

A Meta-model based Approach to Implement Variation Simulation for Sheet Metal Parts using Mesh Morphing Method

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1 Abstract

The virtual process chain is an essential step for the sustainable digital transformation in the manufacturing industry. For the Body-In-White (BIW) sheet metal parts, the manufacturing joining simulation based on finite element method is used to simulate the joining processes in the body shop. The target is to predict the dimensional accuracy of assemblies after using different types of joining technologies. However, the assembly deviation is not only affected by the joining operations, but also by the initial part deviations. Therefore, an integration of geometrical variations in the joining simulation model is necessary to improve the prediction accuracy. The statistical analysis for the geometrical variations also enhances the applicability of the joining simulation, for example in the tolerance analysis. In this paper, a meta-model based approach is developed to implement the variation simulation for the sheet metal parts. The geometrical variations of parts are governed by the tolerance specification and modelled through a mesh morphing method. The nominal FE mesh of the toleranced surface is morphed by using scattered data interpolation followed by direct stiffness method. Joining simulations are implemented with the morphed meshes to build up the meta-model. Afterwards, Monte Carlo (MC) simulations are applied for an efficient statistical analysis. The proposed approach is illustrated along with a prototype part in the body in white. LS-Opt is used to build up the simulation model and LS-Dyna is used to implement the joining simulations. The simulation results are compared with the tolerance simulation results as well as the measurement data.

The proposed simulation model integrates the geometrical variations in the joining simulation model. It enables a statistical tolerance analysis considering the influence of manufacturing process in an early development stage. The sensitivity analysis helps the user to identify the importance of part tolerances, which contribute to the optimization of the product and manufacturing process.

Keywords: Manufacturing joining simulation, variation modelling, meta-model, tolerance analysis

2 Introduction

In the digitalized transformation of the manufacturing industry, variation simulation raises its importance because the variations are unavoidable and they can affect the product quality and increase the cost of production or even lead to rework [1, 2]. For the automotive parts, three types of variations are identified [3]: the part variation, the fixtures variation and the tools variation. For different types of variations, different methods are developed to analyze the influence and ensure the product quality. This paper focuses on the manufacturing process simulation considering the geometrical variations of parts.

Usually, MC based tolerance simulations are implemented to calculate the deviation propagation [4, 5]. Variation analysis based on rigid body assumption for sheet metal parts leads to over-estimation of the assembly deviation. By combining with the linear Finite Element Method (FEM), the method such as the Method of Influence Coefficient (MIC) [6] is developed for the non-rigid parts [7]. Commercial Computer Aided Tolerancing (CAT) tools are used for the tolerance analysis in manufacturing industries [8, 9]. Nevertheless, these tolerance simulation methods have difficulty in simulating the non-linear effects that occurs in the manufacturing processes. Since the manufacturing joining simulation based on non-linear FEM is proved to simulate the assembly deformation of sheet metal assembly accurately [10], a combination of manufacturing joining simulation and tolerance analysis shows its potential in improving the prediction accuracy of the tolerance simulation.

In the digital process chain of product, the manufacturing process simulation based on FEM is implemented to predict the assembly deformation after the manufacturing processes [11]. The simulation model is created based on the nominal mesh that is generated from the nominal CAD model. Therefore, the discrete geometry representation is applied in the simulation model. To simulate the assembly deviation, the variation modelling method is divided into two types: the mesh morphing

methods and the decomposition approaches [12]. The decomposition approaches such as the discrete modal decomposition [13], the Principle Component Analysis [14] are developed to express the form deviations by decomposing the shape error from the measurement data or from the forming simulations. On the other hand, mesh morphing methods integrates the geometrical variations by deviating the control points or directly the surfaces meshes. For example, a third degree polynomial function is used to morph the mesh with and without measurement data for a tolerance analysis [15]. However, the computational intensity limits the application of manufacturing joining simulation in the tolerance analysis, since the statistical analysis requires large amount of samples.

The sampling based tolerance analysis, or so called the meta-model based tolerance analysis shows its potential in the tolerance simulation as well as tolerance optimization [16]. A meta-model is created using the DoE (Design of Experiments) method, which predicts the correlation between input variables and the responses [17]. Advanced sampling methods, such as Latin Hypercube Sampling (LHS), is used to reduce the necessary amount of the FEM simulations [18]. Afterwards, a meta-model based MC simulation can be implemented with little computational effort [19].

Previously, a meta-model based tolerance simulation approach is applied for a reinforcement assembly [17, 20]. Comparing to the rigid and linear FEM based tolerance simulation, the meta-model based tolerance simulation results are closer to the real measurement data. However, the simplifications in the variation simulation model still affects the simulation reliability. The surface profile tolerances of parts are simulated through the linear function and the areas out of the toleranced surfaces are assumed to be nominal. In this paper, the mesh morphing procedure is further improved through natural interpolation and direct stiffness calculation. The new mesh morphing procedure is integrated in the tolerance simulation process through the optimization tool Ls-Opt.

