x)$ represents the j -th constraint in the l -th load case, and $f(x)$ denotes the objective, which can be expressed as follows to include multiple design disciplines,

$$f(x) = \sum_l w_l f_l(x) \quad (2)$$

in which each load case has its own objective to satisfy corresponding design disciplines. w_l represents the weight factor of the load case, and $f_l(x)$ represents the objective of the load case. In this MDO problem, the load case weights are used to solve for a subset of the constraints.

For impact problems, the constraints, of which the derivatives are computed numerically using the multi-point scheme [11-13], are expressed using the spatial kernel functions, which can be referred to literature [14]. With the assistance of the spatial kernels, the MDO problem is transferred into a Lagrangian dual problem, which can be solved using a two-level optimization strategy: an upper level

problem in terms of Lagrange multipliers, load case weights, and spatial kernel variables, and a lower level problem in terms of topology variables. The lower level problem is solved using the projected subgradient method [15], while the upper level problem can be solved using a general mathematical programming method.

During the design optimization process, preferably, the numerical derivatives need to be computed every iteration. But it is very costly for complex problems that require a few hours to complete a FE analysis. To reduce the cost, we compensate the accuracy of the numerical derivatives and compute the derivatives every three or five iterations.

3.2 MDO problem of an automotive hood

The geometry of the engine hood including the outer shell and the solid inner panel (design part) is shown in Fig. 2a, and the finite element model of a child/small adult head form positioning over the outer surface of the hood is shown in Fig. 2b. The impact analysis for evaluating the responses of the head form collision on the engine hood is conducted using LS-DYNA. According to the requirements of the Euro NCAP, there are Pedestrian Safety Protection and Hard Motor Components Protection to be considered in the design optimization. Corresponding to these two protection requirements, two major responses, which are the HIC value and the maximum downward displacement of the hood, are needed to be investigated at an impact location where the collision happens.

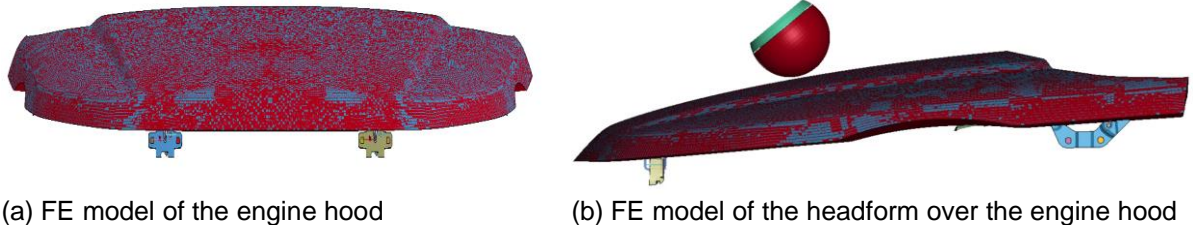


Fig. 2: FE model of the engine hood for head impact analysis

Pedestrian Safety Protection: The HIC value in a head impact analysis is calculated by the effects of head form acceleration and the duration of the acceleration. It is defined as [16]

$$HIC = \max \left\{ \left[\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a(t) dt \right]^{2.5} (t_2 - t_1) \right\} \quad (3)$$

where $a(t)$ indicates the acceleration of the head form at time t , and t_1 and t_2 represent the time interval for the evaluation. The HIC value is to evaluate the maximum average acceleration value over time duration $t_2 - t_1$ which is 15ms in this research. A threshold of 2000 for HIC values indicating life threatening is set as the upper bound of the HIC values.

Hard Motor Components Protection: It is required that the deflection of the hood is within safety region where the deformed surface of the hood doesn't touch or hurt the hard components beneath the hood while a collision between the vehicle and a pedestrian happens. Thus, a threshold of 100 mm is set as the upper bound of the deflection of the hood, which is the maximum downward displacement from the entire structure.

