Towards the Solution of Cross-Talk in Explicit Isogeometric B-Rep Analysis

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1 Introduction

Driven by the increasing need for seamless integration between Computer-Aided Design (CAD) and Computer-Aided Engineering (CAE), Isogeometric Analysis (IGA) emerged with the groundbreaking research conducted by Hughes et al. [1] in 2005. Unlike standard Finite Element Analysis (FEA) that typically employs C^0-continuous Lagrange polynomials as basis functions, IGA utilizes smooth spline-based basis functions, the same as the ones used to describe the CAD geometries. The usage of consistent basis functions offers the potential to bridge the gap between CAD and CAE.

Industrial CAD models are mainly composed of multiple trimmed NURBS patches. Breitenberger et al. [2] proposed Isogeometric B-Rep Analysis (*IBRA*) which directly simulate on trimmed NURBS models. Leidinger et al. [3] extended this methodology and developed *Explicit IBRA*. By showing that arbitrarily small trimmed NURBS elements do not lead to vanishing time step sizes, their work essentially demonstrates the applicability of IGA to various explicit dynamic applications, e. g. vehicle crash simulations. With the availability of *Explicit IBRA*-similar features in LS-Dyna, IGA has gained increasing interest in industry over the past few years. Recently, hybrid IGA-FEA has been successfully applied to full vehicle crash simulations using an IGA body in white (BIW) [4].

Despite these recent advances, industrial applications of IGA yet face a significant challenge. Although higher-order spline-based basis functions are undoubtedly beneficial in terms of enabling higher regularity of the solution field and larger time step sizes, this comes at the cost of each basis function spanning over a larger support domain comparing to FEA. Therefore, a pair of spline basis functions may have overlapping supports even when the underlying control points are distinctly non-adjacent. Consequently, small-scale discontinuities within NURBS models, such as evolving discontinuities arising from progressive element deletions or stationary discontinuities (e. g. notches, holes in the initial geometry) represented by trimming curves, and their corresponding mechanical characteristics cannot be correctly captured with a comparatively coarse mesh. In such situations, the physically disjoint material regions across the discontinuity remain spuriously interconnected. This phenomenon is referred to as Cross-Talk. As an example, consider the seven one-dimensional (1D) basis functions and the trimming interval between ξ_1 and ξ_2 shown in Fig.1. The length of the trimming feature is hereby small in comparison to the element size. As a result, the basis function N_4, represented in red, is supported on both sides of the trimming interval. It therefore introduces an unphysical coupling across the trimming interval, such that the geometric discontinuity is mechanically not correctly captured.

The remainder of this paper is structured as follows. In Section 2, we elaborate on the complexities and challenges associated with modeling discontinuities in trimmed NURBS patches, and provide a detailed explanation of the Cross-Talk theory and the detection algorithm. Section 3 presents our novel control point duplication (CPD) scheme, which is proposed to mitigate or eliminate the effects of Cross-Talk. The feasibility and accuracy of the CPD method is demonstrated through a numerical example. Ultimately, we conclude the paper with a brief summary and an outlook on future research.

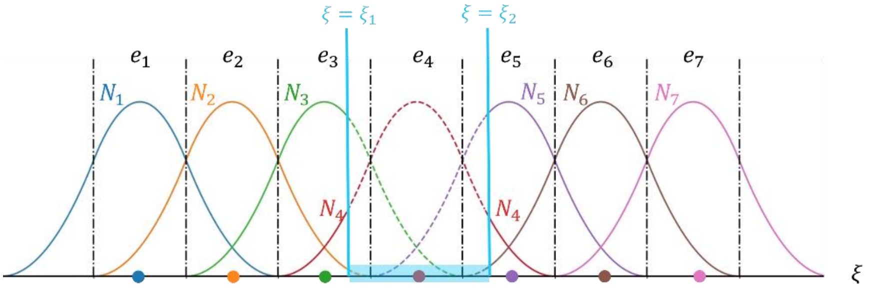


Fig.1: Illustrated are seven quadratic NURBS basis functions in one dimension. The domain $\xi \in [\xi_1, \xi_2]$, highlighted in cyan, is trimmed away.

