Structural Optimization with the Incremental Equivalent Static Load Method for Nonlinear Dynamic Responses

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1 Abstract

This paper presents an efficient approach for optimizing structures under dynamic impact loading conditions. We introduce an improved method called the Incremental Equivalent Static Load Method that enhances the accuracy of the original ESL method. In the original ESL method, equivalent static loads are computed based on the initial geometry and nonlinear displacement results from a nonlinear analysis software. With the Incremental ESL method, we update the stiffness matrix at selected time steps using deformations from a base time step. This enables us to compute and apply equivalent static loads based on incremental displacements for ESL loadcases, resulting in a more precise capture of geometric and material nonlinearity.

The Incremental ESL method demonstrates superior performance by producing better approximations of structural responses and showcasing faster convergence for highly nonlinear analysis problems. To facilitate its practical implementation, we have incorporated the Incremental Equivalent Static Load Method into the OmniQuest GENESIS software, which seamlessly integrates with the existing OmniQuest ESLDYNA software and the ANSYS LS-DYNA software.

To illustrate the effectiveness of the approach, we present several examples of optimization results. These examples use the LS-DYNA software and include the sizing and topometry design of a sill and floor structure under pole crash conditions, topology optimization of a rocker beam cross-section subjected to pole crash scenarios, and topology optimization of a vehicle body experiencing multiple dynamic loading conditions. Through these examples, we demonstrate the ability of the Incremental Equivalent Static method in improving structural performance and optimizing designs under complex dynamic loading conditions.

The method discussed here shares similarities with another improved ESL method named difference-based ESL (DiESL) method, which also uses displacement steps to perform optimization. However, unlike the DiESL method that modifies FE meshes, our implementation is done inside the structural optimization program, GENESIS, and it does not require using multiple models, making it more practical and easier to use.

Overall, this paper demonstrates the significance and benefits of the Incremental Equivalent Static Load Method for structural optimization in nonlinear dynamic responses.

***KEYWORDS**

Equivalent Static Load Method, Incremental ESL Method, topology optimization, sizing optimization and topometry optimization

2 Review of the Equivalent Static Load Method

Structural optimization techniques, coupled with linear finite element analysis, have being used to solve large-scale design optimization problems efficiently with reduced computational cost for decades now. However, when nonlinear finite element analysis is used to analyze the structure, then performing optimization becomes cumbersome and computationally expensive due to the lack of efficient integration between the analysis program and optimization techniques. Optimization methods using the Equivalent Static Loads (ESLs) have been proposed to efficiently perform optimization based on a nonlinear finite element analysis using a linear structural optimization software [1]. A wide variety of nonlinear analysis problems have been optimized using the ESL method [2][3][4][5][6].

ESLs are defined as a set of static load vectors, applied on a model to perform linear static analysis that produces the same displacement field as obtained in the nonlinear analysis. A preliminary nonlinear analysis is performed to evaluate the nonlinear displacement field. This displacement field is used to compute the ESLs. A brief overview of the associated theory is presented here. The governing equation of motion for a transient nonlinear analysis is shown below:

$$M\ddot{z}_N(t) + C\dot{z}_N(t) + K_N\big(z_N(t)\big)z_N(t) = f(t)$$

where $z_N(t)$ is the displacement field over time. The time domain can be divided into a finite number of instances. At each instance, ESLs can be computed by taking a product of the linear stiffness matrix K_L with the nonlinear displacement at that instance, say t_i as shown in the equation below:

$$f_{ESL}(t_i) = K_L z_N(t_i)$$

Each of these ESLs, $f_{ESL}(t_i)$ vectors, are applied on the structure as a loadcase. The structure is optimized considering all these loading conditions. As the optimization modifies the design, the ESLs are no longer equivalent to the nonlinear analysis result. The results from the linear optimization are used to update the nonlinear analysis model and a new nonlinear analysis is done to compute the updated displacement field. This process is repeated until a predefined convergence criteria is satisfied.

3 New Improved Incremental ESL Method

With the original ESL method, for a given time step (loadcase), the displacement solved from the linear static system matches the displacement from nonlinear analysis. However, for the linear system, the stiffness matrix is constant for all the ESL loadcases, and the displacement for each loadcase is always computed with respect to the original shape of the structure. Although the displacement from the linear system matches with the nonlinear analysis at a given time step, the stiffness changes due to the large deformation (geometric and material nonlinearity) are not accounted for. That important information omitted with the original ESL method can lead to insufficient approximation quality for highly nonlinear problems. To overcome this drawback, Triller [8] proposes a difference-based ESL (DiESL) method to improve the approximation quality. With the DiESL method, the finite element mesh at a given time step is modified using the nonlinear displacements from a previous time step, and the ESL loads are computed using the relative displacement between the given time step and the previous time step. This method shows significant improvement of nonlinear approximation quality [8][9]. However, this method requires n_T FE models, where n_T is the number of selected time steps. A multi-model optimization is required to solve the formulated optimization problem in [8][9], which seems inefficient and tedious to set up.

Instead of modifying the FE mesh and having multiple models, we propose to update the stiffness matrix directly using nonlinear displacements. Our implementation is done inside the structural optimization program GENESIS, and it does not require using multiple models, making it more practical and easier to use. An illustration of the incremental ESL method is shown in Figure 1, and the detail of this method is described as follows.