At each impact location, it is ideally required to consider the pedestrian safety protection and the hard motor components protection simultaneously. Since an impact location represents a load case, these design requirements provide two constraints in a single impact load case. Therefore, the optimization problem for multidisciplinary topology optimization of the hood can be expressed as

$$\begin{aligned} \min \quad & f_{mass}(\mathbf{x}) \\ \text{s.t.} \quad & g_{HIC_l1}(\mathbf{x}) \leq 2000 \\ & \vdots \\ & g_{HIC_lM}(\mathbf{x}) \leq 2000 \\ & g_{max_disp_l1}(\mathbf{x}) \leq 100 \\ & \vdots \\ & g_{max_disp_lM}(\mathbf{x}) \leq 100 \\ & x_{min} \leq x_i \leq 1.0 \quad (i = 1, \dots, N) \end{aligned} \quad (4)$$

where x_i denotes the i -th design variable, N is the total number of design variables, and \mathbf{x} denotes the vector of the design variables. x_{min} is the lower bound of the design variable. It is assumed that there

are a total of M impact locations to be considered. $g_{HIC_{l1}}$ and $g_{max_disp_{l1}}$ represent respectively the HIC value and the maximum downward displacement of the hood from the 1st impact location, and $g_{HIC_{lM}}$ and $g_{max_disp_{lM}}$ represent respectively the HIC value and the maximum downward displacement of the hood from the M -th impact location. Thus, the above MDO problem has an objective of a mass fraction and a total of $2 * M$ constraints from M impact locations.

To solve this MDO problem, since the number of global variables (i.e., mass fraction and load case weights) that need to be addressed is noted as $numGlobalVars = 1+(M-1)$, the number of sampling points needed for numerical derivative computation on the usages of the central difference distribution of the DOE equals to $2 * numGlobalVars + 1$, and the total number of FE analyses (e.g., LS-DYNA runs) at the iteration where new numerical derivatives need to be computed equals to $(2 * numGlobalVars + 1) * numLoadCases$. Regardless how often the numerical derivatives are set to be computed and how many optimization iterations are required to converge, the computational cost on the FE analyses has a complexity of $O(n^2)$ with the number of load cases, which is impossible to afford for design optimization of the automotive hood involving with hundreds of impact locations. Thus, it is of great importance to reduce the number of impact locations from hundreds to a few.

4 Worst-Case Design Optimization Strategy

For providing an analogy to the hood design problem with hundreds of impact locations, consider the optimization problem of designing a bridge loaded by a car driving over it. In this problem, every possible location of the car is a load case subject to a constraint, which can be described in Fig. 3. An alternative problem is to design the bridge for a uniform load (the superposition of all the load cases), as shown in Fig. 4a. The worst-case scenario from the initial problem, as shown in Fig. 4b, is added to satisfy the constraints via an increase of the material in the structure. With this design strategy, the resulting structure will withstand all loads and satisfy all constraints.

Determination of worst-case design scenarios is performed by a thorough searching for the absolute worst case possible.

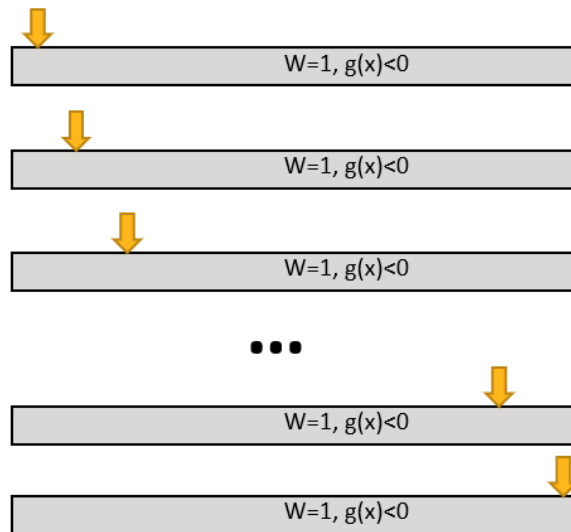


Fig. 3: Description of a bridge with many load cases each with a constraint

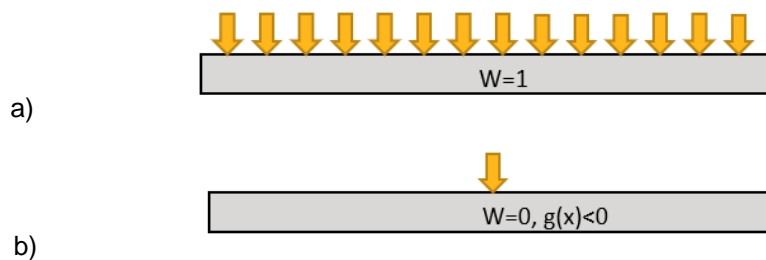


Fig. 4: Worst-case design of a bridge consisting of a) an uniform load and b) the worst load case with a constraint

5 Worst-Case Topology Optimization

Following the description of the worst-case design optimization strategy, we implement this strategy for the MDO of the hood as below.