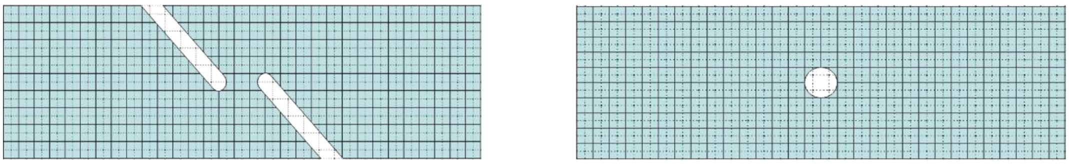
2 Discontinuities and Cross-Talk in Trimmed NURBS Patches

Contemporary CAD models are commonly represented using boundary representations (B-Rep) where an object is represented by its surface, or "skins", rather than its full volume. The B-Rep elements for a two-dimensional (2D) object consist of a set of edges, while those for a three-dimensional (3D) object consist of a set of faces. This is achieved through the design tool known as trimming, which enables designers to remove non-material regions from the underlying NURBS patch. The mathematical description of the underlying patch remains however unchanged.
Although IBRA offers the interoperability between design and analysis of trimmed NURBS patches, there are nonetheless a number of challenges associated with trimming, including multi-patch coupling [3,5,6], numerical integration of trimmed elements [7,8], and more. However, another important challenge has been so far largely overlooked -- Cross-Talk. As such, the mechanisms and detection algorithm for Cross-Talk will be discussed in detail in the following subsections, with a focus on 2D thin shell geometries.

2.1 An Overview of Discontinuity Modeling

The primary issue with modeling discontinuities in IGA arises from the fact that higher-order NURBS basis functions span over a larger support than linear Lagrange polynomials. Discontinuities can manifest in different forms and can be classified into three categories (1) stationary (initial) discontinuity, (2) evolving discontinuity caused by element erosion, and (3) evolving discontinuity in terms of a sharp crack interface.

The first type of discontinuity concerns small-scale geometric features described by trimming curves, such as a thin notch or an open hole within the initial shell geometry, see Fig. 2(a) and 2(b), respectively. While IGA employs an accurate geometry in the analysis, the mechanical behavior of these features may not be captured accurately. As reported in [9], trimming features that are "thin" compared to the neighboring NURBS knot spans might lead to unphysical coupling between the disjoint sides of the discontinuity, known as Cross-Talk. In their work, Cross-Talk is handled via local refinement using hierarchical B-splines.



(a) (b)

Fig.2: Type 1 discontinuity – small-scale geometric features represented via trimming curves. (a) A shear tensile specimen with two R2 notches; (b) A R4 open-hole tensile specimen. Both geometries are descretized using biquadratic NURBS with 4mm element size.

The occurrence of the second type of discontinuity is attributed to eroded elements. In conventional FEA for crash applications, material failure is typically addressed by deleting an element once it meets certain failure criteria. Engineers expect to follow the same approach for IGA models, such that the existing code framework for finite element erosion can be directly adopted. However, the deletion of a single NURBS element does not decouple its neighboring elements from each other due to Cross-Talk. An example of Type 2 discontinuity is demonstrated in Fig. 3.