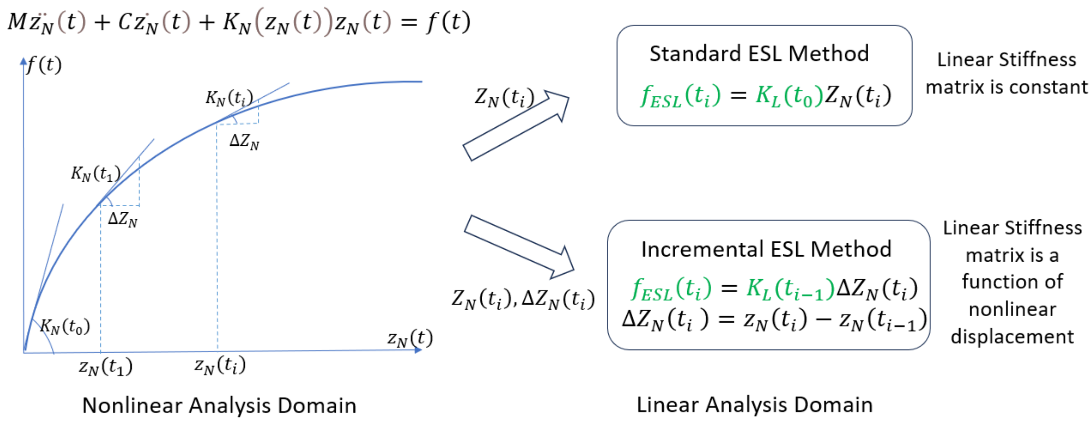

Fig.1: Stiffness matrix updated with Incremental ESL method

For a given ESL loadcase at time step t_i, if a base time step t_{i-1} is specified, then the nonlinear displacement associated with the base time step t_{i-1} will be used to update the stiffness matrix for the ESL loadcase. With the updated stiffness matrix K_L^i, the equivalent static loads for incremental displacement at time step t_i are computed as:

$$f_{ESL}(t_i) = K_L^i * \Delta z_N^i$$

where,

$$\Delta z_N^i = z_N(t_i) - z_N(t_{i-1})$$

The equivalent static loads based on incremental displacements are applied for the given ESL loadcase at time step t_i , which produces displacement same as the relative displacement Δz_N^i between the given time step t_i and the base time step t_{i-1}. If total displacement is intended to be used in optimization, then the total displacement at time step t_i is computed as sum of the displacement from all incremental time steps up to t_i:

$$z_N(t_i) = \sum_{j=1}^{i} \Delta z_N^j$$

Similarly, if the strain energy is intended to be used in optimization, then the total strain energy at time step t_i can be computed as sum of the strain energy from all incremental time steps up to t_i:

$$SE_N(t_i) = \sum_{j=1}^{i} \frac{1}{2} (f_{ESL}(t_j))^T * \Delta z_N^j$$

The overall optimization process with incremental ESL loads is illustrated as in Figure 2. This optimization process has been integrated into ESLDYNA[6] and GENESIS[7]. As a first step, a preliminary LS-DYNA analysis is performed. Then the ESLDYNA software will read the displacements for given time steps and base time steps from LS-DYNA nodout or d3plot files. GENESIS will use the displacements for base time step to update the linear stiffness matrix, and compute the incremental ESLs using the relative displacement between given time step and base time step, and apply them on the structure. Based on the defined design data for GENESIS, an optimization run is performed. Once the optimization is done, the LS-DYNA model is updated with the results from the GENESIS optimization. After updating the model, LS-DYNA analysis is carried out to analyze a new set of displacements. The new set of displacements (LS-DYNA nodout/d3plot files) is used in the subsequent GENESIS optimization run. The loop is repeated until the convergence criteria is met.

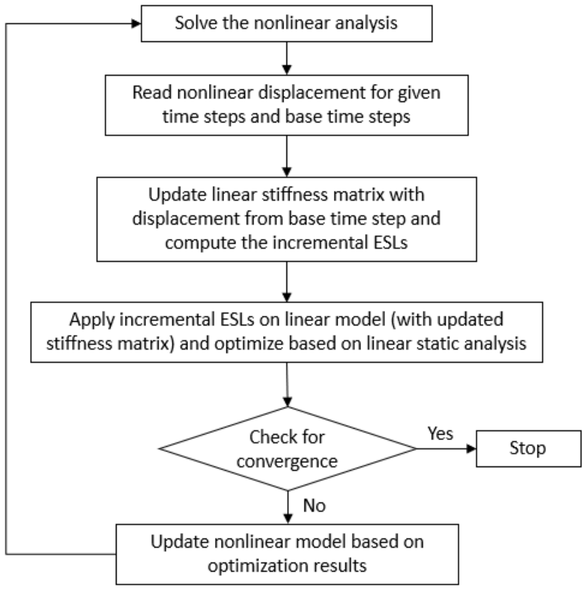

Fig.2: Flowchart for optimization based on incremental ESL Method

4 Examples

4.1 Sizing optimization of sill and floor structure under dynamic impact

In this example, the sill and floor structure are fixed at the inner edge and the structure is impacted by a rigid pole with an initial velocity of 8000 mm/s along the y direction. The end time for the analysis is 0.1 second. The material for the structure is steel, and it is modelled as piecewise linear plastic material in LS-DYNA. The maximum intrusion for the initial design is about 453 mm.