5.1 Specifying impact locations

According to the Euro NCAP pedestrian testing protocol, there are hundreds of impact locations to be tested on the whole surface of the hood. Since the distribution of the impact locations on the hood is symmetric along the red dashed line centered on the hood structure in Fig. 1, only half the number of these impact locations need to be investigated. Due to the limitation of computation resources, 16 impact locations among the half number locations are selected to represent the pedestrian testing protocol by using Design of Experiments (DOE), of which the sampling points are generated using LS-Opt. The DOE of the impact locations is described in Fig. 5. By excluding the three locations at the lower right corner of the DOE that are out of the hood surface, there are 13 representative impact locations to be considered in the MDO problem described in Eq. 3. Each sampling point means a single load case and it requires a full topology optimization design through LS-TaSC. Overall, the outer level DOE goes through all the sampling points one by one followed by the inner level topology optimization for a single impact location. Initial study conducted to check potential load path from multiple impact locations.

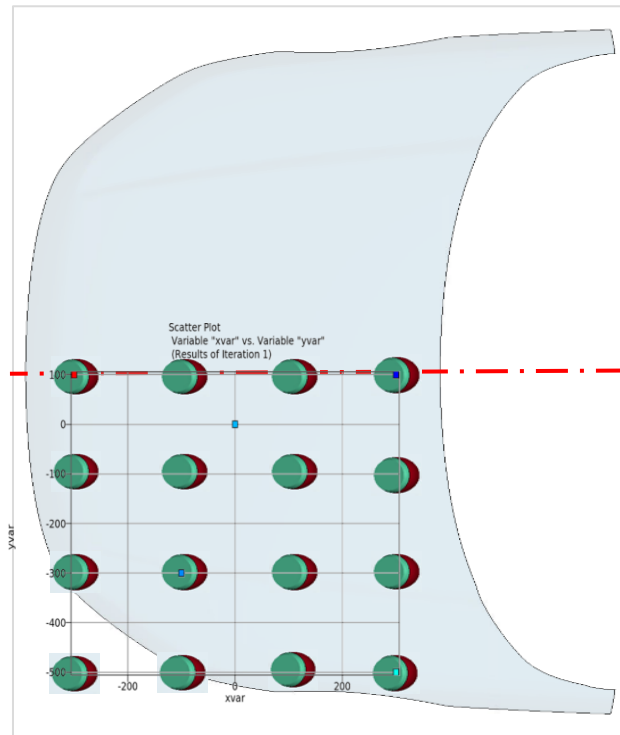


Fig. 5: DOE of the impact locations on the hood

5.2 Specifying worst-case impact location for maximum deflection of the hood

Firstly, with the preselected 13 impact locations, we aim at locating the worst-case impact location that yields the largest deflection of the hood caused by the vehicle-pedestrian collision. To achieve this goal, the constraint applied on the maximum downward displacement at a given impact location is set without bounds, while the HIC value of the hood is required not to exceed 2000, which is the only design requirement to be satisfied in the optimization process. Thus, the MDO problem for the hood on the purpose of investigating the maximum deflection at a single impact location can be expressed as

$$\begin{aligned}
 \min \quad & f_{mass}(\mathbf{x}) \\
 \text{s.t.} \quad & g_{HIC_l1}(\mathbf{x}) \leq 2000 \\
 & -\infty < g_{max_disp_l1}(\mathbf{x}) < +\infty \\
 & x_{min} \leq x_i \leq 1.0 \quad (i = 1, \dots, N)
 \end{aligned} \tag{5}$$

Since only one load case is considered here, it only requires three LS-DYNA runs to provide the sensitivities at each optimization iteration. LS-TaSC solves the above optimization problem, and its

solution provides an optimal design of the hood structure, of which the maximum downward displacement value is collected for later comparison.

This optimization computation conducted by LS-TaSC is repeated for 13 times as the 13 impact locations needs to be considered one by one. Thereafter, 13 optimal designs of the hood structure are provided, and 13 maximum downward displacement values from these 13 designs are collected. At the end, we plot the collected data of the maximum downward displacement values with a colormap, which is shown in Fig. 6 with an overlay of the 13 optimal designs of the hood structure. It shows that the impact location marked in the red square gives the largest deflection of the hood, about 270.2 mm, among 13 impact locations. Thus, this marked impact location is specified as the worst-case for maximum deflection of the hood.