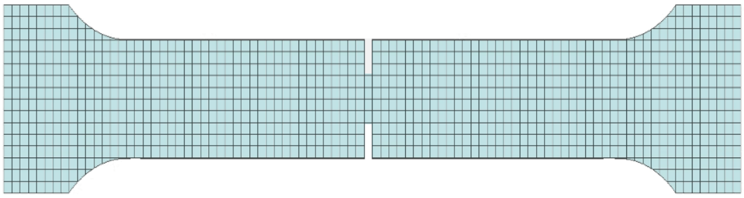
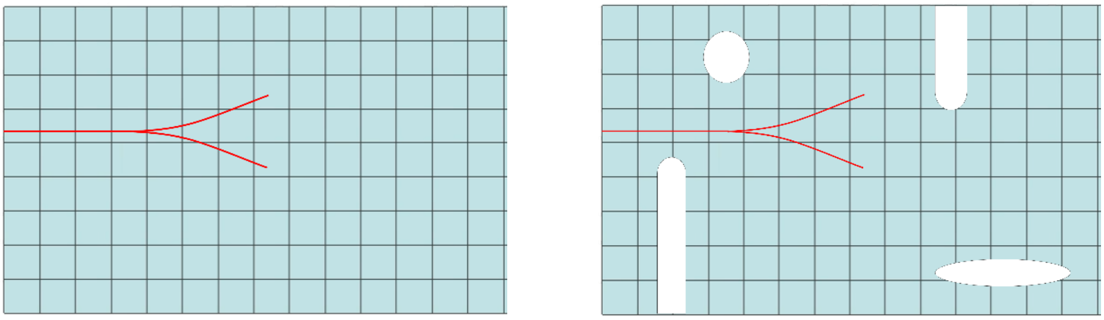


Fig.3: Type 2 discontinuity – element erosion. This figure demonstrates an example of double edge cracks in a tensile specimen, where three NURBS elements are eroded at the top and the bottom side. In this configuration, material on both sides of the vertical cracks will still remain mechanically interconnected to some extent.

In contrast to the first two types of discontinuity, which are specific to IGA, the third type is relevant to both IGA and FEA. This type of discontinuity results from representing crack surfaces as sharp interfaces, see Fig. 4(a) as an example. Modeling this type of discontinuity has always been challenging due to the fact that both FEA and IGA formulations are based on classical continuum mechanics. Significant effort has been devoted to modeling evolving discontinuities related to cracks and interfaces. In the context of IGA, techniques such as eXtended-IGA (XIGA) [10], IGA-meshfree method [11], and peridynamics [12] have been employed to model cracks. Modeling the third type of discontinuity requires not only the explicit representation of the discontinuity across the sharp interface, but also appropriate models for crack initiation and evolution. It is worth noting that both type 2 and type 3 belong to evolving discontinuities that pertain to fracture. Although type 3 offers a key advantage over type 2, as it ensures mass and energy conservation, the latter is still more commonly applied for large industrial models since it is significantly less expensive, particularly in crash.



(a) (b)

Fig.4: (a) Type 3 discontinuity – a branching crack with sharp crack interface; (b) An example with both type 1 and type 3 discontinuity.

Moreover, a trimmed NURBS shell can exhibit both initial discontinuity (type 1) and fracture discontinuity (type 2 or 3). An example where discontinuities of type 1 and 3 both exist is demonstrated in Fig. 4(b).

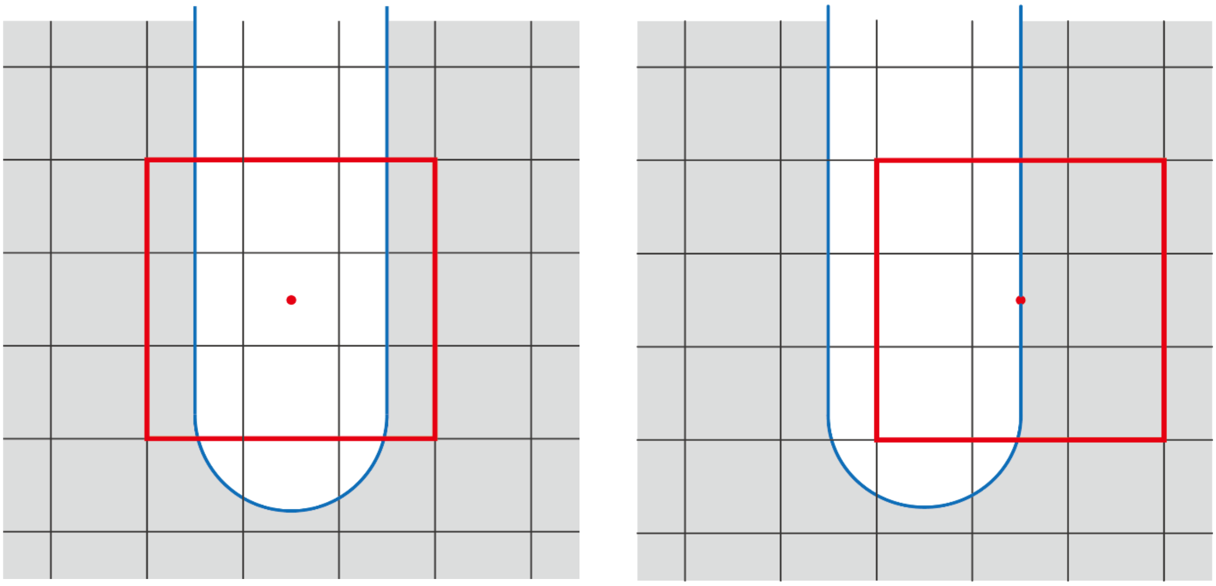
The focus of this paper is the treatment of Cross-Talk due to stationary discontinuities.