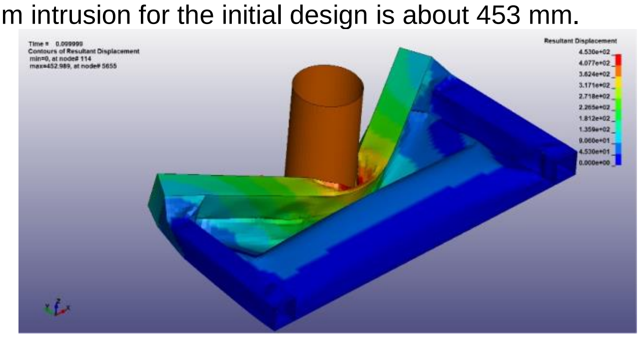

Fig.3: Sill and floor structure under pole impact

For the dynamic loading, 10 equally spaced time steps are selected for defining the ESL loadcases as shown in the table below;

ESL loadcase	1	2	3	4	5	6	7	8	9	10
Base time	0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
Load time	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1

For the sizing optimization, there is a total of 7 designed shell properties as marked in Figure 4. The initial thickness for each designed shell property is 0.8 mm. The allowable value for the shell thickness is from 0.5 mm to 3 mm. We would like to minimize the mass of the structure and reduce the maximum intrusion to under 220 mm. Node 5659 is selected to measure the intrusion, which is in the middle of PSHELL 1007.

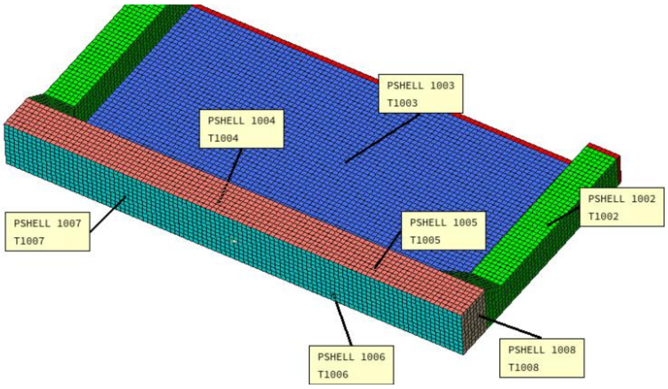

Fig.4: Designed regions (PSHELLS) for sill and floor structure

The optimization results using the ESL method and the incremental ESL method are listed in the table below. The table shows that the incremental ESL method gives an optimal mass that is 20% lower than the original ESL method, and that it also converges using 7 fewer LS-DYNA runs.

Method	Design variable							Mass (kg)	No. ESL cycles (LS-DYNA analyses)
	T1002	T1003	T1004	T1005	T1006	T1007	T1008		
ESL	0.53696	0.74124	0.50001	1.0857	2.9172	2.9172	0.55034	19.2	20
Incremental ESL	0.50000	0.50000	0.50000	2.84004	2.81353	0.50000	0.50000	15.4	13

The thickness plots for the original ESL method and the incremental ESL method are shown in Figure 5. It can be seen that the thickness distribution is quite different. With the original ESL method, the PSHELL 1007 (front) and 1006 (bottom) are reinforced. While with the incremental ESL method PSHELL 1005 (top) and 1006 (bottom) are reinforced. Especially for PSHELL 1007 (in contact with the pole), the difference is quite evident. With the original ESL method, the optimization process chooses to make this panel very thick to increase the bending stiffness because the stiffness matrix is based on the original geometry and this panel is seen in pure bending. The original ESL method is not able to capture the stiffness change due to the large deformation (tension) in this panel. While with the incremental ESL method, the stiffness matrix is corrected using the deformation from the base time step, therefore the optimization is able to get more accurate sensitivities and distribute the material to the panel which in turns helps reducing the intrusion in the most efficient way. The optimal answer given by the incremental ESL method is quite similar to the optimization results obtained by response surface method using 127 analysis points given in [8]. It also reaches a result that closely matches the results using DiESL reported in the same reference.

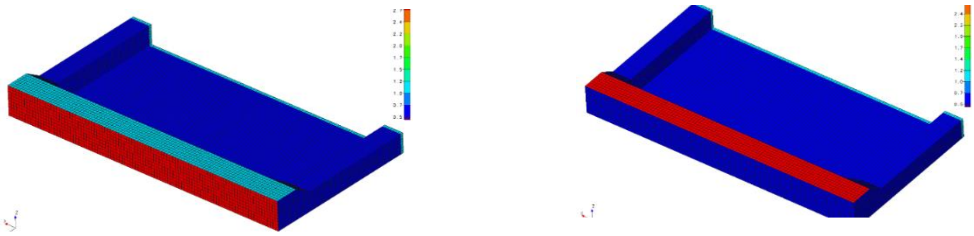

Fig.5: Thickness result with the ESL method (left) and with the incremental ESL method (right)

The objective and constraint history plots are shown in Figure 6. Both the ESL and incremental ESL method were able to satisfy the displacement constraints. The incremental ESL method converged in 7 fewer ESL cycles and 19 (44-25) fewer GENESIS design cycles than the original ESL method.