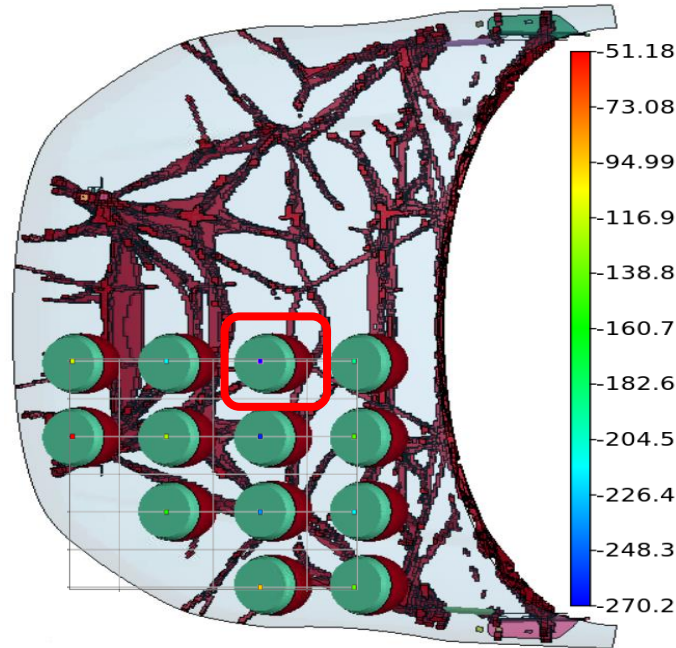


Fig. 6: Overlay of topologies from DOE of worst-case design of displacement

5.3 Specifying worst-case impact location for maximum HIC of the hood

Secondly, we aim at locating the worst-case impact location that yields the largest HIC of the hood caused by the vehicle-pedestrian collision. The same computation strategy as stated above is employed here. For a given impact location, it is required that the maximum downward displacement around the impact area is not to exceed 100 mm while the HIC value obtained from the impact analysis is set without bounds. The MDO problem for the hood on the purpose of investigating the HIC at a single impact location can be expressed as

$$\begin{aligned}
 \min \quad & f_{mass}(\mathbf{x}) \\
 \text{s.t.} \quad & g_{max_disp_l1}(\mathbf{x}) \leq 100 \\
 & -\infty < g_{HIC_l1}(\mathbf{x}) < +\infty \\
 & x_{min} \leq x_i \leq 1.0 \quad (i = 1, \dots, N)
 \end{aligned} \tag{6}$$

By going through the 13 impact locations one by one, LS-TaSC repeats the optimization process of solving the above MDO problem 13 times and provides 13 optimal designs whose HIC values are collected to draw the colormap plot, as shown in Fig. 7. It says that the impact location at the lower right-hand corner, which is marked in the red square, produces the largest HIC value, about 4127, among the 13 impact locations. Thus, this impact location is specified as the worst-case impact location for the maximum HIC of the hood.

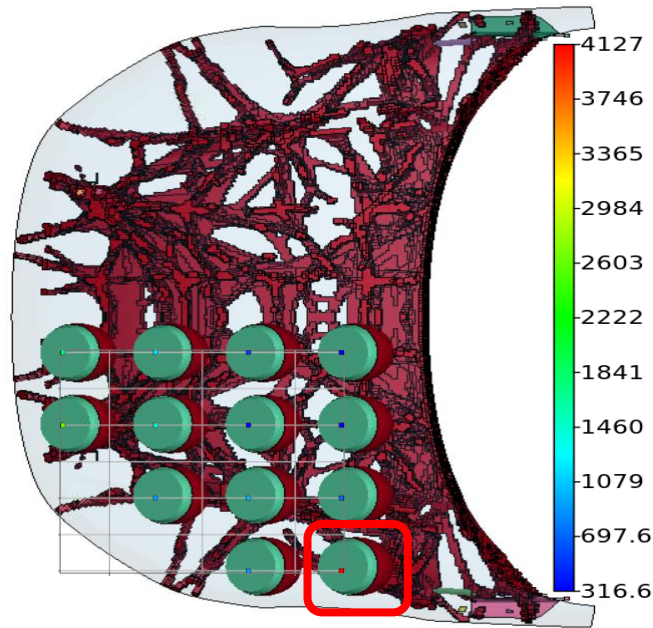


Fig. 7: Overlay of topologies from DOE of worst-case design of HIC

5.4 Optimizing for the worst-case impact locations

Thirdly, with the above specified two worst-case impact locations, we can form a new MDO problem with these two load cases combined with a static load case that applies uniform distributed point loads on the hood. The three load cases are described in Fig. 8.