The proposed simulation model integrates the geometrical variations in the joining simulation model. It enables a statistical tolerance analysis considering the influence of manufacturing process in an early development stage. The sensitivity analysis helps the user to identify the importance of part tolerances, which contributes to the optimization of the product and manufacturing process. This tolerance simulation approach is illustrated with an automotive reinforcement part. As a result, the assembly deviation at four Key Product Characteristics (KPCs) are calculated. A sensitivity analysis for the part tolerances is obtained.

The paper is structured as following: the mesh morphing method based on the natural interpolation and the direct stiffness calculation is proposed in section 3. A meta-model based tolerance simulation model is created with Ls-Opt in section 4. The simulation results are evaluated by comparing with the measurement data as well as the simulation results from previous methods. Section 5 discusses the proposed mesh morphing methods. Conclusions and outlook are summarized in section 6.

3 Generate tolerance affected parts using mesh morphing method

The modelling process of geometrical variation is an essential part of the joining simulation for tolerance analysis. This section of the paper discusses about the generation of tolerance affected parts. The most frequently occurring tolerance for BIW sheet metal parts is the surface profile tolerance and the reason for this lies in the fact that virtually all surfaces in the vehicle construction are free-form surfaces [21]. Thus, this research work focusses on the surface profile tolerance modelling.

For the generation of parts with surface profile tolerance, a FE mesh is derived from the nominal CAD geometry and thereafter the mesh nodes are moved by applying a morphing mesh procedure. The mesh morphing procedure refers to the perturbation of the nominal mesh according to deviations occurring at few control points [15]. To apply the free-form deformation on the FE mesh, the mesh morphing procedure in Fig.1 is suggested in this paper, which includes three main steps: feature extraction, tolerance modelling via scattered data interpolation followed by direct stiffness method.

A part can have many features and each feature can have different tolerances as per their functionality, therefore at first the feature for which the tolerance needs to be modelled is extracted by extracting the nodes comprising that feature in the FE model. This step is called feature extraction and it allows the method to model tolerances for different features independently.

The next step is the tolerance modelling for the extracted feature and for that, scattered data interpolation is used. Scattered data comprises of a set of points X and corresponding values V , where

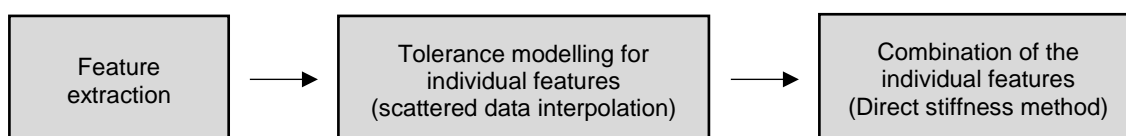


Fig. 1: Overall mesh morphing procedure for sheet metal part with surface profile tolerances

the points have no structure or order between their relative locations. For the interpolation of such scattered data, *scatteredInterpolant* class of MATLAB has been used, which uses a Delaunay triangulation of the scattered sample points to perform interpolation. An interpolating surface is constructed by triangulating the points and lifting the vertices by a magnitude V into a dimension orthogonal to X , as shown in Fig. 2(b). Finally, in order to interpolate the value at a query point X_q , the triangulation data structure is traversed to find the triangle that encloses the query point, as shown in Fig. 2(b). Once the point is found, the subsequent steps to compute the value depend on the interpolation method. In this work, *natural* interpolation method has been used since it provides C^1 continuity at the query points.

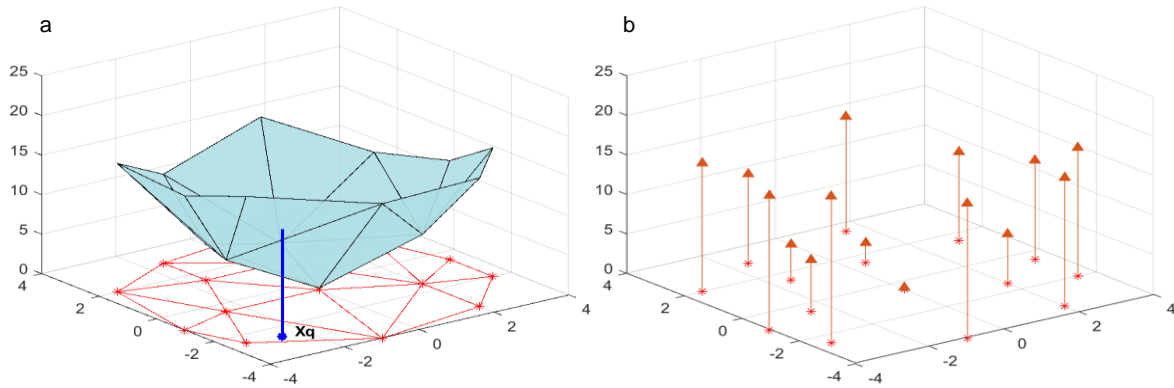


Fig.2: (a) Scattered data set and corresponding values; (b) Delaunay triangulation and query point