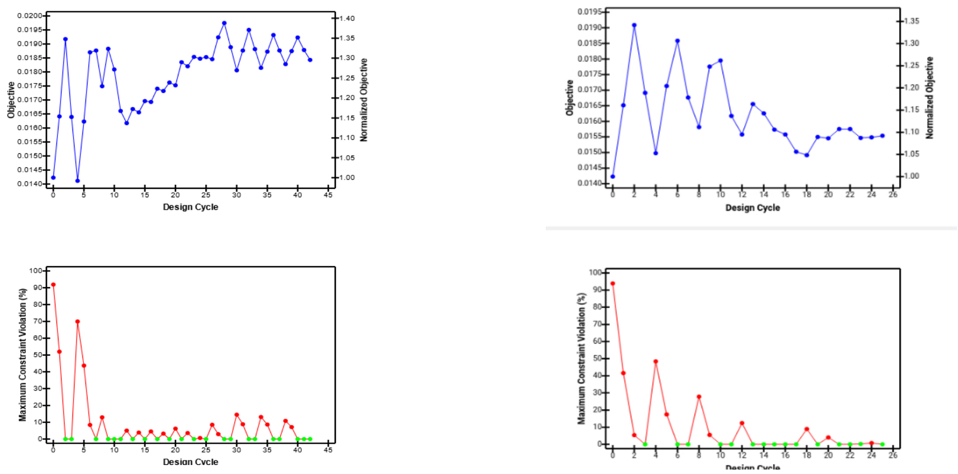

Fig.6: Design history plots with the original (left) and with the incremental (right) ESL method

4.2 Topometry optimization of sill and floor structure under dynamic impact

In this example, the same LS-DYNA example used in section 4.1 is solved here with topometry optimization. The designable region is PSHELL 1003 (the floor panel). The optimization problem is formulated as minimizing the maximum intrusion (maximize the displacement along negative y-direction) at node 5659 without increasing the mass of the structure. The optimization results with the ESL method and the incremental ESL method are listed in the table below. The incremental ESL method finds a design with a much lower intrusion compared to the ESL method.

Method	Maximum intrusion at node 5659 (mm)	Mass (kg)	No. ESL cycles (LS-DYNA analyses)
ESL	382.9	14.2	4
Incremental ESL	221.8	14.2	15

The thickness plots for the ESL method and the incremental ESL method are shown in Figure 7. The incremental ESL method chose to reinforce the connecting edge of the floor panel and the sill, where large plastic strain presents, while the regular ESL method is not able to capture this detail.

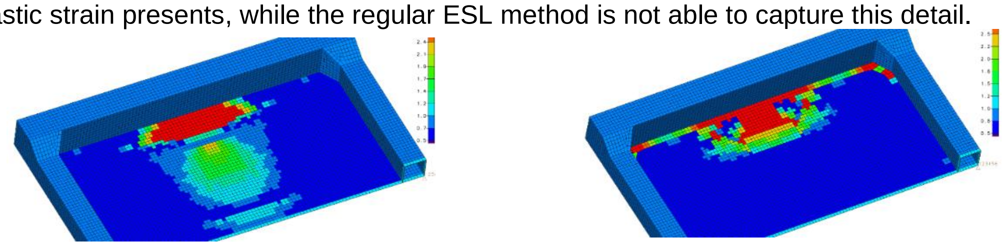

Fig.7: Thickness distribution result with the original (left) and with the incremental (right) ESL method

The objective and constraint histories are shown in Figure 8. Although the Incremental ESL method uses more design cycles in this problem, it is able to find a design that results in a 42% lower intrusion.

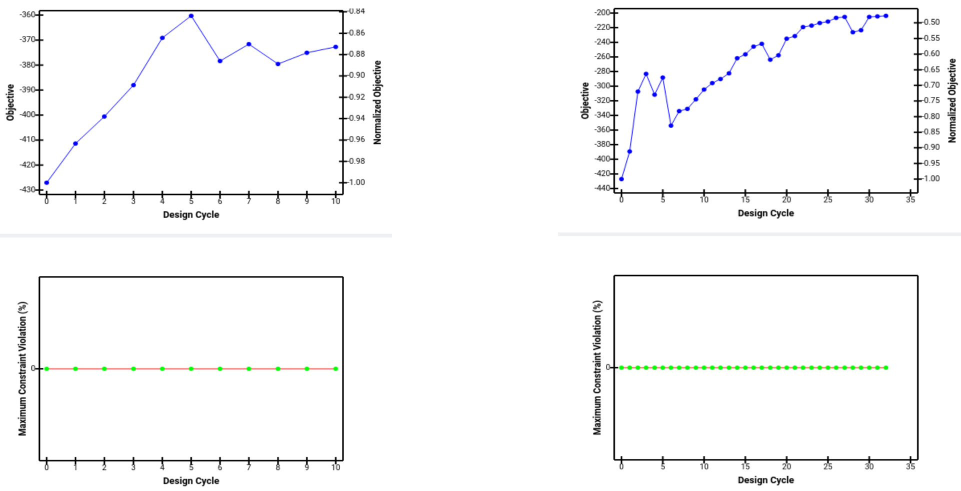

Fig.8: Design history plots with the original (left) and with the incremental (right) ESL method

4.3 Topology optimization of a rocker profile under dynamic impact

In this example, a rocker structure is pushed by a rigid wall with a mass of 85 kg against a rigid pole at an initial velocity of 8056 mm/s. The end time is 12 milliseconds. The material for the rocker beam is aluminum and is modelled as piecewise linear plastic material in LS-DYNA. To absorb the kinetic energy, the structure needs to have enough stiffness. However, the structure cannot be too stiff otherwise the reaction force will be too high. The amount of mass used for the rocker beam can be varied to achieve either a stiffer or softer structure.