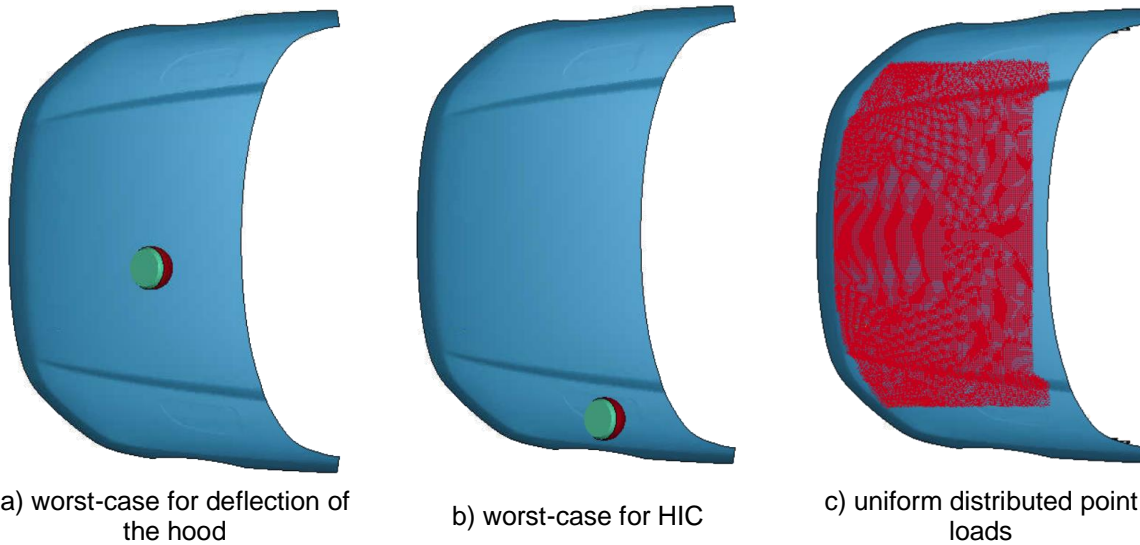


Fig. 8: Three load cases for the worst-case design

In this MDO problem, both the maximum downward displacements of the hood and the HIC value at the two worst-case impact locations need to be considered. The new MDO problem with the worst-case deflection and worst-case HIC of the hood can be expressed as follows,

$$\begin{aligned}
 & \min f_{mass}(\mathbf{x}) \\
 & \text{s.t. } g_{max_disp_l1}(\mathbf{x}) \leq 100 \\
 & \quad g_{HIC_l2}(\mathbf{x}) \leq 2000 \\
 & \quad x_{min} \leq x_i \leq 1.0 \quad (i = 1, \dots, N)
 \end{aligned} \tag{7}$$

To address the MDO problem with multiple load cases, the weighting is active to find the tradeoff between the maximum downward displacement design and the HIC design. For three load cases, seven LS-DYNA runs are needed for each load case and a total of 21 LS-DYNA runs are needed per each

optimization iteration to provide the numerical sensitivities for global variables. The optimization converges after 40 iterations, with the optimal design being shown in Fig. 9 and the histories of maximum downward displacement and HIC being described in Fig. 10. As we can see, both the maximum downward displacement and the HIC requirements are satisfied very well with the optimum structure.