2.2 Definition and detection of Cross-Talk

Since cross-talk often appear as spurious force transmission through the free surfaces of the thin trimming features, it can be defined through the traction-free boundary condition in many cases. However, it is important to note that this definition may not adequately address the Cross-Talk situations when boundary conditions or loadings are directly applied to the trimming curve segment that define thin features. Furthermore, it should be regarded as a criterion for a-posteriori Cross-Talk identification, i.e. after the simulation results have been obtained. Due to the unknown nature of stress field and traction beforehand, it is not suitable for a-priori Cross-Talk detection. Therefore, we provide another definition of Cross-Talk that does not depend upon traction. In this definition, the parametric space of the trimmed NURBS patch is considered, where the set of NURBS basis functions is denoted as $N_{i,j}(\xi,\eta)$ and the active support domain of each basis function is represented by $supp(N_{i,j}(\xi,\eta))$. In the following, we first provide a preliminary definition of conflict control point (CCP).

Def. 1: conflict control point (CCP)
A CP equipped with the basis function $N_{i,j}(\xi,\eta)$ is said to be a CCP if $supp(N_{i,j}(\xi,\eta))$ is union of at least two disjoint subsets.

In Fig. 5, the parametric space of a biquadratic trimmed NURBS patch is displayed with highlighted support domains of two basis functions and their corresponding control point anchors in red. The basis function in Fig. 5(a) qualifies as a CCP based on Def. 1, since the trimming curve splits its support in two disjoint regions. Conversely, the basis function in Fig. 5(b) is not deemed a CCP.



(a) (b)

Fig.5: This figure illustrates the parametric space of a biquadratic NURBS patch. The trimming curve is depicted in blue and the active material region is shown in gray. Panel (a) highlights a CCP while panel (b) displays a non-CCP, along with their support domains.

Def. 2: Cross-Talk
Cross-Talk is present in a trimmed NURBS patch if CCPs exist, and the unphysical coupling takes place in the vicinity of the CCPs.

Hence, the Cross-Talk detection algorithm is designed to analyze the support domain of each control point near the discontinuity and determine whether the trimming curves split the support domain into multiple disjoint material regions. A demonstration of Cross-Talk detection is presented in Fig.6.

In one-dimension (1D), it can be easily proven that the existence of CCP is equivalent to the violation of traction-free boundary condition. However, in 2D, the CCP criterion serves as a necessary, but not sufficient condition for the absence of tractions. Discussions over the details exceed the scope of this paper. Nevertheless, Cross-Talk associated with CCPs is the primary cause of unphysical mechanical responses.