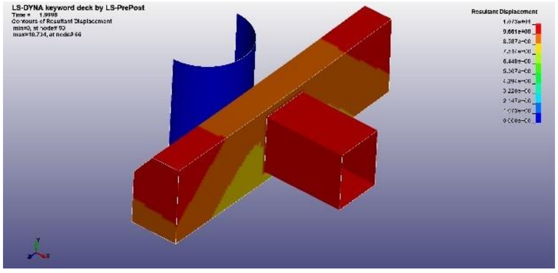

Fig.9: Rocker structure under pole impact

For the dynamic loading, 6 equally spaced time steps are selected (up to time 6 ms) for defining the ESL loadcases as shown in the table below, which is sufficient to cover the time steps with maximum intrusion.

ESL loadcase	1	2	3	4	5	6
Base time	0.0	1.0	2.0	3.0	4.0	5.0
Load time	1.0	2.0	3.0	4.0	5.0	6.0

The objective is to minimize the sum of strain energy of the incremental ESL loadcases. In this example, we tested with mass fraction 30% and 20%. The topology results with the ESL method and with the incremental ESL method are shown in Figure 10 and 11. In the case of using 30% mass, the regular ESL method only produces one I-beam member, while incremental ESL method produces two I-beam members with some angle which is preferred in nonlinear loading conditions to prevent buckling. In the case of using 20% mass, it is clearly shown that the incremental ESL method produces a more well-defined structure compared with the original ESL method. After topology optimization, further sizing optimization could be performed to refine the thickness of the suggested ribs/members.

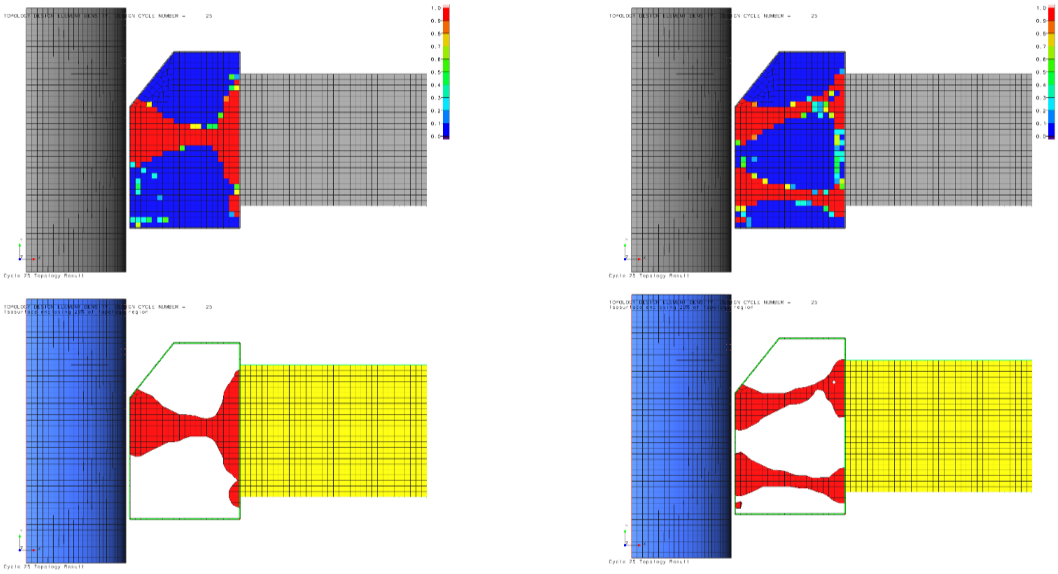

Fig.10: Topology result with the original ESL method (left) and with the incremental ESL method (right) (mass fraction = 30%)