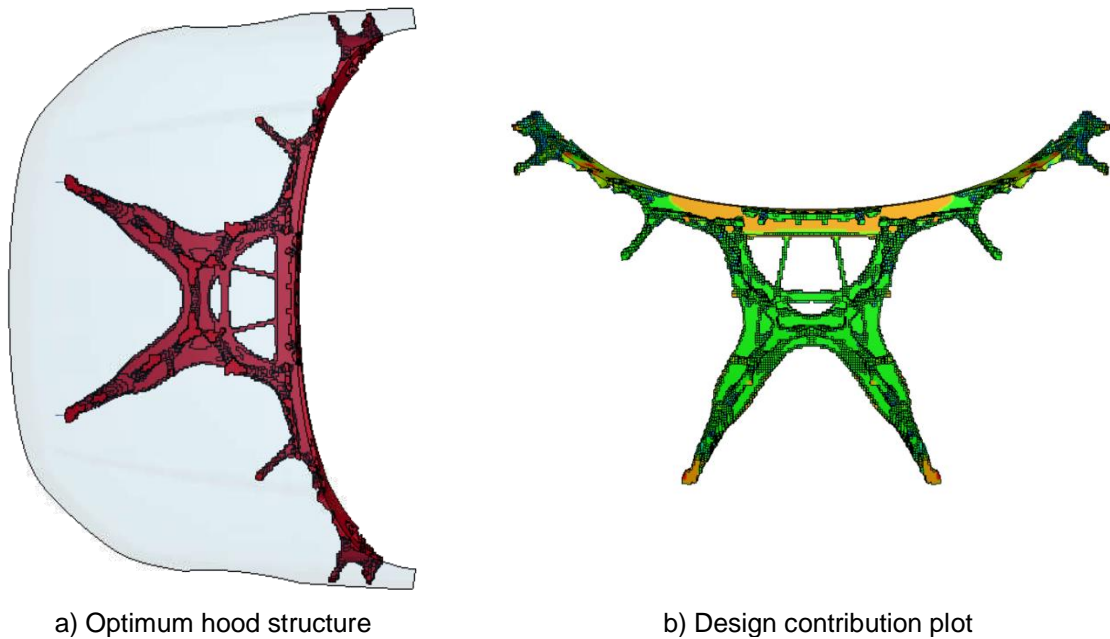


Fig. 9: Optimal hood structure obtained from the worst-case design

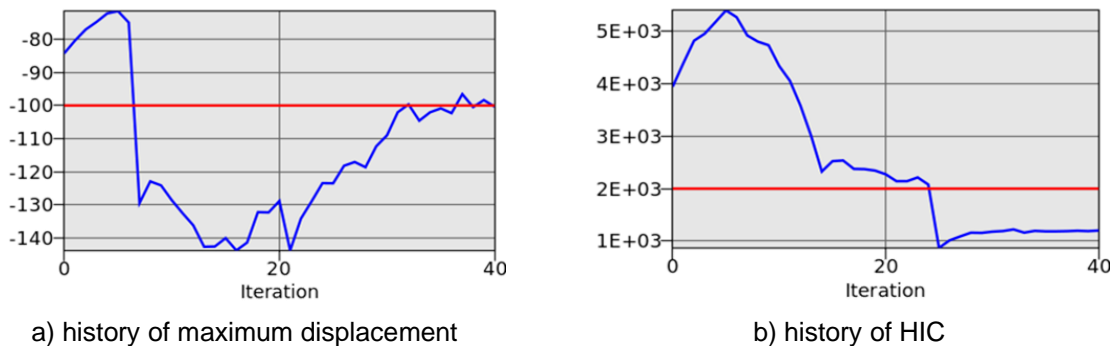


Fig. 10: History plots for the worst-case design

During the worst-case optimization, it is noted that the standard topology optimization design is to stiffen a structure to its maximum, while the topology optimization design that satisfies the HIC constraint requires to soften the load path created for the deflection constraint. This means that simply reducing the load case weight where the HIC constraint is applied is not enough. A negative value must be used as the weight of the load case related to the HIC constraint. This motivates LS-TaSC to have an option of “Soften Structure” in the Objective definition list, which is targeted to handle this situation for our customers who need to solve MDO problems with HIC constraint being defined in one of multiple load cases.

5.5 Verification of the optimal design

At the end, a verification study is conducted to check satisfaction level of the optimal design in terms of the Euro NCAP code. LS-DYNA analyses for the optimal hood structure at the preselected 13 impact locations are conducted individually. The resulting maximum downward displacement of the hood and HIC values of the collision at each impact location are collected and plotted in a colormap as shown in Fig. 11.