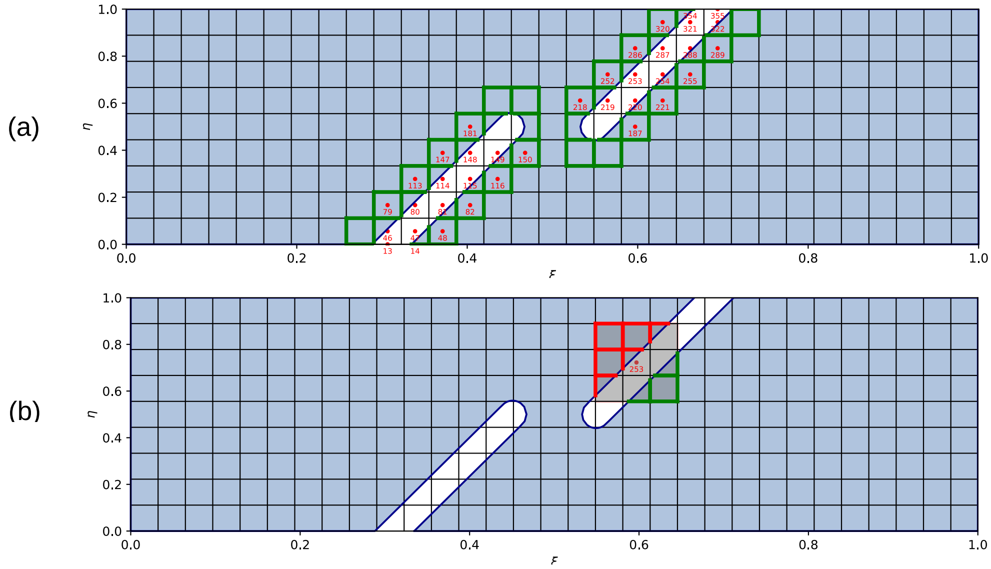


Fig.6: An illustration of Cross-Talk detection. Depicted in blue are the parametric space of a R2 shear tensile specimen, discretized with biquadratic NURBS (4mm element size). (a) The anchor of the detected CCPs are highlighted with red dots; (b) The support domain of the CCP with ID 253 is highlighted, which is clearly composed of two disjoint subsets.

3 Control Point Duplication

The presence of CCPs in trimmed NURBS patches is an indicator of partially excessively coarse discretization. More specifically, multiple disjoint material regions within a common support domain "compete" over one single CP. In light of this situation, we propose the control point duplication as a solution. This involves duplicating each CCP into n duplicates, where n corresponds to the number of disjoint regions present within the support domain of the CCP. Subsequently, each of the n disjoint regions is assigned one duplicate by updating the element connectivity. While this method has been implemented in LS-Dyna via a user-defined interface, it is not firmly integrated in the officially released solver versions at this time. Fig. 7 exemplifies the situations with $n = 2, 3$ and 4 on the support of a typical biquadratic NURBS basis function.

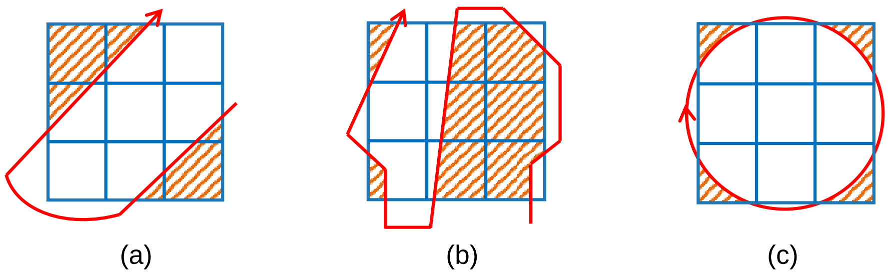


Fig.7: The typical support domain of a biquadratic NURBS consists of 3x3 NURBS elements. In the presence of small-scale trimming features, the support domain of the NURBS basis functions can be split into 2, 3, or 4 disjoint material regions, as depicted in (a), (b), and (c), respectively.

The CP duplication method essentially enriches the basis space locally, therefore counteracting the efffect of the coarse discretization and mitigating the effects of Cross-Talk.

As a numerical example, we consider the aforementioned R2 shear notch specimen. The specimen has a length of $122mm$, a width of $34mm$, and a thickness of $2mm$. We simulate the tensile test of this specimen in LS-Dyna, with four models: (I) a $1mm$ FEA reference model, (II) a $4mm$ FEA model (III) a $4mm$ IGA model, (III) a $4mm$ IGA model that applies CPD. Detailed information regarding the models are outlined in Table 1.