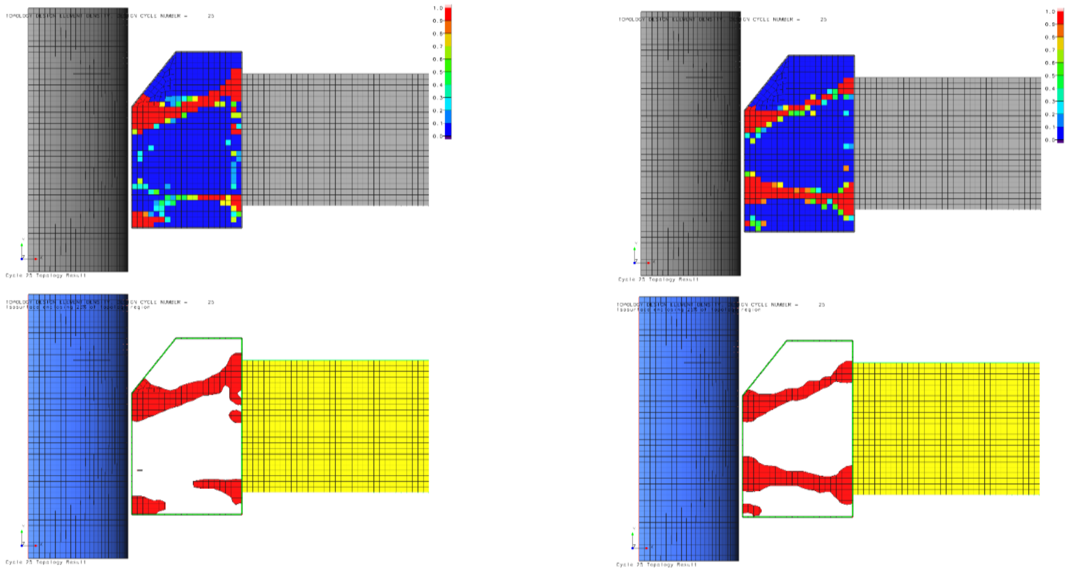

Fig.11: Topology result with the original ESL method (left) and with the incremental ESL method (right) (mass fraction = 20%)

4.4 Topology optimization of a car body with multiple dynamic loading condition

In this example, there are two dynamic loading conditions, frontal impact and side pole impact (Figure 12). The end time for the impact is 0.01 second. The material for the structure is steel and is modelled as piecewise linear plastic material in LS-DYNA. The optimization aims to find a stiff structure while limiting the mass to 20% of the full mass of the design region.

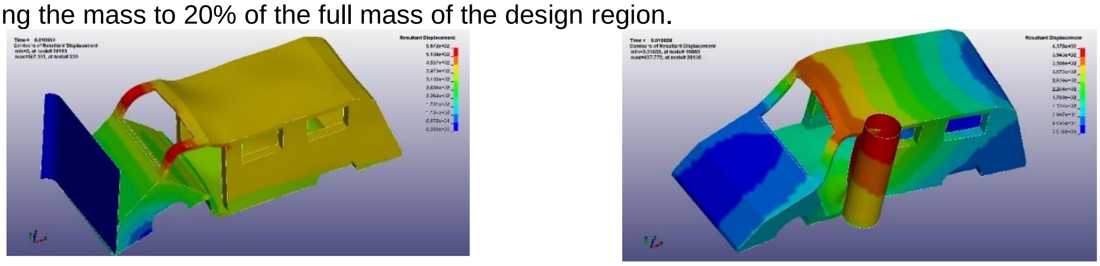

Fig.12: Car body under frontal impact (left) and side pole impact (right)

For the dynamic loading, 5 equally spaced time steps are selected for defining the ESL loadcases for each impact loading conditions as in table below;

ESL loadcases for frontal impact	1	2	3	4	5
Base time	0.0	0.002	0.004	0.006	0.008
Load time	0.002	0.004	0.006	0.008	0.01

ESL loadcases for side impact	6	7	8	9	10
Base time	0.0	0.002	0.004	0.006	0.008
Load time	0.002	0.004	0.006	0.008	0.01

For the optimization problem, there are two objectives defined: the sum of the strain energy of the incremental ESL loadcases (1-5) for the frontal crash and the sum of the strain energy of the incremental ESL loadcases (6-10) for the side pole crash. The two objectives are weighed equally. One constraint is used to restrict the mass to 20% of the full mass of the design region.

The topology results with the original ESL method and with the incremental ESL method are displayed in the images below (Figure 13 and Figure 14). These figures show that the incremental ESL method gives a more well-connected structure compared with the original ESL method. The original ESL method leads to material concentration close to the impact locations. However, the incremental ESL method reveals a clearer load path for: a) the front of the hood, b) the floor from the front to the rear part of the vehicle, and c) the lateral connections between the left and right A pillars in both the roof and the floor of the car. The structural members for the rear portion of the vehicle are also clearer, which is similar to the load path formed by static torsional loadcases. The reason is that now, for each ESL loadcase, a more representative sensitivity is obtained with the updated stiffness matrix using deformation from the base time step; therefore, the incremental ESLs is able to capture more details at different stages during a dynamic simulation. The elements with intermediate density values (green color) can be interpreted as thinner sheets or members.

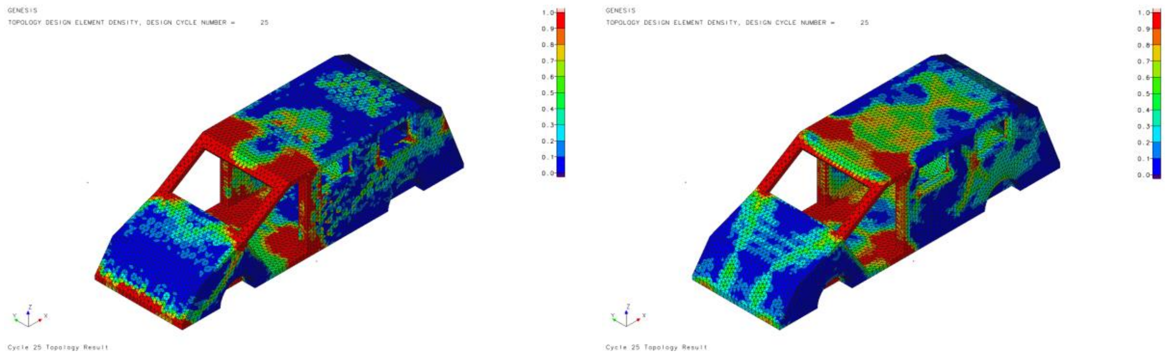

Fig.13: Topology element density result with the original ESL method (left) and with the incremental ESL method (right)