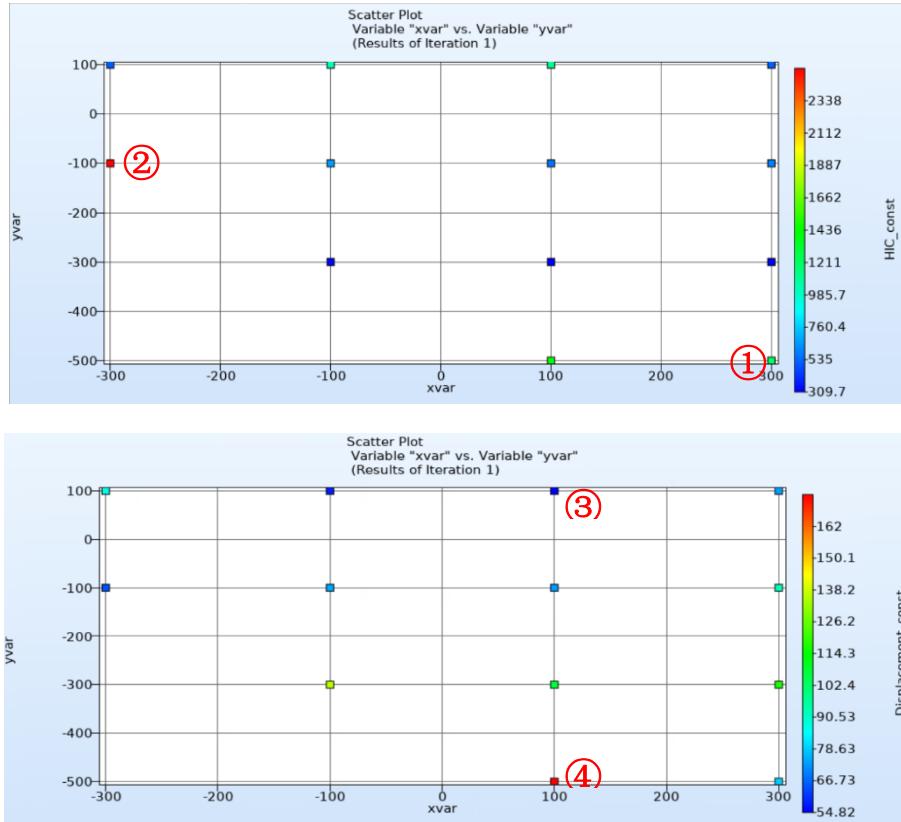


Fig. 11: HIC (upper) and maximum downward displacement(lower) values of the optimum hood at 13 impact locations

The colormaps show that there is one location, #2, being failed for the HIC requirement and a few locations (marked in green and red colors) being failed for the deflection requirement, and these failed locations do not include either of the worst-case locations, #1 and #3, which we specified in Sections 5.2 and 5.3. This means that the worst-case shifts from one location to another during the optimization iterations. The reason for the worst-case shifting is because a stiffer structure yields smaller deformation but causes a higher HIC value due to a big bouncing back of the head form received in the collision. By removing materials from the structure, the structure gets softened and produces smaller HIC value but larger deformations at places where materials are removed. For the MDO problem in Eq. 7, to decrease the HIC value at impact location #1, materials are removed from the nearby area to soften the structure during the optimization, and this cause large deformations at impact locations surrounding #4. To satisfy the maximum downward displacement constraint at location #3, materials are added to strengthen that area through the loading path. However, this causes the HIC value at location #2 to be a little bit over the limit. A possible solution to avoid the worst-case location shifting is to use an iterative design methodology identifying multiple worst-case locations.

6 Summary

We introduced a framework of the worst-case topology optimization that requires solving constrained, multi-disciplinary optimization problems to satisfy industry manufacturing codes such as the Euro NACP pedestrian testing protocol. It is a new maturity level for topology optimization to design for these industry codes. A computation strategy is proposed to select representative impact locations using DOE and to determine the worst-case impact locations through estimating the worst performances. This computation strategy reduces the burden to conduct hundreds and thousands of LS-DYNA runs required for numerical derivative computation involving with hundreds of load cases. It enables the design process to select and focus on the "important" load cases for an efficient performance-oriented design. This design framework can be used in complex design of other structural components considering multiple disciplines and multiple load cases beyond its application in the hood design.

The future work will be an iterative strategy addressing the worst-case shifting issue. This issue should be solved by including the worst-case impact locations iteratively during the optimization process, which implies implementing an active load case strategy.

7 Literature

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