Time Integration	Material	Damage model	Boundary conditions (left/ right)	FEA Model	IGA Model
explicit	*MAT_024	DIEM	*BOUDNARY_SPC *BOUNDARY_PRESCRIBED_MOTION	• ELFORM = 3	• ELFORM = 2 • IRL = 0 • No stabilization

Table 1: An overview of the model details.

The detection algorithm shows the presence of 36 CCPs in model III, each of which has a support domain that contains two disjoint regions, i.e. $n = 2$, see Fig. 6. Consequently, in model IV, each of the 36 original CCPs are duplicated into 2 duplicates, resulting in additional 36 CPs compared to model III. Note that during the computation, the duplicated control points are treated as regular control points and interact with the simulation process accordingly. There is no distinction made between the original control points and the duplicated ones in LS-Dyna.

Fig. 8 illustrates the von Mises stress fields in the four different models shortly before failure. Both model I (the FEA reference model) and model II (the $4mm$ FEA model) show zero von Mises stress in the notch area depicted with arrows, suggesting the fulfillment of the traction-free boundary condition there. Model III, the conventional $4mm$ IGA model, exhibits a completely different behavior due to Cross-Talk. Firstly, spuriously high stresses are observed along the free notch boundaries, indicating unphysical force transmission across the notches. Secondly, both sharp corners are distorted towards the opposite side of the discontinuity. These two observations are both indications of non-zero traction, or Cross-Talk. As a result, both sides of the discontinuity remain interconnected to some extent behind the stage, leading to overstiffening the entire model.

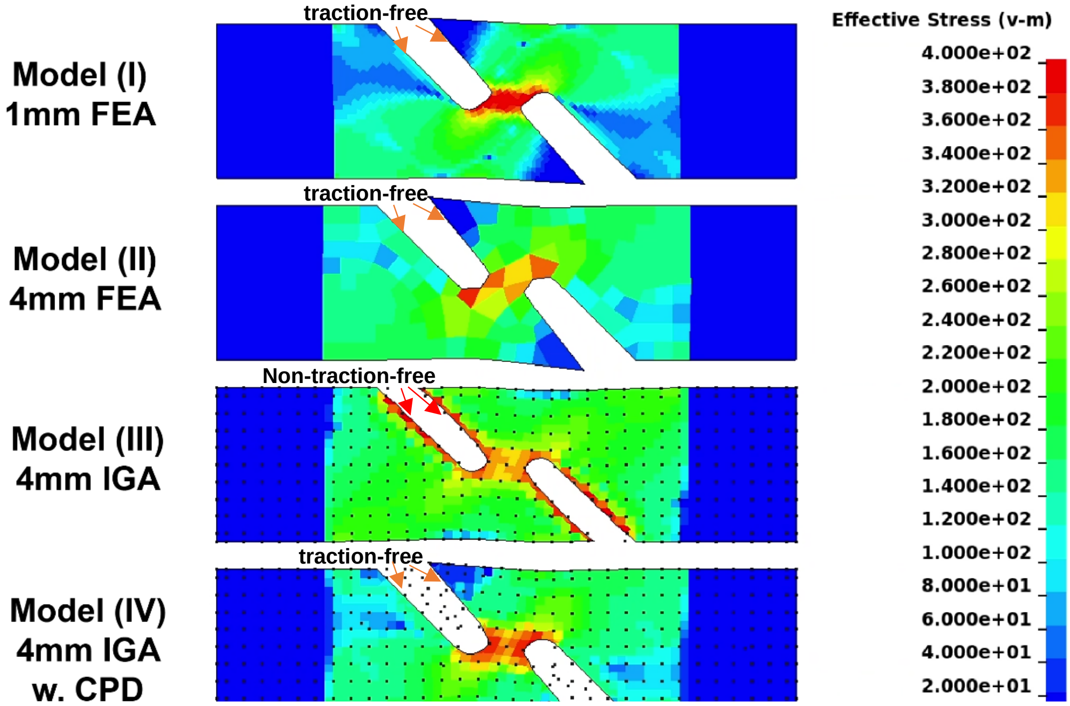


Fig.8: Von Mises Stress fields from model I, II, III and IV.