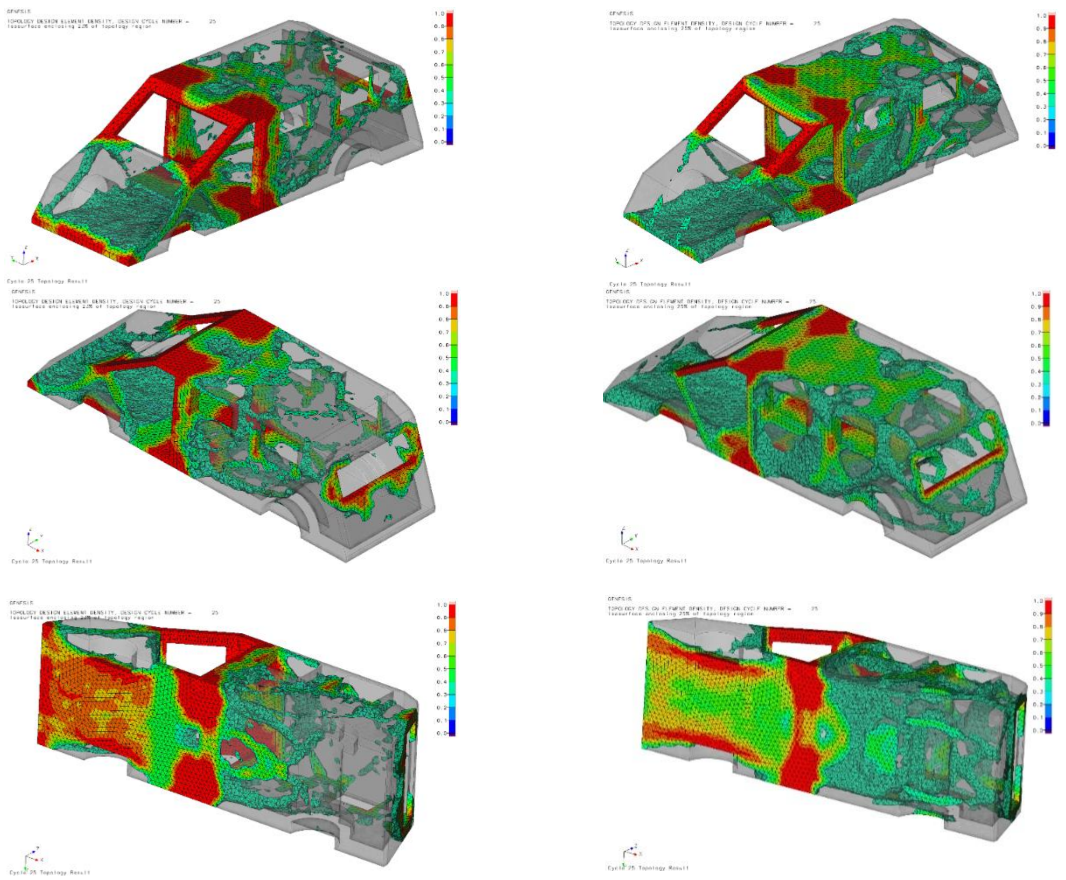

Fig.14: Topology isosurface results with the original ESL method (left) and with the incremental ESL method (right)

4.5 Topology optimization of a car body with combined dynamic and static loading conditions

In this example, topology optimization is performed using 2 dynamic and 2 static loadcases. The dynamic loadcases are the same frontal and lateral LS-DYNA dynamic loadcases used in section 4.4. The static loadcases are one bending and one torsional loadcase (Figure 15). There are four objectives: a) minimize the sum of strain energy of the incremental ESL loadcases (1-5) for frontal crash, b) sum of strain energy of the incremental ESL loadcases (6-10) for side pole crash, c) the strain energy of the bending loadcase, and d) the strain energy of torsional loadcase. The four objectives are weighed equally.

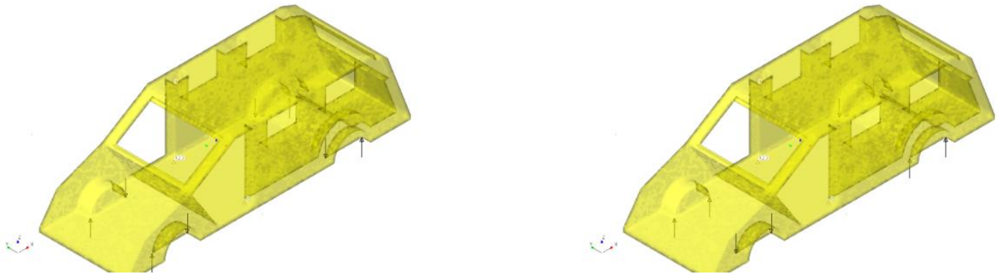

Fig.15: Static bending loadcase (left) and static torsional loadcase (right)

The topology results using the four loadcases are shown in Figure 16. Compared with the result with the incremental ESL loadcases only (example 4.4), a few different members are formed around the front wheel and in the rear portion due to the static bending and torsional loadcases.