In order to utilize the concept of scattered data interpolation for the mesh morphing, few control points on the surface of the extracted feature are selected and defined as the control points (X) for the creation of interpolant object of *scatteredInterpolant* class. Since we are using the FE model, these control points are specific nodes on the feature and they are chosen as shown in Table 1. To ensure that the query nodes lie inside the convex hull of control nodes in order to avoid the extrapolation possibility, the feature nodes that form the convex hull for the entire feature are chosen as the first set of the control nodes. This set is assigned a random value within the tolerance range. The second set of control nodes is derived from the information about measurement points on the feature. The nodes of the FE model resembling the measurement points on the CAD geometry are chosen and assigned a value equal to the deviation value that needs to be modelled. Finally, the third set of control nodes is derived from the available datum information. The nodes corresponding to the datum targets are selected and given a zero-deviation value.

Once all the above specified nodes are selected and assigned the values, they are utilized as control

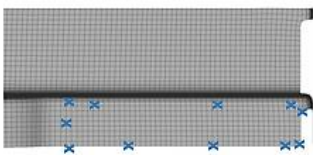
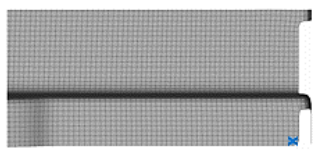

Control points	Control points in the FE model	Assigned values
Convex hull nodes		Random values within the tolerance range
Measurement nodes		Deviation value
Datum nodes		Zero

Table 1: Selection of control points and values assignment for scattered data interpolation

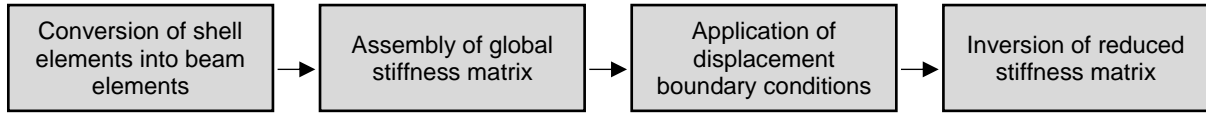


Fig.3: Implementation of the direct stiffness method

points for the creation of the interpolant. The interpolant function is then used to interpolate the values of all the remaining feature nodes and finally, the interpolated values are applied at the feature nodes in the normal direction as the surface profile tolerance is always defined in the normal direction. Hence, by using the approach of scattered data interpolation, the surface profile tolerance can be effectively modelled for the individual extracted features using few control points (mesh morphing procedure).

The last step is combining the individual extracted features again into the complete model. As mentioned above, a part can have many features and these features often share boundaries with each other, thus in order to maintain mesh connectivity, it is mandatory that the shared boundary nodes have the same position once the surface deviations for the features have been modelled independently. In addition to this, mesh smoothness also needs to be ensured in order to avoid sharp deviations. As stated above, *natural* interpolation method ensures C^1 continuity only at the query points but not at the control points and therefore, the deviations at the control points need to be smoothed. However, the deviations at the measurement nodes need to be retained since these deviations are explicit deviations and their values must be same to the deviation values that need to be modelled. In addition to it, the deviations at the datum nodes also need to be retained at zero, since during real part measurement, the datum points are always aligned onto their nominal position and hence have zero deviations. To achieve this, deviations at all the boundary nodes, which are nodes at the boundary of the feature and at all the convex hull nodes are forced to zero and these deviations are re-calculated through direct stiffness method.

Direct stiffness method uses the interpolated values from the scattered data interpolation as the displacement boundary conditions and the unknown deviation values at the remaining FE-model nodes are calculated by inversion of the reduced stiffness matrix for the complete model [22]. Fig. 3 shows the step-by-step implementation of the direct stiffness method. The FE model consists of shell elements and since they are computationally expensive, each shell element in the model is converted into maximum possible combination of beam elements e.g. QUAD shell element into six beam elements and TRI shell element into three beam elements. In the next step, the beam element stiffness matrices are assembled for the entire model to create global stiffness matrix. Thereafter the interpolated values at the feature nodes are applied as displacement boundary conditions and the global stiffness matrix is the