This is supported by the force-displacement diagram depicted in Fig. 9, where model III (red curve) exhibits an ultimate tensile strength (UTS) that is twice as high as in model I (black curve). The experimental results show that Model IV with a $4mm$ mesh size, which incorporates the CPD method, exhibits an excellent level of consistency with the $1mm$ reference model in terms of both the stress field and the global force-displacement curve. This validates the effectiveness of the CPD method in eliminating major Cross-Talk and ensuring accurate mechanical responses in trimmed NURBS patches. Moreover, our investigations indicate that the CPD method has a negligible effect on the critical time step size, thereby demonstrating its efficiency and practicality as a solution to mitigate the effects of Cross-Talk.

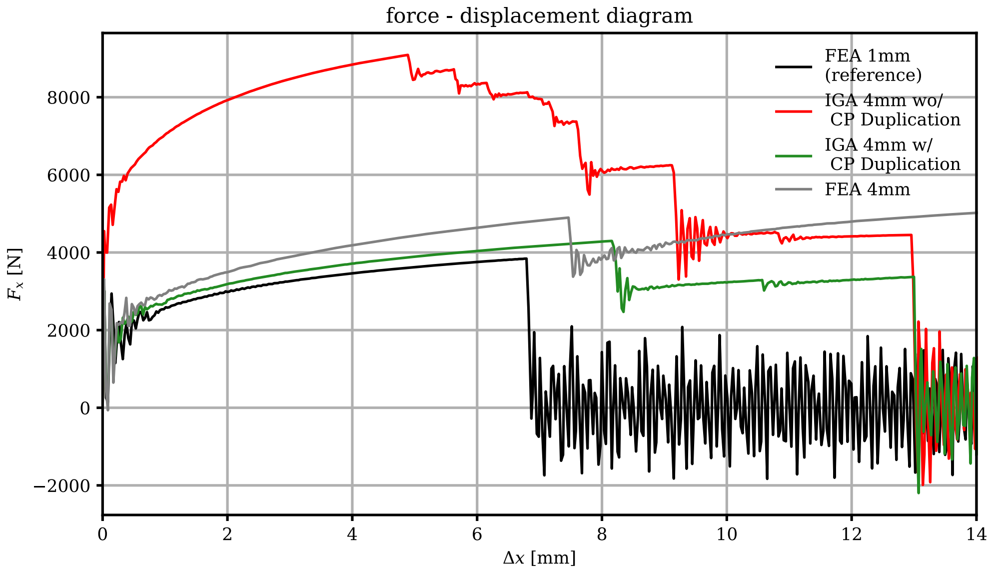


Fig.9: The resultant x-forces measured at the SPC boundaries (LHS) plotted over x-displacement of the right boundary.

4 Conclusion and Outlook

In conclusion, Cross-Talk is a significant challenge in industrial applications of IGA, particularly when modeling discontinuities in trimmed NURBS patches. This paper elucidates the theory behind Cross-Talk and proposes a CPD scheme to mitigate or eliminate the effects of Cross-Talk. The effectiveness of the method is demonstrated through a numerical example of a R2 shear notch specimen. Results show that the CPD method successfully eliminates major Cross-Talk and ensures reliable simulation results using trimmed NURBS patches. Notably, the extra computational overhead associated with the CPD method is minimal when compared to the local refinement approach, and is therefore of great relevance for industrial applications, where excessively fine mesh is in general unaffordable.

We are currently establishing the groundworks of extending the CPD method to the treatment of type 2 discontinuities, i.e. evolving discontinuities arising from element erosions. This type of discontinuity is fundamentally similar to type 1, since deleting a NURBS element is akin to trimming out the element using an inner trimming loop along its boundary. The key distinction between the two types is that the stationary discontinuity can be potentially resolved offline, i.e., prior to commencing the simulation, whereas type 2 requires repeated treatments at each time step when element deletion occurs.

5 Literature

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