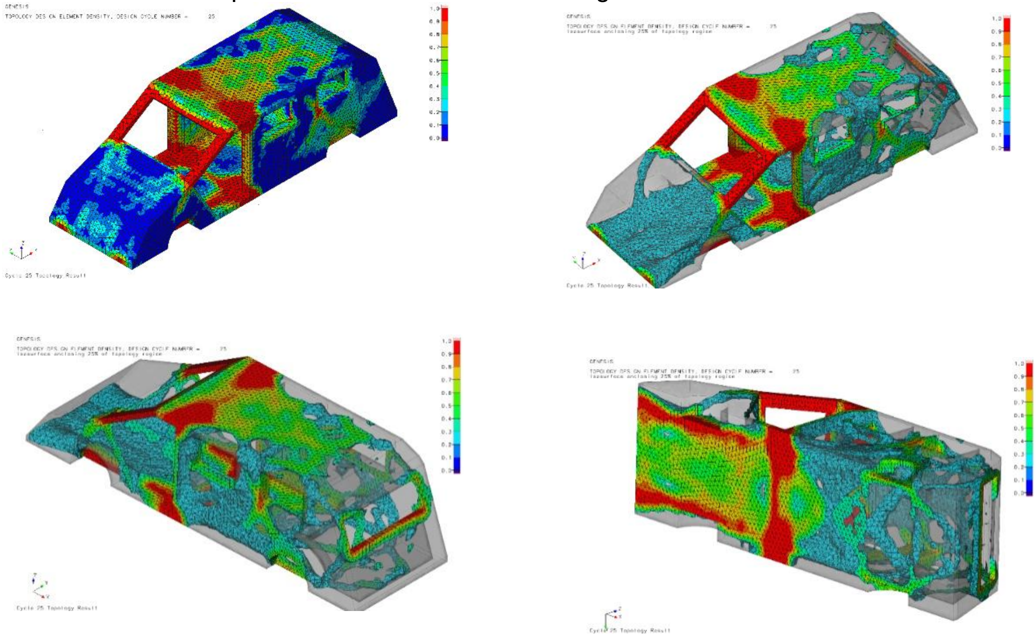

Fig.16: Topology results with the incremental ESL method for combined dynamic and static loading conditions. The top-left figure shows the density results. The other 3 figures show 3 different views of the isosurface results

To serve as a reference answer, the topology results when only the two static loadcases are used are shown in Figure 17.

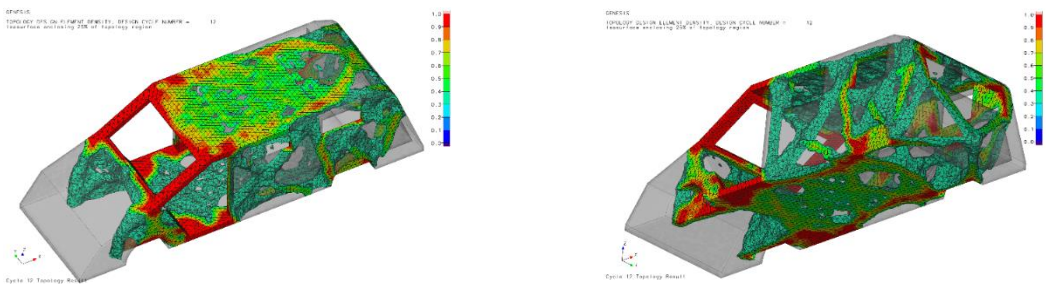

Fig.17: Two different views of the topology optimization result with static loading conditions only

5 Summary

This paper presents an improved Incremental Equivalent Static Load Method that enhances the accuracy of the original ESL method. With the Incremental ESL method, the stiffness matrix is updated at selected time steps using deformations from a base time step. This enables the computation of equivalent static loads based on incremental displacements for ESL load cases, resulting in a more precise capture of geometric and material nonlinearity.
To illustrate the effectiveness of the approach, several examples have been presented. Through these examples, the ability of the Incremental Equivalent Static method in improving structural optimization approximation quality under complex dynamic loading conditions is clearly showcased: The first example showed that the Incremental ESL method produced a lighter structure; in the second example, the incremental ESL method produced a structure that is stiffer; in the third example, the incremental ESL method showed clearer results when using one dynamic load case; in the fourth example, the Increment ESL method also produces clear answers, but in this case, using two dynamic load cases; while in the last example, the flexibility of the Increment ESL method demonstrated the ability to handle multiple dynamic and static load cases simultaneously.
The method discussed here shares similarities with another improved ESL method named the DiESL method, which also uses displacement steps to perform optimization. However, unlike the DiESL method that modifies FE meshes, the presented implementation is done inside the structural optimization program, GENESIS, and it does not require the use of multiple models, making it more practical and easier to use.
Overall, this paper demonstrates the significance and benefits of the Incremental Equivalent Static Load Method for solving practical structural optimization in nonlinear dynamic responses.

6 Literature

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