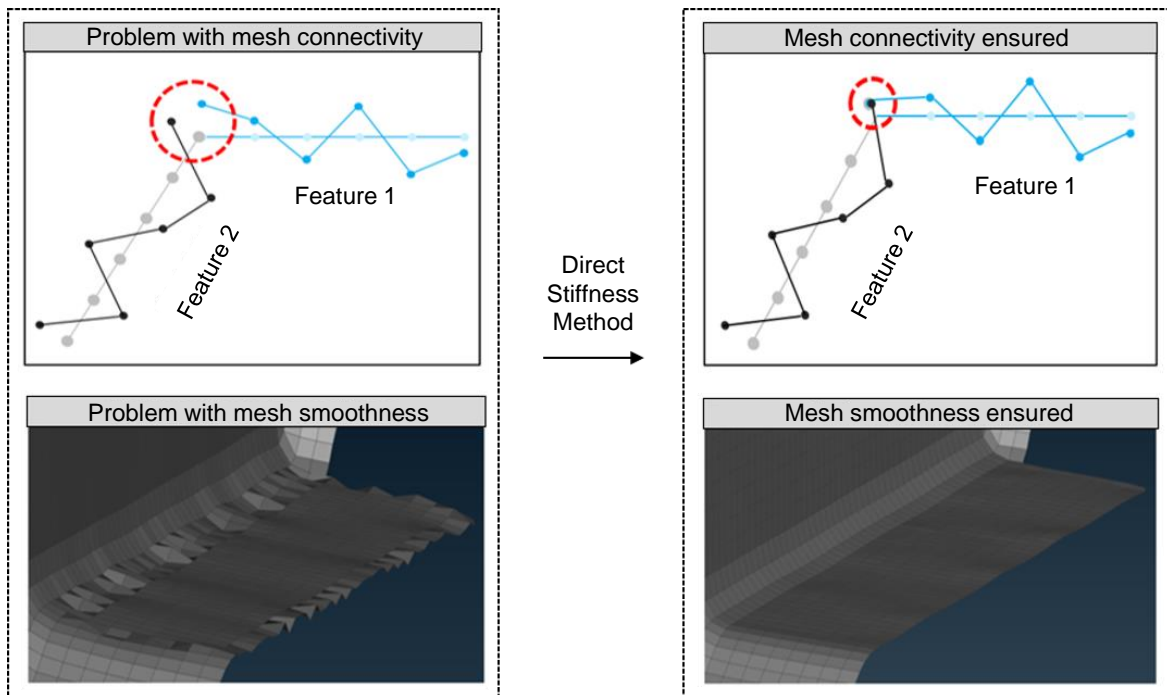


Fig.4: Direct stiffness method ensures the mesh connectivity and mesh smoothness

feature nodes are applied as displacement boundary conditions and the global stiffness matrix is reduced accordingly. The reduced stiffness matrix is inverted to obtain the unknown deviation values. Finally, these calculated nodal deviations are applied at the corresponding nodes to obtain the final morphed geometry. Fig. 4 shows the issues of mesh connectivity and mesh smoothness along with the problem rectification by the use of direct stiffness method. The direct stiffness method prevents the shared boundary node between feature 1 (blue) and feature 2 (black) from losing the connectivity due to the modelling of the deviations in the normal direction. In addition, the re-calculation of the deviations at the control points using the direct stiffness method ensures mesh smoothness. In the next section, the introduced mesh morphing procedure will be integrated as a simulation step for the tolerance analysis of an automotive sheet metal assembly.

4 Meta-model based tolerance simulation for sheet metal parts

With the developed mesh morphing procedure, a meta-model based tolerance simulation model can be built up. In the following sections, an automotive reinforcement assembly is simulated as an illustration.

4.1 Use-case description

In the production line, the reinforcement part R shown in Fig. 5 is joined on the underbody structure of the car. To avoid large simulation scale, a joining partner P is designed for the experiment. Both parts are made of steel CR340LA with the thickness of 0.89 mm. As an important joining technology for sheet metal parts in the BIW manufacturing process, the clinching process is used to join the reinforcement part and the joining partner.

The initial tolerance specification is provided in Fig. 5 based on the engineering experience. The datum system and the geometrical tolerances are indicated based on the company standard MBN [23], which is correlated with the international standard ISO [24]. A joining station with four clamps is designed for the clinching process. The parts are joined together at the clinching position 1 – 12. To evaluate the assembly dimensional quality, four measurement points MP1 – MP4 are selected as the KPCs on both flanges.

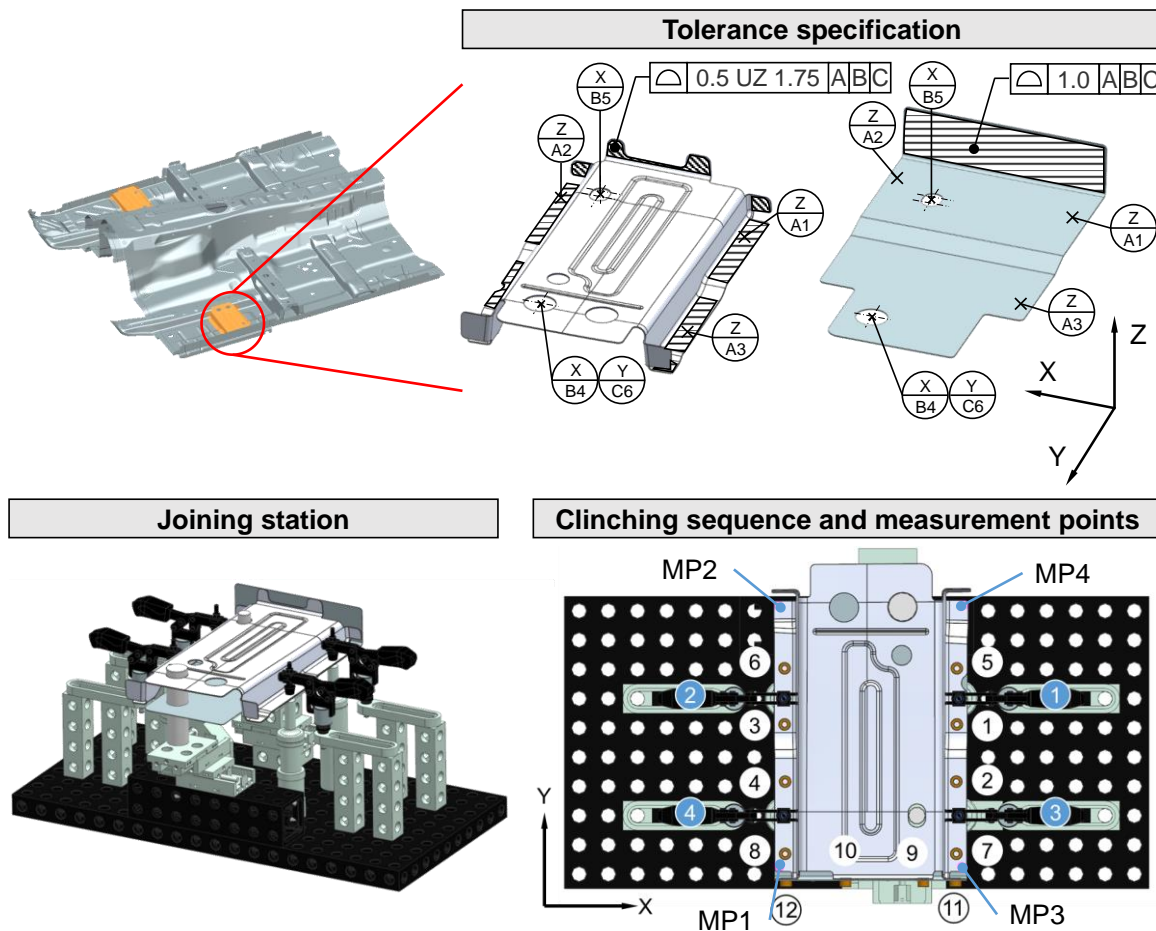


Fig.5: Product manufacturing information for the reinforcement assembly

4.2 Simulation model in Ls-Opt

The manufacturing clinching process is simulated based on the concept of Position, Clamp, Fasten and Release (PCFR) [25] with the Ls-Dyna. To implement the tolerance analysis considering the joining process of two parts, a meta-model based tolerance simulation model is created in Ls-Opt Version 7.0.0.

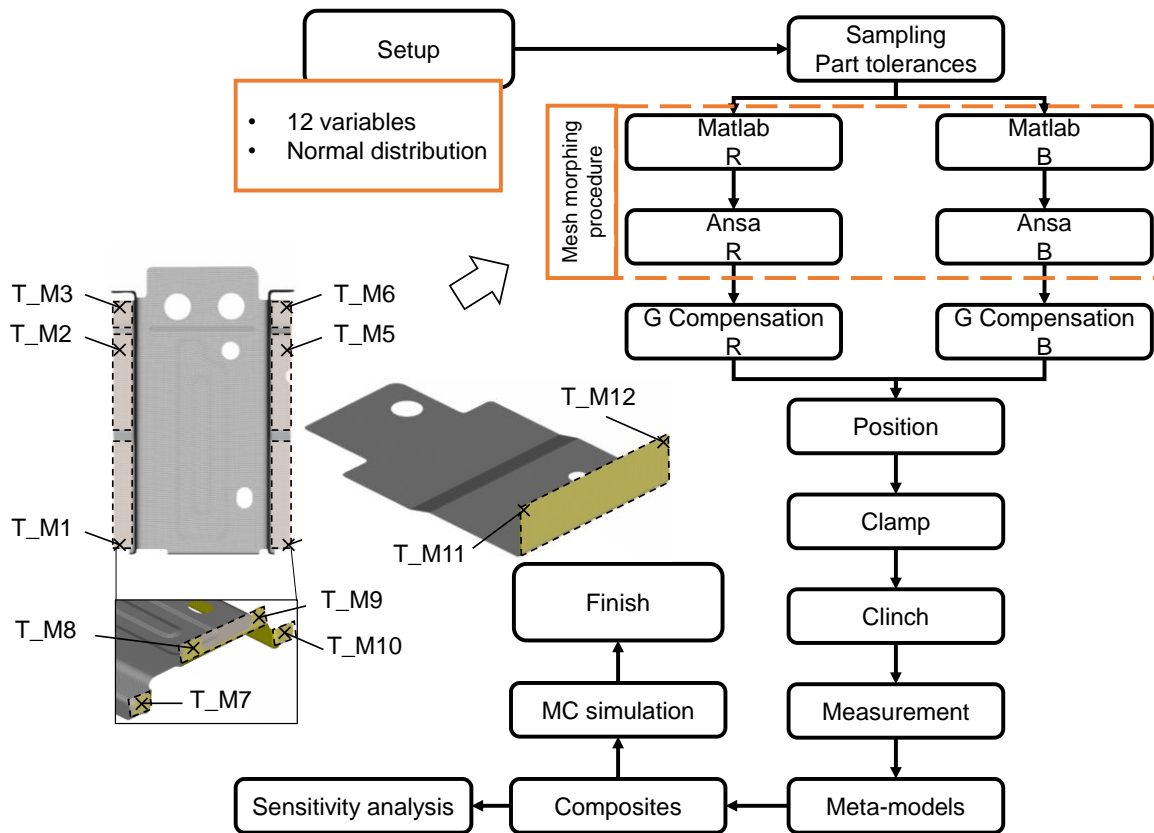


Fig.6: Meta-model based tolerance simulation model for reinforcement part in Ls-Opt

According to the mesh morphing procedure in last section, 12 variables are defined for all tolerated surfaces in the system. In this research, both linear and quadratic types of meta-model will be evaluated. Therefore, the minimum required amount of experiments is 137 according to Equation (1). Considering the computational capacity, 200 experiments are implemented for this case study. The space filling sampling approach based on LHS is used to sample the variables.

$$\text{Quadratic meta-model: } \text{int} (0.75(n + 1)(n + 2)) + 1 \quad (1)$$

The deviation ranges of variables are determined by the tolerance specification. To build up the response surfaces, the design space needs to be covered as complete as possible. Therefore, the uniform distribution type is applied to all variables instead of the defined normal distribution. The normal distribution type will be applied later in the meta-model based simulation. After the sampling stage, 200 sets of deviation values are generated for all variables. With the 200 morphed meshes, 200 joining simulations are implemented by using the FE solver Ls-Dyna. Ls-Opt calls the Ls-Dyna input deck in batch mode. The displacement at every KPC after every operation is recorded. Finally, the assembly deviation is calculated by the summation of all recorded displacements.

As soon as all joining simulations are finished, the 200 sets of variables and the corresponding responses are used to train the meta-models. First, a proper type of meta-model needs to be determined. As a general criterion to evaluate the adequacy, the maximal Root Mean Square Error (RMSE) and the Coefficient of Determination (CoD) is compared between three types of meta-models are compared in Table 2. Comparing to the other meta-model types, the Feedforward Neural Network (FFNN) have the minimal RMS error, which implies generally is has a higher predict accuracy for this system. Therefore, the FFNN meta-model is used to present the correlation between part tolerances and assembly deviations for the further MC simulation.

To analyze the deviation propagation after manufacturing joining process considering the distribution of part tolerances, MC simulations are implemented in this step. The normal distribution types are applied

Type	Max RMSE	Min CoD
Linear polynomial	0.126	0.463
Quadratic polynomial	0.082	0.640
RBF	0.126	0.463
FFNN	0.018	0.912

Table 2: Adequacy check for different meta-models through RMS error and CoD

to all variables. All the variables are sampled 1500 times according to their deviation range and normal distribution type. With the FFNN meta-model, the statistical results for the assembly deviations are calculated within one minute.

4.3 Evaluation of simulation results

Comparing with the rigid and non-rigid tolerance simulation, the meta-model based tolerance simulation implies a smaller assembly deviation after the joining process, which is consistent as the real measurements. The deviation ranges at all KPCs are also verified through the joining simulation based tolerance simulation. The mean values at MP1 and MP3 have a larger difference to the measurement, because these two positions are close to clinching surfaces that have unequal disposed profile tolerances. It leads to large gaps between two clinching partners and the contact calculation at this area are affected. Nevertheless, the difference of the mean shift of the meta-model based tolerance simulation are closer to the measurement results.

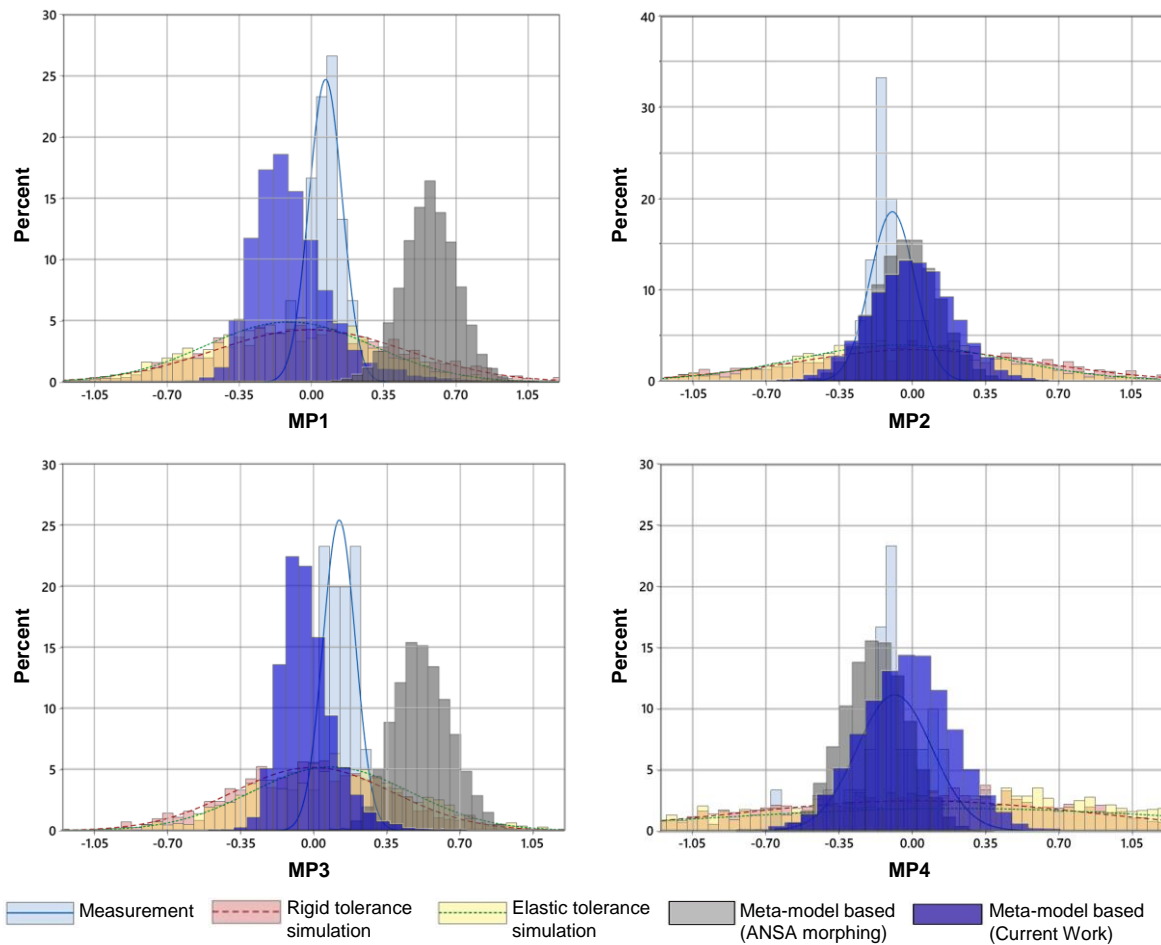


Fig.7: Comparison of simulation results with measurements and previous simulation results

Generally, the prediction accuracy of meta-model based tolerance simulation in the early digital concept phase (without measurement data) is improved comparing to the rigid and elastic tolerance simulations. Moreover, the simulation results based on the mesh morphing procedure proposed in this paper has further improved the simulation results.

The qualitative results in Fig. 7 clearly show that the meta-model based tolerance simulation is efficient than the conventional tolerance simulation techniques (rigid and elastic) since the former results into smaller assembly deviation. For quantitative comparison (Table 3 and Table 4), it can be seen that the standard deviation value for the assembly deviation at FKC M1 ranges from 0.406-0.466 for the rigid and elastic tolerance simulation while for the meta-model based tolerance simulation, the standard deviation is much smaller and ranges from 0.13 to 0.172. Thus, the standard deviation value obtained from the meta-model based tolerance simulation is closer to the real measurement data with 0.08 value.

Response	Measurement	Rigid	Elastic	Meta-model based (ANSA morphing)	Meta-model based (Developed method)
MP1	0.070	-0.0037	-0.107	0.57	-0.023
MP2	-0.010	0.0183	-0.055	-0.03	-0.005
MP3	0.122	-0.0035	0.084	0.52	-0.022
MP4	-0.084	0.0166	0.21	-0.17	-0.054

Table 3: Simulation and measurement results for mean value at KPCs

Furthermore, the meta-model based tolerance simulation in the current work shows a closer mean value to the measurement results than the manual ANSA morphing method. However, the standard deviation of assembly deviations obtained in the current work is higher than the manual ANSA morphing method and this is due to the implicit nature of the manual method and the simplicity of the modelled geometric variations. The developed variation modelling method in this work covers the entire design space due to its explicit nature and efficiency.

Response	Measurement	Rigid	Elastic	Meta-model based (ANSA morphing)	Meta-model based (Developed method)
MP1	0.080	0.466	0.406	0.13	0.172
MP2	0.107	0.578	0.504	0.15	0.191
MP3	0.078	0.385	0.382	0.13	0.120
MP4	0.180	0.820	1.107	0.15	0.206

Table 4: Simulation and measurement results for standard deviation at KPCs

However, the quantitative comparison of the measurement and the current work results shows slight differences in statistical terms. This can be accounted to the non-availability of large amount of the measurement data. Only 30 samples of measurement data are available, while for the current work 1500 MC samples have been used.

5 Discussion of the variation modelling method

This section of the paper focusses about the highlights and limitations of the developed method. During the development and implementation of the proposed method which have been summarized in Table 5 based on two criteria i.e. the application of the method and the implementation of the method.

Highlights and limitations of the developed geometric variation modelling method

Focusing on the *application of the method* criterion, the geometric variation modelling method developed in this work has explicit nature, which means that the exact deviation value for a feature can be modelled at the measurement point for that feature. Such explicit nature allows the modelled tolerances to cover the entire design space and the effect of individual part tolerances on the assembly deviations can be effectively analyzed. In addition to this, the modelling has been carried out by varying few selected control points, which facilitates the method to be used in the prediction stage, where only information about tolerance range is available. The chosen control points can be moved randomly within their

Criteria	Highlights	Limitations
Application of the method	<ul style="list-style-type: none"> a. Explicit modelling of the profile tolerance in the prediction stage. b. Modelling of complex free-form deviations. c. Independent tolerance modelling for the features sharing boundaries. 	<ul style="list-style-type: none"> a. Forming history not considered. b. Real deformation is more complex than the interpolated pattern.
Implementation of the method	<ul style="list-style-type: none"> a. Method automation through batch call. 	<ul style="list-style-type: none"> a. Manual inputs e.g. data about feature nodes, measurement points and datum points required. b. Data loss or data mishandling may result due to multiple software platforms.

Table 5: Highlights and limitations of the developed geometric variation modelling method

tolerance range, thus resulting in different morphed models. Furthermore, the scattered data interpolation has been utilized to perform mesh morphing, which results in complex free-form deviations, instead of simplified linear or angular deformations. Finally, the inclusion of the direct stiffness method as the last modelling step facilitates the method to treat each feature, even the features sharing boundaries, independently. However, the forming history e.g. thickness variation of sheet metal parts, internal stress state due to forming process, etc. is not considered in this modelling method. Also, the deformations have been modelled on the basis of tolerance data and deviation value, which are not identical to the real deformations.

Considering the *method of implementation* criterion, the method is implemented via MATLAB and Python scripts, which automates the entire process of variation modelling. LS-Opt has been used to batch call these scripts to model the tolerances, followed by batch call to LS-Dyna to carry out the joining simulations. This automation facilitates the direct implementation of the method in the statistical tolerance simulation. However, the method still requires few manual inputs e.g. data about feature nodes, measurement points and datum points. Furthermore, since various software are involved in the method implementation, the data transfer between various software platforms may cause data loss or data mishandling, if not implemented in a cautious way.

6 Conclusion and outlook

In this paper, a mesh morphing method is proposed to simulate the geometrical variations of the sheet metal parts. It contributes to the meta-model based tolerance simulation. Free-form deformation is integrated to the parts in an early development stage according to the tolerance specification.

Using the proposed geometrical variation modelling method, a meta-model based tolerance simulation is implemented for a sheet metal assembly. Comparing to the traditional tolerance simulation for rigid parts and non-rigid parts, the simulation results in this paper are more consistent with the hardware measurement data.

In this research, the toleranced features still need to be defined and selected manually. In the future research work, an interface can be constructed between the mesh morphing process and the CAD database, which will enable an automated retrieval of the tolerance data. Furthermore, the sensitivity analysis also provides the optimization potential for the part tolerances, which can be used for the tolerance optimization.

7 Literature

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